STANDARD SERVICE ARITHMETICS

GRADE SIX

KNIGHT
STUDEBAKER
RUCH

Standard Mathematical Service

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Edited by George W. Myers

Standard Service Arithmetics

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PREFACE

The *Standard Mathematical Service* is an organized enterprise, the first of its kind, undertaking to provide adequate tools for assisting the mathematics teacher in his fourfold professional task: "teach-practice-measure-remedy." The *Standard Service Series of Arithmetics*, of which the present volume is for the sixth grade, is a main unit of this Service.

The *Standard Service Arithmetics* are a consistent attempt to apply the known facts about child-learning to arithmetic teaching and learning. These facts have been drawn from the classroom experience of hundreds of teachers and from careful experimentation, carried out over a period of several years. The material and organization of all books of the Series have been tried out and digested into final form after classroom use by pupils from all parts of the country.

This text for the sixth grade employs consistently the following ten features:

(1) The replacement of the customary interest-killing review of the work of preceding grades by carefully built diagnostic inventory tests of essentials and ideas (see pp. 1-24), with an appendix of reteaching units (Self-Help Study and Practice Lessons, pp. 359-439). By these provisions for diagnosis and remedial work each pupil may restudy and practice only and precisely what he needs, and no pupil is forced through what he already knows well enough. This is a recognition of individual differences that many teachers will prize.

(2) Gradual, detailed development of new topics, together with close regard to the vocabulary, illustrations, and explanations employed.

(3) Scrupulously calculated distribution of drill and review (*a*) to insure proper amounts of periodic practice on the basic number facts and processes, and (*b*) to prevent the forgetting of what has been previously learned. "Bunched" drills for **skill-building** and distributed drills for **skill-holding** have both received expert attention.

(4) The systematic use of motivating devices such as: (*a*) legitimate appeal to the spirit of competition with the pupil's own past records and with the standards achieved in tests by thousands of other sixth-grade children; (*b*) persistent use of the principle of "awareness of success or failure at the time of learning" through the Self-Testing Drills, Problem Scales, and other self-measuring exercises; (*c*) prevention of initial discouragement through easy mastery, attained by a leisurely, step-by-step unfolding of topics; and (*d*) variety of presentation of drill materials (see Articles 35, 91, 100, 102, 108, 114, and 148) and variety in the types of problem provided (see Articles 24, 61, 75, 93, 145, 168, 204, 205, and 206).

(5) Provision for securing an active, critical, social, and self-reliant attitude toward the learning of arithmetic.

(6) Definitely planned lessons to give children an insight into problem study and problem analysis, with detailed guidance in the formation of habits of estimating and of judging intelligently.

(7) Systematic measurement and recording of achievement which reveal individual differences, through Self-Testing Drills, Problem Scales, and other standardized material. These units are closely correlated with the general subject-matter and are distributed throughout the year.

(8) The Self-Help Study and Practice Lessons, beginning on page 359, a distinctive feature of scientific procedure for attaining the following objectives, viz.:

(*a*) Relearning of forgotten or poorly learned essentials of work done in preceding years. Valid, reliable, and easily administered

diagnostic tests reveal precisely which pupils need the relearning and precisely which things need to be relearned.

(*b*) Obviating the usual interest-killing drills that are included in most current textbooks as reviews of the previous years' essentials. Current texts require that all pupils, regardless of individual need, be forced through the same review drills. The Self-Help Study and Practice Lessons provide organized and specifically helpful review material in the text to be used by only such pupils as need it.

(*c*) Affording a scientific remastery and holding technique. This technique is so keyed into the initial diagnostic test and the regular body of sixth-grade instruction units as to constitute a practical recognition of individual needs and differences, and it is embodied in a plan which easily meets the demands of practical classroom procedures.

(9) Provision for training the pupil into habits of using his book as a means of discovering his own needs and of doing for himself the things that will meet those needs—in short, using his book as a self-teaching tool. (See the Self-Help Study and Practice Lessons, the Self-Help Index beginning on page 457, and pages 1-2, 4, 24-27, 35, 44, 125, 174-175, 359, and 457.)

(10) Special consideration of common causes of errors. Recent arithmetical research has revealed certain widespread causes of computational errors. These findings have been utilized in setting the pupil on guard against the most common blunders. (See pp. 36-39, 133-135, 213-217, 230-233.)

One outstanding departure of the *Standard Service Series* of arithmetics is that an entire book is given to each of the grades. The impelling reason for this lies in the fact that a genuinely modern treatment of arithmetic, based on the scientific findings of the last decade, demands leisurely and expansive procedures. The authors proved to themselves through years of intensive experimental study that a fully adequate pedagogical treatment demanded the space of a generous volume for each grade. There

could be no thought of trimming the final product to the dimensions and character of the customary reference-book type of text, thereby discarding the results of careful investigations made over a period of years. The decision was forced by the necessities of the undertaking. The main items of space that require a separate volume for each grade fall into three groups:

I. Definite pedagogical demands for expansive treatments. (1) Development, step by step, of scientifically itemized stages of difficulty in new arithmetical processes and topics, by advances so gradual and so copiously illustrated, exemplified, and practiced as to be easily within the grasp of the child. The carefully built and detailed treatments of denominate numbers, decimals, and percentage are illustrations of the space required by such procedure. (See pp. 49-50, 82-86, 91-92, 140-143, 155-156, 157, 159-161, 219-222, 238-239.)

(2) A number of silent-reading exercises in chatty and informal language for child-appeal. These lessons, sometimes with pictorial and diagrammatic aids, provide stimulating instruments for self-teaching.

(3) Carefully devised lessons which assist the learner in problem solving, with problem settings drawn from real life. These units also train the pupil in estimating results and in the technique of checking. (See pages 36-39, 165-167, 169, 264-265.)

(4) Twenty-nine standardized Self-Testing Drills for the maintenance of acquired skills, the systematic self-measurement of pupil progress, and the discovery of individual needs for immediate relearning.

(5) Twenty-one standardized Problem Scales, which furnish special challenges to measurable achievement and a persistent attack on the problem-solving difficulties of children. These "scales," requiring from two to three pages each, clearly make for bulk; but experimentation has proved that this use of space is well justified.

(6) Ample drill material, particularly on the more difficult combinations and types of examples. This book actually contains the drill necessary to accomplish the teaching job, without relying on the customary supplementing that must be made by the teacher from other textbook sources.

II. Special material for recognizing individual differences and provision for optional work. (1) Graded content which meets the varying abilities present in typical classrooms. All authorities agree that it is high time for competent textbook writers to begin giving systematic consideration to superior children in the public schools.

(2) Scientifically built diagnostic tests reviewing the work of preceding years, followed by remedial exercises in the form of Self-Help Study and Practice Lessons. These guarantee the pupil a working hold on the essentials of previous work, essentials that are fundamental to his successful progress in the work of the sixth grade.

III. Mechanical demands. (1) Considerable space devoted to making clear the organization of the material, both to teachers and pupils. Chapter headings (showing large units of the work), centered section topics (exhibiting the large sub-units of each chapter), and numbered article titles (indicating the arithmetical substance of the lesson under study at the moment) combine to make clear the mathematical "flow" of the text. To these are added parenthetical suggestions for each article, that indicate the authors' advice to teachers concerning the most effective way of handling these units.

(2) Summaries at the ends of chapters, that provide convenient and definite means for a quick test-survey of the progress made by the pupil.

(3) An unusual amount of space devoted to pictorial and diagrammatic aids to understanding.

The foregoing list of space requirements indicates the inherent necessity of more generous texts in arithmetic than have customarily been provided.

Experimental use of this material in a large number of sixth-grade classes widely scattered over the country has demonstrated that expansive treatment takes not more but **less time** than the customary condensed text material. The average sixth-grade class, as a matter of fact, normally uses as many pages of material as this book contains, taking about 150 pages from its adopted text and getting the remainder of the material from sources that are almost certain to be ill-fitted to the text. This book for **class use** does not need such supplementing. The authors would lay heavy stress on the fact that, while expansive treatment increases bulk, it **increases** also the **rate of progress**; and it **decreases** both the **difficulties** and **failures.**

Finally, the *Standard Service Arithmetics* have been validated through experimental use and critical study of hundreds of teachers. Such matters as the interest value of the materials, time allotments, size of the book, and effectiveness of the teaching devices, etc., have been studied in the light of the actual reactions of thousands of typical school children.

The authors' acknowledgments are herewith made to the administrators and teachers, unnamed here only because their number runs into hundreds, who generously aided in passing this material through the refining fires of classroom experimentation, thereby assisting in bringing it to the final form.

THE AUTHORS.

CONTENTS

PART ONE

CHAPTER I

BASIC TESTS IN THE FUNDAMENTALS. HOW TO USE YOUR BOOK

PAGE

Deciding upon Needed Restudy 1
Basic Tests in Whole Numbers 3
Basic Tests in Fractions 11
Informal Tests in Denominate Numbers 14
Informal Tests in Mensuration 18
Informal Tests on Useful Facts 21
Important Facts about This Book 24
Summary 27

CHAPTER II

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS. MULTIPLICATION OF DECIMALS. MULTIPLYING BY 10, 100, and 1000

Problem Work 28
The Self-Testing Drills 30
Self-Testing Drill No. 1 34
Improving Your Work 35
Problem Solving 36
The Problem Scales 39
Problem Scale No. I 40
Inventory of Important Ideas 41
Multiplication of Denominate Numbers 44
Division of Denominate Numbers 49
Problem Scale No. II 51
Self-Testing Drill No. 2 53
Multiplication of Decimals 54
Problem Scale No. III 60

PAGE
Self-Testing Drill No. 3 62
Multiplying by 10, 100, and 1000 63
Self-Testing Drill No. 4 68
Summary 69

CHAPTER III

DIVIDING BY 10, 100, and 1000. DRAWING TO SCALE. DIVISION OF DECIMALS. THE DECIMAL FORM OF REMAINDERS

Multiplying and Dividing by 10, 100, and 1000 71
Drawing to Scale 75
Problem Scale No. IV 78
Self-Testing Drill No. 5 81
Dividing by Decimals 82
Review of Fractions 89
Self-Testing Drill No. 6 90
Remainders as Decimals 91
Problem Work 93
Problem Scale No. V 97
Self-Testing Drill No. 7 99
Summary 100

CHAPTER IV

GRAPHS. BANKING PRACTICES AND FORMS. ALIQUOT PARTS. BUSINESS ARITHMETIC

Graphs 102
Problem Scale No. VI 103
Making and Reading Graphs 106
Self-Testing Drill No. 8 108
Bank Accounts, Checks, and Bank Forms 109
Self-Testing Drill No. 9 121
Aliquot Parts 123
Review of Fractions 125
Problems in Business Arithmetic 126
The Savings Bank 128
Problem Scale No. VII 130
Some Causes of Errors 133
Self-Testing Drill No. 10 136
Summary 137

CHAPTER V

INTRODUCTION TO PERCENTAGE. INSTRUCTION IN PROBLEM SOLVING

PAGE

Learning How to Work Percentage 140
Self-Testing Drill No. 11 153
Percentage, Continued 154
Finding a Per Cent of Any Number 157
Multiplying by Large Mixed Numbers 159
Self-Testing Drill No. 12 163
Percentage, Continued 164
Instruction in Problem Solving 165
Self-Testing Drill No. 13 168
Estimating Answers 169
Problem Scale No. VIII 170
Summary 172

PART TWO

CHAPTER VI

REVIEW TESTS ON ESSENTIALS

Testing Your Ability in Arithmetic 174
Basic Tests in Whole Numbers 175
Basic Tests in Fractions 180
Informal Tests in Denominate Numbers 182
Informal Tests in Mensuration 186
Informal Tests in Decimals 188
Informal Test in Checking 191
Informal Tests in Percentage 192
Informal Test on Banking 194

CHAPTER VII

DISCOUNT AS AN APPLICATION OF PERCENTAGE. REVIEW OF ZERO DIFFICULTIES. FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

Percentage in Discount 197
Self-Testing Drill No. 14 204
Review of Percentage 205

PAGE
Problem Scale No. IX 207
Self-Testing Drill No. 15 210
Review for Mastery 211
Review of Zero Difficulties 213
Self-Testing Drill No. 16 218
Finding What Per Cent One Number Is of Another 219
Drills and Problems Using Percentage 227
Some Causes of Errors 230
Self-Testing Drill No. 17 234
Using Percentage 235
Problem Scale No. X 236
Self-Testing Drill No. 18 240
Summary 241

CHAPTER VIII

MENSURATION OF AREAS AND VOLUMES. REVIEW OF PERCENTAGE

Mensuration of Areas and Volumes 243
Problem Scale No. XI 248
Self-Testing Drill No. 19 251
Practice on Fractions and Decimals 252
Review of Two Uses of Percentage 254
Problem Scale No. XII. 255
Self-Testing Drill No. 20. 257
Summary 258

CHAPTER IX

ACCOUNTS. ESSENTIALS OF INTEREST

The Personal Account 260
Problem Solving 264
Problem Scale No. XIII 266
Day-books 268
Self-Testing Drill No. 21 271
Interest as Percentage 272
Review of Percentage 278
Problem Scale No. XIV 280
Self-Testing Drill No. 22 283
Summary 284

CHAPTER X

MENSURATION OF TRIANGLES AND PARALLELOGRAMS

PAGE
Triangles and Parallelograms 287
Area of a Right Triangle 290
Estimating Answers 294
Area of a Parallelogram 295
Self-Testing Drill No. 23 301
Area of Any Triangle 302
Problem Scale No. XV 308

CHAPTER XI

GETTING READY FOR THE SEVENTH GRADE

Careless Mistakes 310
Self-Testing Drill No. 24 311
Good Habits for Solving Problems 312
Problem Scale No. XVI 316
Summary of Addition 318
Summary of Subtraction 319
Summary of Multiplication 320
Summary of Division 321
Large Numbers 322
Problem Scale No. XVII 324
Summary of Fractions 326
An Important Project (Division of Decimals) 328
Self-Testing Drill No. 25 330
Summary of Percentage 331
Vocabulary Practice 335
Problem Scale No. XVIII 337
A Useful Problem-Solving Habit (Estimating) 339
Problem Scale No. XIX 340
Self-Testing Drill No. 26 342
Summary of Mensuration 343
Self-Testing Drill No. 27 346
Problem Scale No. XX 347
Self-Testing Drill No. 28 349
Problem Scale No. XXI 350
Self-Testing Drill No. 29 351
A Bird's-Eye View of Arithmetic 352

SELF-HELP STUDY AND PRACTICE LESSONS

PAGE

How to Use the Self-Help Study and Practice Lessons . . . 359
Addition of Whole Numbers 360
Subtraction of Whole Numbers 364
Multiplication of Whole Numbers 369
Division of Whole Numbers 376
Checking for Accuracy 388
Words Used in Arithmetic 389
Addition of Fractions and Small Mixed Numbers 391
Subtraction of Fractions and Small Mixed Numbers 399
Multiplication of Fractions and Small Mixed Numbers . . . 403
Division of Fractions and Mixed Numbers 411
Addition and Subtraction of Denominate Numbers 417
Mensuration 424
Decimal Fractions 428

Answer Key to Self-Help Lessons 431

Extra Lessons for Good Workers 440

Answer Key to Extra Lessons 452

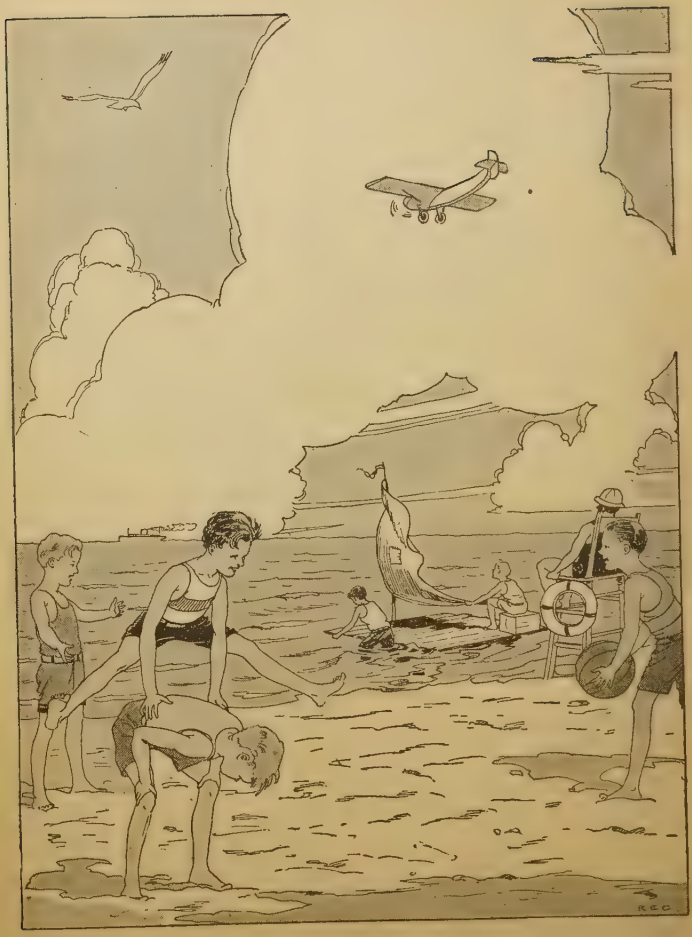
Tables of Measure 456

Pupil's Self-Help Index 457

Progress Charts 463

General Index 467

Standard Service Arithmetics



SIXTH-GRADE ARITHMETIC

PART ONE

CHAPTER I

BASIC TESTS IN THE FUNDAMENTALS. HOW TO USE YOUR BOOK

DECIDING UPON NEEDED RESTUDY

1. Planning Review Work. (Class Work.) During the summer vacation you have been thinking about summer work, play, hikes, camping trips, and other things which make vacation so pleasant.

But during three months of vacation you may have lost some of the skill you worked hard to gain in your fifth-grade study. Many of you will find that you work examples more slowly and make more mistakes than you did last May or June. Some of you may even find that you have forgotten **how** to do certain kinds of work. You can get your old knowledge and skill back quickly if you do careful work during the next few days.

Of course you do not need to review any skill in arithmetic just for the sake of reviewing. You should, however, test yourself on important things and restudy those things upon which your tests show that you really need to review.

During the next few days you will be given this opportunity to test yourself on the important things you have studied in arithmetic thus far in school. There will be tests

in adding, subtracting, multiplying, and dividing whole numbers and fractions and tests in other parts of arithmetic work. After each test there are standards which will tell you just how well you know this particular part of arithmetic. If you want to do good work in your arithmetic this year, it is important that you know thoroughly the arithmetic that is covered by these tests.

PLANNING REVIEW WORK

Our Scores on Review Tests

	WHOLE NUMBERS				FRACTIONS			
	+	−	×	÷	+	−	×	÷
John	7	20	10	3	13	12	18	13
Mary	9	28	12	5	15	19	14	16
Albert	12	27	14	8	16	17	9	18
Ruth	10	31	16	8	12	20	15	19

If you will answer the questions below, you will understand better what these tests are.

Use the chart above in answering the questions.

1. Do you think that John needs to restudy addition of whole numbers? Compare his score with the standards for the Addition Test on page 4.

2. Should Albert restudy multiplication of whole numbers? Compare his score with the standards for the Multiplication Test on page 7.

3. Should Mary restudy the subtraction of fractions? See the standards on page 12.

4. Ruth's scores show that it would be wise for her to restudy what topics in arithmetic? Can you find all the standards to use in deciding about Ruth's work?

BASIC TESTS IN WHOLE NUMBERS

2. Testing Your Ability in Addition. (WRITTEN WORK.) Your work on the test below will show you whether you need more practice on addition before going on. You should have your paper and pencil ready before your teacher tells you to begin work. Number the examples in order. You need not copy them unless told to do so.

When time is up, your teacher will read the correct answers, and you will put a "C" after each correct answer on your paper. Then count your correct answers and, from the standards on page 4, find how well you have done.

Time allowed: 6 minutes.

BASIC ADDITION TEST

1.	2.	3.	4.	5.	6.
37	84	37	68	814	456
58	44	44	5	382	684
		66	46	906	828
			99		157
			28		

7.	8.	9.	10.	11.
26	3671	2776	5068	940
57	9207	637	710	9769
71	3948	5851	8995	2180
11	3806	4659	74	6671
74		9967	93	6408
95			7388	8792
38				5374

Go on to the next page.

12. *Copy and add:* $516+180+555+502+662$

13. *Copy and add:* $34+44+55+48+89+74$

STANDARDS

8 examples correct→fair fifth-grade work
9 examples correct→good fifth-grade work
10 examples correct→fair sixth-grade work
11 examples correct→good sixth-grade work
More than 11 correct→excellent sixth-grade work

3. Do You Need to Restudy Addition? (CLASS WORK.) Your score on the Addition Test may have shown you that you have forgotten much about adding whole numbers or that you never thoroughly learned how to add. Of course, it will not do you any good to know that you are weak unless you do something to help yourself improve.

It is important to restudy the addition of whole numbers right now if you did not do at least "fair sixth-grade work" on the test. Of course, some pupils who did do fair work will want to restudy addition, because they are ambitious to excel in arithmetic. By talking over in class the matter of review, you can decide for yourself what to do.

If you have decided to restudy addition, the following suggestions will be of help:

1. Spend a few minutes looking over the **Self-Help Study and Practice Lessons** on addition, beginning on page 360. Read Articles 235 and 236 carefully.

2. Use any of these Lessons which you and your teacher think are the ones you need.

Suggestions for pupils who do not need to review addition. If you do not need to restudy addition, what should you do during the time that other pupils are busy with their review? The next two questions will suggest good ways of spending your time.

Do you need to do extra work on any other school subject?

Is there any interesting book on travel or some other subject you like which you might get started on now?

There are **Extra Lessons for Good Workers,** beginning on page 440. You will find these interesting if you want to do extra work in arithmetic.

4. Testing Your Ability in Subtraction. (WRITTEN WORK.) Your work on the test below will show whether you need more practice on subtraction now. Have your paper and pencil ready when time is called to begin. You need not copy examples unless told to do so.

Time allowed: 8 minutes. Compare your work with the standards below the test.

BASIC SUBTRACTION TEST

1.	2.	3.	4.	5.
68	39	93	35	73
35	6	84	28	73
6.	**7.**	**8.**	**9.**	**10.**
59	44	676	419	809
50	20	652	414	807
11.	**12.**	**13.**	**14.**	**15.**
967	963	828	725	1360
910	825	363	657	857
16.	**17.**	**18.**	**19.**	**20.**
2566	6698	2083	8594	8322
2370	6574	447	2659	6968

Go on to the next page.

21.	22.	23.	24.	25.
3970	6041	9240	5917	5000
3892	4348	4491	3258	1126

26.	27.	28.	29.	30.
4637	4260	10624	9009	13254
1059	2980	8138	1065	7275

31. *Copy and work:* 1394 − 606

32. *Copy and work:* 8131 − 1934

STANDARDS

25–26 examples correct → fair fifth-grade work
27 examples correct → good fifth-grade work
28 examples correct → fair sixth-grade work
29 examples correct → good sixth-grade work
30–32 examples correct → excellent sixth-grade work

5. Do You Need to Restudy Subtraction? (Class Work.) It would be unwise for you to go on with sixth-grade work without an earnest attempt to improve, if your work in subtraction is less than "fair sixth-grade work" as measured by the Subtraction Test.

The Self-Help Study and Practice Lessons on subtraction, which begin on page 364, are for pupils who want to make perfectly sure that their ability to subtract is in good working order before going on. These Lessons are to be used as the Self-Help Lessons on addition were used.

Suggestions for pupils who do not need to restudy subtraction. It does not pay to be "easy" on yourself. Make very sure that a restudy of subtraction would not help you, before you decide to spend your time on other things while the rest of the class is reviewing subtraction. If you decide that you do not

need any work on subtraction, spend your time on other school subjects, in reading a valuable book, or studying the Extra Lessons for Good Workers, pages 440-455.

6. Testing Your Ability in Multiplication. (WRITTEN WORK.) In this test be sure to copy the examples correctly. Allow plenty of space on your paper.

Time allowed: 14 minutes. Compare your work with the standards below the test.

BASIC MULTIPLICATION TEST

1.	2.	3.	4.	5.	6.
458	671	90	83	594	847
1	5	8	40	9	7

7.	8.	9.	10.	11.
73	876	826	5349	9009
13	682	539	2680	4173

12.	13.	14.	15.	16.
605	370	4509	1487	3256
497	409	6005	8479	7008

STANDARDS

8 examples correct→fair fifth-grade work
9–10 examples correct→good fifth-grade work
11–12 examples correct→fair sixth-grade work
13 examples correct→good sixth-grade work
14–16 examples correct→excellent sixth-grade work

7. Do You Need to Restudy Multiplication? (CLASS WORK.) Your teacher and parents want you to learn to make decisions for yourself and to form habits of inde-

pendent work. These habits are just as important for your success in life as any of the other things you learn. Can you decide for yourself whether you need a restudy of multiplication? Is it good sense to be "easy" on yourself?

Self-Help Lessons on multiplication begin on page 369.

If you have time to spend while other pupils are reviewing multiplication, which of the plans listed below are useful ones?

1. Sit around and do nothing.

2. Attack some topic in another school subject which you do not understand very well.

3. Use the Extra Lessons for Good Workers (pages 440-455).

4. Take time to write an interesting letter to a grandparent or other relative who will be very much pleased to hear from you.

5. Start reading some worth-while book.

8. Testing Your Ability in Division. (WRITTEN WORK.) Be careful to copy each example correctly. Allow plenty of space upon your paper. **Time allowed:** 20 minutes.

BASIC DIVISION TEST

1. $8\overline{)26003}$	4. $60\overline{)190207}$	7. $670\overline{)138020}$
2. $73\overline{)2935}$	5. $780\overline{)222300}$	8. $407\overline{)260073}$
3. $66\overline{)26555}$	6. $304\overline{)2432912}$	9. $58\overline{)52673}$

STANDARDS

3 examples correct→fair fifth-grade work
4–5 examples correct→good fifth-grade work
6 examples correct→fair sixth-grade work
7 examples correct→good sixth-grade work
8–9 examples correct→excellent sixth-grade work

9. Do You Need to Restudy Division? (CLASS WORK.) In deciding whether you need to restudy division, you should remember that, on the whole, division is harder for most pupils than either addition, subtraction, or multiplication. You had better be too hard than too easy on yourself in the matter of spending at least a little time in restudying division.

The Self-Help Lessons on division begin on page 376.

SOME NUMBER RELATIONS

10. Finding Missing Numbers. (ORAL WORK.) You have been spending the last few days in testing your ability to work with whole numbers and doing important restudying. Tomorrow you will begin to test your ability to work with fractions. For today you need do no written work.

The exercises below and on the next page are good fun. Some of the exercises are not so easy as they look.

EXERCISE (A)

What number comes next in each example below?

SAMPLE: 1 3 5 7 9 11 .....

In the sample, each number is obtained by adding 2 to the number before it. 13 comes next.

1.	$\frac{1}{2}$	2	$3\frac{1}{2}$	5	$6\frac{1}{2}$	8 .....
2.	31	27	23	19	15	11 .....
3.	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$ .....
4.	1	3	9	27	81	243 .....
5.	8	$7\frac{1}{3}$	$6\frac{2}{3}$	6	$5\frac{1}{3}$	$4\frac{2}{3}$ .....

EXERCISE (B)

Study silently for 5 minutes; then be ready to tell which numbers are wrong in the 13 examples which follow. Use pencil and paper if needed. Do not mark in the book.

SAMPLE: If you begin with 4 and multiply by 3, then multiply the product (12) by 3, and so on, you get: $4\times3=12$, $12\times3=36$, $36\times3=108$, $108\times3=324$, $324\times3=972$, $972\times3=2916$, etc.

If 394 were given in the sample instead of 324, it would be wrong, because it could not be obtained in the same way as the other numbers in this sample.

In each example below all the numbers are obtained in the same way. **There are at least 4 mistakes.** Find them.

1. $\times3$ starting with 3: 9, 27, 81, 243, 729, 2187.
2. $\times4$ starting with 2: 8, 32, 128, 512, 2048, 8192.
3. $\div3$ starting with 1458: 486, 162, 54, 18, 6, 2.
4. $\div2$ starting with 192: 96, 48, 24, 12, 6, 2.
5. $\div4$ starting with 8192: 2048, 512, 128, 38, 8, 2.
6. $+27$ starting with 9: 36, 63, 90, 117, 144, 161.
7. -39 starting with 2000: 1961, 1922, 1883, 1844, 1805, 1766, 1727.
8. $\times8$ starting with $\frac{1}{2}$: 4, 32, 256, 2048, 16384, 131072.
9. $\times7$ starting with $\frac{1}{49}$: $\frac{1}{7}$, 1, 7, 49, 343, 2201.
10. $+4\frac{3}{4}$ starting with 0: $4\frac{3}{4}$, $9\frac{1}{2}$, $14\frac{1}{4}$, 19, $23\frac{3}{4}$, $28\frac{1}{2}$.
11. $-1\frac{1}{2}$ starting with 15: $13\frac{1}{2}$, 12, $11\frac{1}{2}$, 9, $7\frac{1}{2}$, 6.
12. $\times\frac{1}{2}$ starting with 168: 84, 42, 21, $10\frac{1}{2}$, $5\frac{1}{4}$, $2\frac{5}{8}$.
13. $\div\frac{1}{3}$ starting with 1: 3, 9, 27, 81, 243, 729.

BASIC TESTS IN FRACTIONS

11. Testing Your Ability to Add Fractions. (WRITTEN WORK.) During the next few days, you will have an opportunity to find how well you can work with fractions. You will be able to decide whether you need to restudy the addition of fractions by comparing your score on the test below with the standards.

Be sure to copy examples carefully. **Time allowed:** 20 minutes. **Answers must be in reduced form to count as correct.**

BASIC TEST IN ADDITION OF FRACTIONS

Find the sums:

1. $\frac{5}{8}+\frac{3}{8}$
2. $\frac{3}{10}+\frac{1}{2}$
3. $2\frac{5}{12}+1\frac{1}{2}$
4. $\frac{2}{5}+\frac{1}{2}$
5. $5\frac{3}{4}+2\frac{1}{2}$
6. $\frac{5}{6}+\frac{3}{4}$
7. $7\frac{1}{12}+3\frac{7}{12}$
8. $\frac{4}{9}+11\frac{5}{6}$
9. $4\frac{2}{3}+8\frac{1}{2}$
10. $5\frac{3}{4}$ / 7 / $1\frac{2}{5}$ / 3
11. $2\frac{1}{8}$ / $\frac{5}{12}$ / $7\frac{1}{3}$
12. $5\frac{1}{4}$ / $3\frac{13}{20}$ / $2\frac{1}{10}$
13. $13\frac{7}{9}$ / $7\frac{2}{3}$ / $23\frac{4}{9}$
14. $3\frac{5}{16}$ / $2\frac{3}{8}$ / $5\frac{1}{4}$
15. $7\frac{3}{5}$ / $8\frac{1}{3}$ / $1\frac{1}{5}$
16. $\frac{3}{8}$ / $4\frac{5}{6}$ / $3\frac{1}{3}$
17. $13\frac{5}{12}$ / $9\frac{1}{4}$ / $11\frac{2}{3}$ / $22\frac{5}{6}$

STANDARDS

11 examples correct→fair fifth-grade work
12–13 examples correct→good fifth-grade work
14–15 examples correct→fair sixth-grade work
16 examples correct→good sixth-grade work
17 examples correct→excellent sixth-grade work

12. Do You Need to Restudy the Addition of Fractions? (CLASS WORK.) If you did good work on fractions last year, you will be surprised how quickly you can get your old skill back even if you did not do very well on the Addition Test. The best way to do this is to use the Self-Help Lessons on addition of fractions. They begin on page 391.

If your work on the Addition Test shows that you do not need to restudy the addition of fractions, plan to spend your time wisely while the other pupils are restudying. There are some suggestions on pages 4 and 8.

13. Testing Your Ability to Subtract Fractions. (WRITTEN WORK.) Make sure that you copy examples carefully and that you **leave answers in reduced form.**

Time allowed: 20 minutes.

BASIC TEST IN SUBTRACTION OF FRACTIONS

1. $\frac{4}{5}-\frac{2}{5}$	**8.** $6\frac{3}{4}-2\frac{1}{6}$	**15.** $12\frac{5}{6}-5\frac{3}{8}$
2. $\frac{11}{12}-\frac{7}{12}$	**9.** $4\frac{3}{8}-2\frac{3}{8}$	**16.** $7\frac{1}{9}-3\frac{7}{12}$
3. $\frac{3}{4}-\frac{1}{2}$	**10.** $\frac{9}{10}-\frac{3}{4}$	**17.** $7\frac{1}{16}-6\frac{1}{2}$
4. $\frac{4}{5}-\frac{1}{2}$	**11.** $5\frac{1}{3}-2\frac{5}{6}$	**18.** $9\frac{5}{6}-\frac{4}{5}$
5. $3\frac{1}{3}-1\frac{1}{2}$	**12.** $7\frac{1}{2}-3\frac{4}{5}$	**19.** $4\frac{5}{8}-2\frac{1}{3}$
6. $7-2\frac{3}{4}$	**13.** $11\frac{1}{3}-4\frac{3}{4}$	**20.** $6-3\frac{7}{36}$
7. $15\frac{2}{9}-12$	**14.** $8\frac{1}{5}-4\frac{1}{4}$	**21.** $15\frac{11}{12}-12\frac{3}{8}$

STANDARDS

16–17 examples correct→fair fifth-grade work
18 examples correct→good fifth-grade work
19 examples correct→fair sixth-grade work
20 examples correct→good sixth-grade work
21 examples correct→excellent sixth-grade work

14. Do You Need to Restudy Subtraction of Fractions? (CLASS WORK.) It is wise to make sure **now** that you can work examples like those in the test on page 12 easily and correctly. The Self-Help Lessons on subtraction of fractions, which begin on page 399, will help in a restudy.

15. Testing Your Ability in the Multiplication of Fractions. (WRITTEN WORK.) A good score on the test below means that you can handle the multiplication of fractions well enough to do good work. Poor scores mean that you should take time to restudy.

Make sure to copy examples accurately and to reduce answers when needed. **Time allowed:** 15 minutes.

BASIC TEST IN MULTIPLICATION OF FRACTIONS

1. $\frac{1}{5}\times 3$	2. $\frac{5}{6}\times\frac{8}{15}$	3. $\frac{3}{4}\times 3\frac{1}{2}$	4. $2\frac{1}{2}\times 0$
5. $1\frac{1}{4}\times\frac{4}{5}$	6. $5\frac{5}{6}\times 3\frac{3}{5}$	7. $1\frac{1}{24}\times 1\frac{4}{5}$	8. $\frac{7}{8}$ of 3
9. $3\times\frac{1}{8}$	10. $4\frac{1}{4}\times 15$	11. $\frac{4}{9}\times\frac{1}{4}\times\frac{4}{9}$	12. $4\frac{2}{3}\times 5\frac{1}{4}$
13. $\frac{1}{4}$ of 2	14. $3\frac{1}{2}\times 1\frac{1}{2}$	15. $2\frac{1}{6}\times 1\frac{1}{2}$	16. $2\frac{5}{8}\times\frac{4}{9}$
17. $\frac{3}{5}\times\frac{3}{10}$	18. $\frac{1}{2}$ of $3\frac{1}{3}$	19. $7\times 3\frac{1}{6}$	20. $1\frac{7}{12}\times\frac{4}{15}$

STANDARDS

10–11 examples correct→fair fifth-grade work
12–13 examples correct→good fifth-grade work
14–15 examples correct→fair sixth-grade work
16–17 examples correct→good sixth-grade work
18–20 examples correct→excellent sixth-grade work

The Self-Help Lessons on multiplication of fractions begin on page 403. Do you need to use these Lessons?

16. Testing Your Ability in Division of Fractions. (WRITTEN WORK.) Use the test below to help you decide whether you need to restudy division of fractions. In taking the test, be sure to copy examples carefully and to reduce answers when necessary. **Time allowed:** 20 minutes.

BASIC TEST IN DIVISION OF FRACTIONS

1. $\frac{1}{3} \div \frac{1}{2}$	2. $\frac{1}{2} \div \frac{2}{5}$	3. $10\frac{1}{2} \div 1\frac{1}{6}$	4. $3\frac{3}{5} \div 9$
5. $2\frac{1}{3} \div 2\frac{1}{2}$	6. $\frac{1}{20} \div \frac{3}{8}$	7. $15 \div 1\frac{2}{3}$	8. $6 \div \frac{1}{3}$
9. $\frac{4}{9} \div \frac{6}{9}$	10. $30 \div \frac{1}{2}$	11. $3\frac{3}{4} \div \frac{3}{8}$	12. $\frac{4}{5} \div 1\frac{1}{15}$
13. $3\frac{1}{6} \div \frac{3}{4}$	14. $\frac{7}{16} \div \frac{5}{8}$	15. $2\frac{1}{10} \div 1\frac{1}{2}$	16. $1\frac{1}{12} \div 4$
17. $1\frac{1}{6} \div \frac{3}{5}$	18. $2\frac{1}{12} \div 2\frac{1}{5}$	19. $\frac{3}{4} \div \frac{3}{4}$	20. $7 \div 1\frac{1}{4}$

STANDARDS

10–12 examples correct → fair fifth-grade work
13–15 examples correct → good fifth-grade work
16–17 examples correct → fair sixth-grade work
18–19 examples correct → good sixth-grade work
20 examples correct → excellent sixth-grade work

If you need to restudy division of fractions, you will find Self-Help Lessons beginning on page 411.

REVIEW TESTS IN DENOMINATE NUMBERS

17. Informal Tests in Denominate Numbers. (WRITTEN WORK.) Examples using denominate numbers are not hard if you keep certain facts about them in mind. All sixth-grade pupils should be able to work all the examples in Tests (*A*) and (*B*) on the next two pages. These tests

will show you whether you have forgotten what you ought to know about changing the forms of denominate numbers.

TEST (A)

Changing the Form of Denominate Numbers

Write the number that should be on each blank in the 28 examples below. Number your papers carefully. **Suggested working time:** 15 minutes.

1. 16 in. = ____ ft. ____ in.
2. 6 qt. = ____ gal. ____ qt.
3. 1 cu. yd. = ____ cu. ft.
4. 7 ft. = ____ yd. ____ ft.
5. $1\frac{1}{2}$ lb. = ____ oz.
6. $1\frac{1}{3}$ sq. yd. = ____ sq. ft.
7. 1 bu. = ____ pk.
8. $8\frac{1}{4}$ ft. = ____ rd.
9. 16 qt. = ____ pk.
10. 6 ft. = ____ yd.
11. $3\frac{1}{2}' =$ ____ ″.
12. 32 oz. = ____ lb.
13. 1 sq. ft. = ____ sq. in.
14. 3 gal. = ____ qt.
15. 180 sec. = ____ min.
16. 1 mi. = ____ ft.
17. 19 in. = ____ ft. ____ in.
18. 320 rd. = ____ mi.
19. $2\frac{1}{2}$ pk. = ____ qt.
20. 1 sq. mi. = ____ A.
21. 1 T. = ____ lb.
22. $16\frac{1}{2}$ ft. = ____ rd.
23. 80 min. = ____ hr. ____ min.
24. $\frac{1}{4}$ lb. = ____ oz.
25. 11 yd. = ____ rd.
26. 288 sq. in. = ____ sq. ft.
27. 18 sq. ft. = ____ sq. yd.
28. 10 pk. = ____ bu. ____ pk.

If examples like those in Test (*A*) are hard for you, use the Self-Help Lesson on page 417 (Art. 305) and the Tables of Measure on page 456.

TEST (B)

The Form of Answers

Answers using denominate numbers should always be reduced when possible, as shown by the following samples. **Suggested working time:** 15 minutes.

First Found Answer	Good Form of Answer
(*a*) 4 ft. 14 in.	5 ft. 2 in.
(*b*) 4 yd. 7 ft.	6 yd. 1 ft.
(*c*) 3 lb. 18 oz.	4 lb. 2 oz.
(*d*) 4 bu. 11 pk.	6 bu. 3 pk.

Change each answer below to the correct form.

1. 2 yd. 6 ft. 15 in.
2. 4 hr. 120 min.
3. 1 gal. 6 qt. 5 pt.
4. 1 sq. yd. 12 sq. ft.
5. 3 lb. 20 oz.
6. 3 bu. 11 pk. 6 qt.
7. 4 sq. ft. 248 sq. in.
8. 27 cu. ft.

If Test (*B*) was hard for you, use the Self-Help Lesson on page 418 (Art. 306) and the Tables of Measure on page 456.

18. Testing Your Ability to Add Denominate Numbers. (WRITTEN WORK.) Use the test below to decide whether you should restudy addition of denominate numbers. Answers must be in reduced form. **Time allowed:** 4 minutes.

TEST IN ADDITION OF DENOMINATE NUMBERS

1. 3 ft. 4 in.
   6 ft. 9 in.
   1 ft. 3 in.

2. 6 gal. 1 qt.
   2 qt.
   4 gal. 3 qt.

3. 4 lb. 8 oz.
   1 lb. 7 oz.
   3 lb. 11 oz.
   6 lb. 6 oz.

4.	5.	6.
4 hr. 15 min.	8 yd.	4 bu.
1 hr. 25 min.	7 yd. 2 ft.	1 bu. 3 pk.
3 hr. 10 min.	4 yd. 1 ft.	3 bu. 1 pk.
2 hr. 30 min.	2 yd. 2 ft.	3 pk.
		7 bu. 2 pk.

STANDARDS

3 examples correct→good fifth-grade work
4 examples correct→fair sixth-grade work
5 examples correct→good sixth-grade work
6 examples correct→excellent sixth-grade work

There are Self-Help Study and Practice Lessons beginning on page 419 for your use if you need them.

19. Testing Your Ability to Subtract Denominate Numbers. (WRITTEN WORK.) Your score on the Subtraction Test below will show whether you need to spend a little time in restudying the subtraction of denominate numbers. Be sure to copy examples correctly as you take the test. **Time allowed:** 3 minutes.

TEST IN SUBTRACTION OF DENOMINATE NUMBERS

1.	2.	3.
3 qt. 1 pt.	5′ 4″	15 lb.
1 qt.	2′ 8″	4 lb. 10 oz.

4.	5.	6.
8 gal. 1 qt.	2 hr. 10 min.	6 yd. 1 ft.
2 gal. 3 qt.	1 hr. 20 min.	2 yd. 2 ft.

Go on to the next page.

7.	8.	9.
18 lb.	8 T. 1025 lb.	4 bu. 1 pk.
10 lb. 14 oz.	1500 lb.	2 bu. 3 pk.

STANDARDS

5 examples correct→fair fifth-grade work
6 examples correct→good fifth-grade work
7 examples correct→fair sixth-grade work
8 examples correct→good sixth-grade work
9 examples correct→excellent sixth-grade work

Use the Self-Help Lesson on page 421 if you need to.

MENSURATION

20. Informal Tests in Mensuration. (WRITTEN WORK.) Here are three little tests in measuring. They may take more than one class period. There are no standards for these tests. Decide for yourself about your need for review work, or talk with your teacher about it.

TEST (A). WORDS USED IN MENSURATION

Number your answers on your paper carefully. Write (as your answers) the word or words which should be on each blank line. **Suggested working time:** 5 minutes.

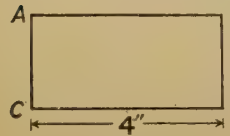


FIG. 1

1. Fig. 1 is called a.....
2. Fig. 1 has a.....of 12″.
3. Fig. 1 has an..... of 8 sq. in.
4. In Fig. 1 the 4″ and the 2″ are the.....
5. In Fig. 1 lines *AB* and *CD* are of.....length.

6. In Fig. 1 lines AC and CD form a.....angle.

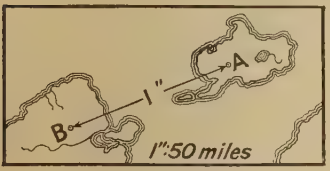


FIG. 2

7. In Fig. 2, the 1″: 50 miles is the.....of the map.

8. In Fig. 2, a distance of $1\frac{1}{2}$″ on the map represents .....miles.

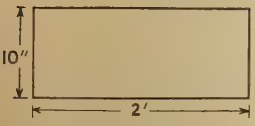


FIG. 3

9. In Fig. 2, the distance between points A and B is..... miles.

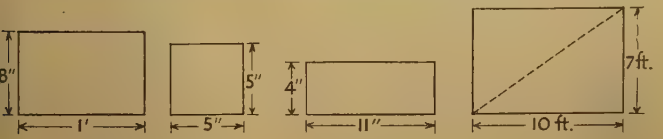
10. Before finding the area of Fig. 3, the dimensions would first be changed to the.....unit.

If the work in Test (*A*) troubled you, use the Self-Help Lesson on page 424 (Art. 314).

TEST (B). FINDING AREAS

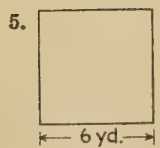
Time suggested: 10 minutes. Number each example. You need write only the answers. Name answers carefully.

Find the areas of the figures below and at the top of page 20.



1. 2. 3. 4.

Go on to the next page.

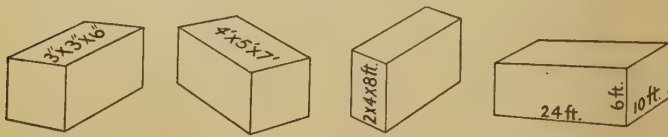


If you need to restudy finding areas, use the Self-Help Study and Practice Lessons on pages 425-427.

TEST (C). FINDING VOLUMES OF RECTANGULAR SOLIDS

You need write only the answers. Number each answer on your paper. **Time suggested:** 10 minutes.

Find the volumes of the figures below.



1. 2. 3. 4.

5. Find the number of cubic yards in a rectangular bin which is $9' \times 12' \times 18'$.

6. Find the capacity in cubic feet of a rectangular swimming tank whose dimensions are $6' \times 22' \times 46'$. (The tank is the same depth throughout.)

7. What is the capacity in cubic feet of a freight car whose inside dimensions are $9\frac{1}{3}' \times 12' \times 40'$?

Self-Help Lessons on finding volumes begin on page 427.

USEFUL FACTS

21. Informal Tests on Useful Facts. (WRITTEN WORK.) Your work on Tests (*A*), (*B*), and (*C*) in this article will show you how well you know certain useful facts. It is important that you correct any mistakes, because you will need the facts many times in your sixth-grade work.

TEST (A). DECIMAL NUMBERS

(Time suggested: 15 minutes)

1. Make up a single sentence using all the words below. Use any other words you need to complete the sentence.

decimals adding decimal points subtracting straight

2. Complete the answers in the work shown below.

(*a*) \$35.78 (*b*) \$6.78 (*c*) \$500.00 − \$250.50 = 24950 *Ans.*
15.34 2.45
2044 *Ans.* 4.70
.45
1438 *Ans.*

3. *Find the answers:*

(*a*) \$3.27 + \$10.04 + \$.37 = (*b*) \$27.35 − \$14.28 =

4. In the number 3678.49:

(*a*) The 4 is in the.....place. (*d*) The 7 is in the.....place.
(*b*) The 3 is in the.....place. (*e*) The 6 is in the.....place.
(*c*) The 9 is in the.....place. (*f*) The 8 is in the.....place.

5. In the number 7594.375:

(*a*) The (.) is called a..... .....
(*b*) All figures on the left of the . are.....numbers.
(*c*) All figures on the right of the . are.....fractions.

Go on to the next page.

6. Express as decimals: (*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{8}$ (*d*) $\frac{3}{4}$.

7. Write the next numbers in fraction form.

(*a*) $.33\frac{1}{3}$ (*b*) .5 (*c*) 1.75 (*d*) .125.

See Art. 320 for Self-Help Lessons if you need them.

TEST (B). CHECKING

(Time suggested: 10 minutes.)

Do not work the examples below, but check them. Show the actual work you do in checking the four examples.

1.	2.	3.	4.
31	4183	807	$133\frac{24}{61}$
67	−1476	115	61)8137
4	2707	4035	61
81		807	203
19		807	183
202		92805	207
			183
			24

Use the Self-Help Lesson on page 388 if you need to.

TEST (C). FRACTION AND WHOLE NUMBER VOCABULARY

(Time suggested: 15 minutes.)

Be careful to number each answer on your paper.

1. Changing $\frac{3}{6}$ to $\frac{1}{2}$ is called.....

2. In multiplication the number $4\frac{1}{4}$ would be changed to $\frac{17}{4}$ before multiplying. $4\frac{1}{4}$ is called a.....number, and $\frac{17}{4}$ is called a (an).....fraction.

3. In the work at the right, the multiplication has been made easier by using..... .

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}} = \frac{2}{9}$$

4. The number 12 is a..... denominator of $\frac{1}{4}+\frac{1}{3}$.

5. In solving, $\frac{2}{5} \div \frac{4}{9}$ would be changed to $\frac{2}{5} \times \frac{9}{4}$. A rule used here is: "To divide by a fraction, ...the divisor and multiply."

6. In an answer, $\frac{5}{2}$ would be changed to $2\frac{1}{2}$ because answers should be left in..... .....

7. $\frac{4}{5} = \frac{12}{15}$ because $\frac{4\times3}{5\times3} = \frac{12}{15}$. This shows that..... both terms of a fraction by the.....number does not change the.....of the fraction.

8. $\frac{9}{8}$ is the.....of $\frac{8}{9}$.

9. *What should be on the blanks in the statements that follow the next four examples?*

(*A*)	(*B*)	(*C*)	(*D*)
47	317	$20\frac{1}{2}$	$4+8+3=15$
$\times 3$	-102	$4\overline{)82}$	
141	215		

(*a*) In Ex. (*A*) the 47 is called the.....

(*b*) In Ex. (*B*) the answer 215 is called the.....

(*c*) In Ex. (*A*) the 3 is called the.....

(*d*) In Ex. (*C*) the fraction $\frac{1}{2}$ is called the.....

(*e*) In Ex. (*A*) the answer 141 is called the.....

(*f*) In Ex. (*C*) the 4 is called the.....

(*g*) In Ex. (*C*) the 82 is called the.....

Go on to the next page.

(*h*) In Ex. (*D*) the answer 15 is called the.....

(*i*) In Ex. (*C*) the answer $20\frac{1}{2}$ is called the .....

You will find Self-Help work on the following pages:

Example 1—Page 393
Example 2—Page 394
Example 3—Page 408
Example 4—Page 391
Example 5—Page 412
Example 6—Page 394
Example 7—Page 393
Example 8—Page 411
Example 9—Page 389

IMPORTANT FACTS ABOUT THIS BOOK

Pupils using the Standard Service Arithmetic for the first time should study Art. 22. Pupils who have used this arithmetic in Grade V will need to spend but a moment on Art. 22.

22. Learning to Use Your Arithmetic Book. (Class Work.) It is important for you to know how this book is to be used. The following facts will help you.

First: This book is divided into **chapters.** You are now studying Chapter I. Turn back to the beginning of the book and find where Chapter I starts. What is the main purpose of Chapter I? On what page does Chapter II begin? What topics are taken up in Chapter II? What is taught in Chapter III? In Chapter IV?

Second: Each chapter is divided into **articles.** Each article has been given a number in heavy type at the left of the first line of the article. These numbers are called **article numbers.**

You are now reading Article 22. Each article has a name or title. Article 22 is called "Learning to Use Your

Arithmetic Book." Find Article 28. On what page is it? What is the title (name) of Article 28?

Notice that after the title of each article there are always some words in parenthesis. Thus, Art. 22 has as its title, **Learning to Use Your Arithmetic Book.** (CLASS WORK.) Again, Art. 25 begins **Measuring Your Progress in Arithmetic.** (SILENT STUDY.) The heading of Art. 35 is: **Rapid Drill.** (ORAL WORK.) Art. 37 reads: **Practice in Dividing Denominate Numbers.** (WRITTEN WORK.)

The words "Class Work," "Silent Study," "Oral Work," "Written Work," etc., give you directions for using the articles.

"Class Work" means that your teacher will probably have you talk this article over in class. "Silent Study," "Silent Reading," and "Study Period" mean that your teacher will probably ask you to study quietly before recitation. "Oral Work" will usually be done without pencil or paper. "Written Work" should be done neatly with pencil and paper.

Third: Sometimes you will need to **restudy** certain articles which you have previously used. These articles are easily found.

(*a*) Some articles refer you to another article. Do you see how Art. 34 refers you to Arts. 32 and 33?

(*b*) *The Self-Help Index.* Sometimes you will want to review something without being told to do so. You may want to restudy what is in some article you have studied this year, or you may need to review something you should have learned in fifth-grade arithmetic. The **Self-Help Index,** beginning on page 457, will be a great help to you in finding just the article or articles you may need to review.

Turn to page 457 and see how useful the Self-Help Index can be in directing you to articles you want to review. Most pupils find this Index a very helpful "tool" for good work in arithmetic. Be sure to read Article 337 carefully.

Spend a little time right now in getting familiar with the Self-Help Index. If you can answer the next questions correctly you will understand the Index pretty well.

1. Find in the Self-Help Index the numbers of the articles that would help you in multiplication of mixed numbers.

2. Suppose long-division examples like $576\overline{)34567}$ were troubling you and you wanted to restudy long division; what articles would you use?

3. Suppose you needed some more work on multiplication of decimals such as shown in the example at the right. What articles would you find of help?

$$\begin{array}{r} 34.67 \\ \times .82 \\ \hline \end{array}$$

4. What articles could you use at the end of the year to review on division of decimals?

5. Discuss in class the next question: "Should a pupil wait to be made to use the Self-Help Index or is it a good plan to use it whenever a pupil needs some review work?"

6. Find some references to the Index anywhere in the book. See pages 35, 44, and 54. Why are you often referred to the Self-Help Index during the year?

Fourth: Whenever you come to a **New-Work** sign such as you see at the beginning of Arts. 22, 32, and 36, you will know that something is to be studied which you should

make sure to master, because it is new work. Every sixth-grade pupil knows that it is wise to take especial pains to master new work the first time it is studied.

SUMMARY

23. Summary of Chapter One. (Class Work.) The questions below are good ones to talk over in class.

QUESTIONS

1. Why is it a good plan to take tests like those in Chapter I at the beginning of the school year?

2. Why are Self-Help Study and Practice Lessons printed in this book?

3. Can the Self-Help Lessons be used at any time?

4. Can you find any examples or questions in Chapter I that are really too hard for pupils who expect to do good work in the sixth grade?

5. On what pages is the Self-Help Index?

6. Which of the statements below is the best answer to this question: "When can a pupil use the Self-Help Index?

(*a*) Only when his teacher asks him to do so.

(*b*) Only at the end of the year when he is trying to make up for poor work during the year.

(*c*) Any time during the year when a little independent study will help him.

CHAPTER II

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS. MULTIPLICATION OF DECIMALS. MULTIPLYING BY 10, 100, AND 1000

24. Problem Work. (A Class Competition.) Read the following 10 problems before you are told to begin actual work. Do you think that you could work them all correctly? Here are the records of four other sixth-grade classes. **Only problems worked correctly were counted.**

Class (*A*). $\frac{7}{8}$ of the class worked all.
Class (*B*). $\frac{2}{3}$ of the class worked all.
Class (*C*). $\frac{1}{2}$ of the class worked all.
Class (*D*). Only $\frac{1}{3}$ of the class worked all.

Help your class to stand high. Have paper and pencil ready. **Time allowed:** 20 min. Write neatly.

1. A group of 5 girls are going together to a fair. Tickets cost 50 cents each. How much will 5 tickets cost?

2. I sent by mail a birthday present to my cousin. The postal clerk put on the package 3 five-cent stamps and 1 three-cent stamp. What was the cost of the stamps?

3. Jimmie bought 300 of the marbles advertised at the right. How much did they cost him?

Marbles 25c a Hundred!

4. At a school picnic 36 lemons were used. How much was spent for lemons if they cost 40¢ a dozen?

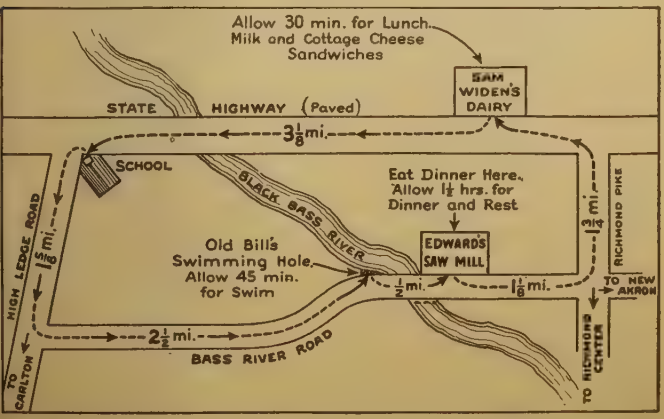
5. Which is longer, 42 da. or 9 wk.? How much longer?

6. I want to buy a radio set costing \$7.50, and \$1.25 worth of wire. I have saved up \$6.25. How much more money do I need?

7. "Snowhite" soap at a special sale was offered at 3 bars for 25¢. What would a dozen bars cost at the sale price?

8. How much money must a Boys' Club save in order to pay for 9 baseball suits at \$3.25 each, and 9 pairs of baseball shoes at \$3.00 a pair?

9. I bought 15 articles at 5 for \$1.00. How much did the 15 articles cost?



10. The Hiker's Club is planning an all-day Saturday hike. The hike starts and finishes at the School. If the Club starts at 9 o'clock and averages $2\frac{1}{2}$ miles an hour while actually walking, at what time should the Club get back to the school?

How many of the 10 problems did you do correctly? Is your class as good as Class (*A*)? Are there some problems which you should work again?

THE SELF-TESTING DRILLS

25. Measuring Your Progress in Arithmetic. (SILENT STUDY.) Theodore Roosevelt once said that the great trouble with many men and women is that they are too satisfied with themselves. They live each day and year without ever thinking whether the record they are making is better than their past record.

But successful people are always trying to make each year's work better than that of the year before. They watch their past records and try to beat them. Business men and women are constantly asking, "How is our business record this year as compared with last year?" Clubs, churches, and all sorts of organizations are ever trying to beat their past membership records and to do more and more things for their members.

You know that athletes watch their records closely. Baseball players are always trying to better their batting and fielding averages. Track athletes train and practice hard in order to clip another second off their best past record.

Why do successful men and women in all walks of life keep records and watch their progress so closely? It is because they want to improve, and their past records tell them whether the work they are doing is better or poorer than what they have done.

Success in arithmetic is much like success in anything else. If you are going to learn to add, subtract, multiply, and divide rapidly and accurately, **you must know your past records and try to beat them.**

PRESIDENT OF THE COMPANY: The records show a steady improvement in your department this year. You are cutting down the costs of doing business, and yet are turning out more work. I congratulate you.

About once a week during this year, you will be given the chance to run an arithmetic race **against your own past records.** Your score will be the **rating** which you earn. If you will turn to Art. 26, *Self-Testing Drill No. 1*, you will see that there are 20 examples and problems to be worked. You will be given exactly 20 minutes. After you finish, and your papers have been corrected, you will look up your rating in the *Standards* given after the problems and examples. Ratings of 7-10 are very good. Ratings of 4-6 are fair or average, but below 4 are poor.

Now look at the *Progress Chart* on page 33. This shows the record of Robert Grinnell, a sixth-grade pupil in the Elm Place School, for all 29 *Self-Testing Drills* of the sixth grade. Every time that Robert took one of the drills, he recorded his rating on the *Progress Chart.* The chart is in a form called a **graph.** In the back of the book you will find a chart for keeping your own records. **Look at it**

now and make up your mind to keep your record neatly. In order to show that you understand how to keep your record, answer for your teacher the questions below.

QUESTIONS

1. What was Robert's rating on Self-Testing Drill No. 1? On Drill No. 8? Drill No. 20? Drill No. 29?

2. Did he beat his record on Drill 5 when he took Drill 6?

3. Between which drills did he gain 2 points in his rating? Between which did he lose 3 points?

4. Look at Robert's graph from Drills 11 to 14. Do you think he tried very hard here? Are you going to try hard every time?

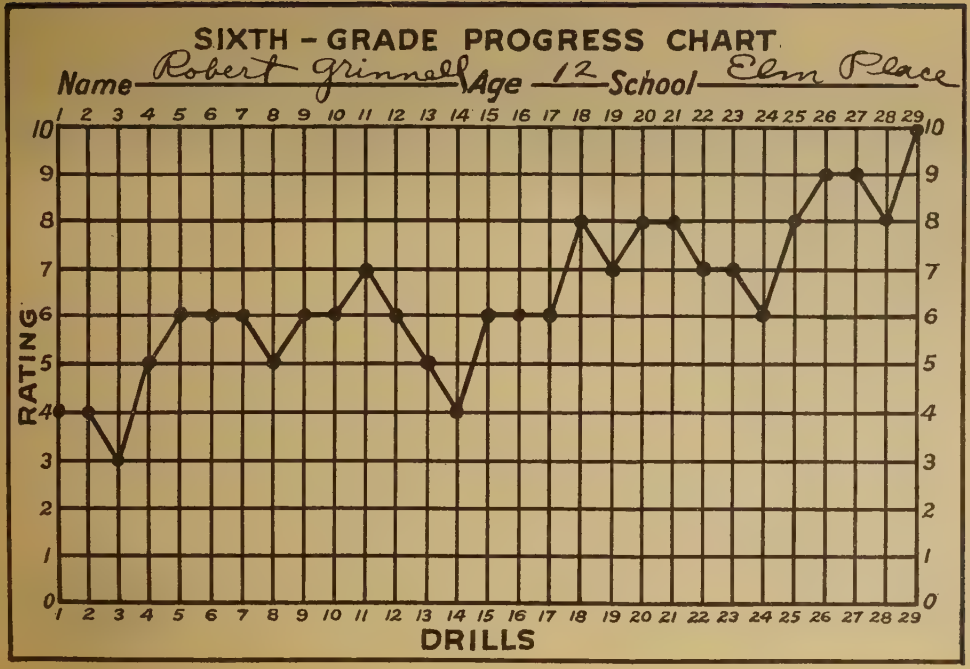
5. Did he improve much during his year in sixth grade?

6. How many examples and problems did Robert have correct on Drill No. 1?

HINT: His rating was 1. Look in the table of standards at the end of Drill 1 (page 35). How many did he need to have correct to earn a rating of 1?

7. Can you explain the standards for Drill 6, page 90?

Now get ready to try Drill No. 1. Read the directions. Be sure that you understand these directions. Wait for the teacher to give the signal "Go!" Work hard and try to make a good record the very first time. After you have finished, your teacher will give you the right answers. Find how many examples you worked correctly, and look up your rating in the standards. Record your rating on your own Progress Chart. Always record your **first** trial on the chart. Be sure to correct examples you get wrong on your first trial. This will help on the next drills.


SIXTH - GRADE PROGRESS CHART
Name Robert Grinnell Age 12 School Elm Place
RATING
DRILLS
0 1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

SELF-TESTING DRILL NO. 1

26. Directions. Wait for the signal "Go!" Wherever possible save copying by placing your paper right below the example. Copy only when necessary. Number examples as you work them.

You will be allowed exactly 20 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order.

Try to make a good rating on your first sixth-grade drill. Rate yourself by the standards at the end of the drill.

1. *Subtract:*

$$\begin{array}{r} 94327 \\ -\ 65234 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 437 \\ 602 \\ 591 \\ 832 \\ +\ 475 \\ \hline \end{array}$$

3.

$$\begin{array}{r} 37.432 \\ -29.008 \\ \hline \end{array}$$

4. $\frac{4}{5} \times \frac{2}{3} = ?$

5.

$$\begin{array}{r} .06 \\ 5.01 \\ .23 \\ 2.96 \\ +\ 5.72 \\ \hline \end{array}$$

6. *Multiply:*

$$\begin{array}{r} 432 \\ \times\ 67 \\ \hline \end{array}$$

7. $\frac{3}{7} \div \frac{9}{14} = ?$

8. $37\overline{)2405}$

9. $18\overline{)2004}$

10.

$$\begin{array}{r} 7528 \\ \times 269 \\ \hline \end{array}$$

11.

$$\begin{array}{r} 6104 \\ \times 3724 \\ \hline \end{array}$$

12. $12.3 + 1.47 + 4.364 = ?$

13. $3\frac{1}{4} - 2\frac{2}{3} = ?$

14. $\frac{7}{18}$ of $5 = ?$

15.

$$\begin{array}{r} 4\frac{1}{2} \\ 2\frac{2}{3} \\ 17\frac{5}{6} \\ +\ 9\frac{3}{4} \\ \hline \end{array}$$

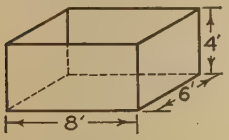
16.

$$\begin{array}{r} 6 \text{ qt.} \\ -\ 3 \text{ qt. } 1 \text{ pt.} \\ \hline \end{array}$$

17. $\frac{6}{24} + \frac{5}{8} = ?$

18. $82\overline{)1435}$

19. A garden plot was 12 ft. long and 4 ft. wide. What was its area?



20. A boy made a box of the same dimensions as those shown in the drawing. Find the volume of the box.

STANDARDS

Number correct	0–4	5	6	7–8	9	10	11	12	13	14–15	16–19	20
Rating	0	1		2	3	4	5	6	7	8	9	10

Correct errors at once. If your rating does not satisfy you, use the Self-Help Index on page 457. Practice on this drill in spare moments.

IMPROVING YOUR WORK

27. Self-Help in Reviewing. (STUDY PERIOD.) A sixth-grade pupil need not wait for his teacher to tell him everything that he is to do. He is old enough to take responsibility for his own work. This article shows you how to take such responsibility for your own progress in arithmetic.

What examples did you find hard in Drill No. 1?

You can easily see that you should practice on the kinds of work that are hardest for you. Such practice will pay you, because, in the Self-Testing Drills to be worked throughout the year, you will make higher ratings if you practice **now** on what is hard for you.

On page 457 you will find the **Pupil's Self-Help Index.** Whenever you find an example or problem that troubles you, use the Self-Help Index. Suppose that you had trouble in working Example 14 in Self-Testing Drill No. 1. Example 14 is on the multiplication of fractions. This

may be a hard subject for you. How can you learn to multiply fractions better?

First: Turn to the Self-Help Index (page 457).

Second: Find "Multiplication of Fractions."

Third: Jot down on a slip of paper all the article numbers listed for this topic.

Fourth: Turn to some or all of these articles. Study them carefully and work a large number of the examples in them.

Remember that the Self-Help Index is a useful reference whenever you need review work.

PROBLEM SOLVING

28. Causes of Mistakes in Problem Solving. (Study Period.) During the sixth grade you will be asked to solve many problems. Some of the problem work has been made so that you can measure or judge your ability to solve problems. Spend a minute or two now looking over Arts. 29 and 30.

Few if any pupils in the sixth grade solve **all** the problems they work correctly the first time they try them. Pupils are not always perfect, you know. But some pupils are **so careless** that they make **many more mistakes** than they need to make. If you get clearly in mind the main causes for mistakes in problem solving, you will know what things to avoid when you are working problems yourself. You can think of the next paragraphs as some of the most important ones for you to study this year. In fact, there are no paragraphs in this book which are more important for you to understand thoroughly.

Teachers who have made a careful study of sixth-grade pupils and their work in arithmetic have found four im-

portant facts about mistakes that pupils make. Think over each fact carefully as you study it.

Fact One. **Careless reading of a problem causes 3 out of every 10 mistakes in problem solving.**

Every time you begin to solve an arithmetic problem, you have to use your ability to read carefully and accurately. In fact, the best way to keep down mistakes in problem solving is to make sure that you read the problem so carefully that you know exactly what you are asked to do and just what facts you need to use.

You have spent a great deal of time in school learning how to read. Solving arithmetic problems gives you a good chance to use your ability to read. Is it fair for every sixth-grade teacher to expect her pupils to read all arithmetic problems carefully? What does Fact One say about causes of mistakes in problem solving?

Fact Two. **Careless mistakes in figuring are responsible for 2 out of every 10 wrong answers to problems.**

You know that you will not get the correct answer to the example 49×345 if you think that 9×5 makes 47. In solving a problem in arithmetic, you must do the actual figuring accurately if you are to get correct answers.

You have spent much time learning how to add, subtract, multiply, and divide accurately. During the year you will be taking many oral drills and Self-Testing Drills so that you will not forget how to work with numbers. You must use your ability to work with numbers every time you solve a problem. Would you blame a teacher for being disappointed in a pupil who did drill work well but who made careless mistakes in figuring when solving problems? What does Fact Two tell you about causes of mistakes in problem solving?

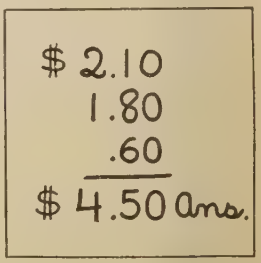
Fact Three. **Two out of every 10 mistakes in problem solving are caused by a pupil doing part of a problem correctly and then forgetting to finish the work.**

Here is an example of forgetting to finish the work or failing to get the answer called for:

John and Benjamin were partners. They worked together and divided their earnings equally. They earned the following amounts by cutting lawns.

From Mrs. Wagner, $2.10
From Dr. Albright, 1.80
From Mr. Beveridge, .60

How much did each boy earn?



Read the problem again and see if the pupil whose work is shown was really through or if he made a mistake by not doing everything that was asked for.

Do you think that Fact Three is so very hard to remember that sixth-grade pupils should be "excused" if they forget to do everything a problem asks for?

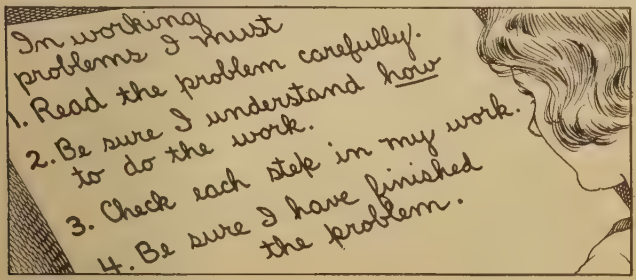
Fact Four. **One mistake out of every 10 is caused by a pupil simply not knowing how to do some work which he has had a chance to master.**

Suppose a problem can be correctly solved by working this example: $6\overline{)11 \text{ ft. } 10 \text{ in.}}$ If a pupil does not know how to divide denominate numbers, he will not be just lucky and guess the correct answer.

You are never asked to work a problem until you have been taught how to do the figuring which must be done. Of course, if you trust to luck and do not really master the articles which are labelled New Work, you will have

no one to blame but yourself if you find problem solving too hard for you. The Self-Help Index is made so that pupils who have their wits about them can restudy any topic which they may need in order to solve a problem which troubles them.

You may not be able to solve **every** problem in this book correctly, but you are expected to solve most of them, and **you can** if you will watch out for the causes of mistakes


In working problems I must
1. Read the problem carefully.
2. Be sure I understand how to do the work.
3. Check each step in my work.
4. Be sure I have finished the problem.

which are explained by the Four Facts. The picture above shows how one sixth-grade girl took advantage of this discussion on problem solving when she started work on Problem Scale No. 1 on page 40.

THE PROBLEM SCALES

29. Measuring Your Ability to Solve Problems. (SILENT STUDY.) In your sixth-grade work you are going to take twenty-one **Problem Scales.** The next article is **Problem Scale No. I.** Look at Art. 30 right now. Notice the **standards** on page 41. Notice also that each problem or part of a problem has a number printed at its right. This number tells you how many points you will earn if you solve that problem or part of a problem correctly.

In Problem Scale No. I you can earn 40 points by working every problem correctly.

How many points must you earn to do **Superior** work? How many points to do **Average** work? Do you think you can earn enough points to rate as doing **Good** work?

PROBLEM SCALE NO. I

30. Measuring Your Problem-Solving Ability. (WRITTEN WORK.) You can earn 40 points by working every problem correctly. Find your score by adding the numbers that are printed at the right of each problem you worked correctly.

Rate yourself by the standards given at the end of this article.

1. If I multiply a number by 5, I get 20 as the product. What is the number? (2)

2. A man weighed 185 lb. with his winter overcoat on, and 170 lb. without his overcoat. How much did the overcoat weigh? (2)

3. How many inches equal $1\frac{1}{2}$ ft.? (3)

4. The Overland Express was due at 2:15 P. M. It was 3 hours late. At what time did it arrive? (4)

5. At a school party 12 sandwiches were to be allowed to every 4 people. How many sandwiches would be needed for 12 people? (4)

6. If a train goes at the average rate of 38 miles an hour, how far will it go in $3\frac{1}{2}$ hr.? (4)

7. How many quarts are there in $2\frac{1}{4}$ gallons? (4)

8. Mrs. Brown buys soap at 4 bars for 25¢. How much do 24 bars cost? (5)

9. If $\frac{1}{20}$ of milk is cream, how many quarts of cream will there be in 5 gallons of milk? (5)

10. Henry Smith bought some hogs for $200 and sold them for $250. He made $5 on each hog. How many hogs did he buy? (7)

STANDARDS

Superior work:	34–40 points	Slow work:	11–19 points
Good work:	29–33 points	Inferior work:	0–10 points
Average work:	20–28 points		

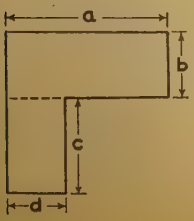
1. Should mistakes in problem solving be corrected?

2. Do you read problems carefully before working them?

3. What standard in problem solving have you set as your goal for the year?

INVENTORY OF IMPORTANT IDEAS

31. General Review. (Study Period.) Study this exercise silently. You will find it pretty easy if you did good work on Chapter I. Be ready to give correct answers.



1. How would you find the area of this floor plan? Suppose you knew only the dimensions which are lettered.

2. Name three units of measure which are used in expressing area.

3. Which of these numbers can be divided exactly by 2: 11, 24, 8, 41, 126, 33, 58?

4. The inside dimensions of a safe are $2' \times 4' \times 4'$. What is the volume in cubic inches?

5. How many hundredths are there in the number 2489.0143? How many tenths? How many units or ones? How many thousandths?

6. *Give the differences:*

(*a*)	(*b*)	(*c*)	(*d*)
24.7	340.12	.384	81.93
3.2	1.47	.007	62.47

7. The fraction $\frac{20}{4}$ would be changed to what form in an answer?

8. Which of these numbers are denominate numbers:

4.17 24 T. 16 doz. $16\frac{2}{3}$ 32 hr.?

9. What sort of work is shown here? →

$$\frac{\overset{1}{\cancel{2}}}{\cancel{3}} \times \frac{\cancel{3}}{7}$$
$$1$$

10. In solving, $\frac{2}{3} \div \frac{4}{7}$ would be changed to $\frac{2}{3} \times \frac{7}{4}$. The rule used is: "To divide by a fraction, you multiply by its....."

11. Here are six answers to problems. If any are not in the simplest form, reduce them. (*a*) 13 in. (*b*) 168¢ (*c*) $\frac{7}{4}$ (*d*) $\frac{1}{2}$ mi. (*e*) $\frac{9}{12}$ (*f*) 2 qt. 3 pt. (*g*) 1 ft. 6 in.

12. What common denominator would you use in adding $\frac{}{2}+\frac{}{3}+\frac{}{6}$? In subtracting $\frac{}{2}$ from $\frac{}{14}$? In adding $\frac{}{2}+\frac{}{6}+\frac{}{3}+\frac{}{4}$? In adding $\frac{}{8}+\frac{}{2}+\frac{}{16}+\frac{}{4}$?

13. What is the first thing you would do in working this example: $4\frac{1}{2} \times 3\frac{2}{5}$?

14. What is wrong in the answer? $4.17 + 3.18 = 735$.

15. Be ready to give the answer to each of these:

(*a*)	$16-9$	(*e*)	$6\overline{)52}$	(*i*)	$13-9$	(*m*)	$14+9$	(*q*)	5×0
(*b*)	$6+0$	(*f*)	$11-9$	(*j*)	$28+4$	(*n*)	$7\overline{)54}$	(*r*)	$24+7$
(*c*)	$8\overline{)30}$	(*g*)	$9-0$	(*k*)	$9\overline{)26}$	(*o*)	$14-6$	(*s*)	$13+6$
(*d*)	$18-9$	(*h*)	$13-4$	(*l*)	$7\overline{)53}$	(*p*)	$55+6$	(*t*)	$9\overline{)40}$

16. Change the forms of the fractions below to the forms having the given denominators.

$\frac{1}{2}=\frac{\ }{8}$ $\frac{2}{3}=\frac{\ }{12}$ $\frac{3}{5}=\frac{\ }{15}$ $\frac{3}{4}=\frac{\ }{16}$ $1=\frac{\ }{5}$

17. Write and work an example showing that multiplying both terms of a fraction by the same number does not change the value of the fraction.

18. Write and work an example showing that dividing both terms of a fraction by the same number does not change the value of the fraction.

19. (*a*) Write 5 numbers that can be divided by 2 without remainder; (*b*) 5 other numbers that can be divided by 3 without remainder; (*c*) 5 other numbers that can be divided exactly by 5. (Each number should have more than one figure.)

20. Write the reciprocals of: $\frac{2}{3}$, $\frac{4}{7}$, $\frac{5}{2}$, 4, 6, $\frac{1}{5}$.

21. Write these decimals as fractions: .25 .5 .75 .4

22. What belongs on each blank below?

(*a*) $\frac{1}{2}$ lb. equals.....oz. (*d*) 4 yd. equal.....ft.
(*b*) 6 pt. equal.....qt. (*e*) 9 sq. ft. equal.....sq. yd.
(*c*) 1 mi. equals.....ft. (*f*) 476¢ equal $.....

23. Find the average of these numbers: $248+508+721$.

24. Find the difference: 42.6 minus 17.03.

25. Find in Art. 26 some numbers which have these names: multiplicand, divisor, dividend, fraction, mixed number, denominate number, multiplier.

26. Which examples in Art. 26 had answers named as follows: difference, product, sum, quotient?

27. Give the place value of each figure in 72493.016.

28. How could you check Examples 1, 2, 8, and 11 in Art. 26?

➡If any of the work in this article bothered you, remember the Self-Help Index on page 457.

MULTIPLICATION OF DENOMINATE NUMBERS

32. Multiplying Denominate Numbers. (STUDY PERIOD.) The three samples in this article show you how to work the examples in Article 33. These samples explain a good way to multiply denominate numbers. When you have to multiply hours and minutes or pounds and ounces or feet and inches, it is a good plan to do the work like the samples.

SAMPLE I. A carpenter needed 6 boards, each 5 ft. 9 in. long. Find the total length of the boards needed.

Multiply:

5 ft. 9 in.
×6

4	6
30	

34 ft. 6 in., *Ans.*

(*a*) Think 6×9 in. as 54 in.; then reduce to 4 ft. 6 in.

Thus:

$$12\overline{)54}\text{ in.} = 4\text{ ft. }6\text{ in.}$$

48

6

(*b*) Write the 4 ft. 6 in. as in the sample.

(*c*) Multiply the 5 ft. by 6 as done in the sample.

(*d*) Add to get the product (answer).

SAMPLE II. How much maple syrup could a Vermont farmer put in 10 cans, if each can holds 3 gal. 3 qt.?

3 gal.	3 qt.
	10
7	2
30	
37 gal.	2 qt., *Ans.*

(*a*) Think 30 qt. in reduced form as 7 gal. 2 qt.
(*b*) Write 7 gal. 2 qt. as in the sample.
(*c*) Multiply the 3 gal. as shown in the sample.
(*d*) Add to get the product as shown.

SAMPLE III. A delivery man for the Fair Grocery Store averages 2 hr. 45 min. a trip. He makes three trips a day. How much time does he average per day on his deliveries?

2 hr.	45 min.
	3
2	15
6	
8 hr.	15 min. *Ans.*

1. The 2 and the 15 shown in the work stand for 2 hr. and 15 min. How were these figures gotten?

2. Why would it have been careless to do the work like this? ⟶

2 hr.	45 min.
	3
6 hr.	135 min. *Ans.*

3. In what way would the practice in Article 306 help you in multiplying denominate numbers?

The one "hard spot" in multiplying denominate numbers is reducing numbers to good forms for answers.

➡In multiplying denominate numbers, we often have to reduce amounts. Do you know all the facts printed in the Table of Measures on page 456? You have to use these facts very frequently.

33. Practice in Multiplying Denominate Numbers. (WRITTEN WORK.) Every sixth-grade pupil should be able to work the exercise below without making any mistakes.

EXERCISE

Find the products:

1.	2.	3.	4.
4 ft. 7 in.	4 gal. 1 qt.	6 lb. 11 oz.	4 pk. 2 qt.
7	8	5	3
5.	**6.**	**7.**	**8.**
3 hr. 15 min.	10 ft. 4 in.	1 gal. 3 qt.	4 lb. 2 oz.
4	8	8	9
9.	**10.**	**11.**	**12.**
4 min. 30 sec.	3 qt. 1 pt.	4 bu. 2 pk.	4 yd. $2\frac{1}{2}$ ft.
7	12	9	6

34. Problems Using Multiplication of Denominate Numbers. (WRITTEN WORK.) The problems below use the "New Work" that you have learned in Articles 32 and 33.

1. If 45 min. a day is spent upon arithmetic, how much time is spent on arithmetic per school week (5 da.)?

2. If 3 hr. and 45 min. is spent upon arithmetic each week, how much time is spent on arithmetic during a school year of 36 weeks?

3. Dorothy is making some towels. She needs 3 ft. 4 in. of linen for each. If she makes 4 towels, how much linen will she need in all?

4. Mr. Adams lives in Danvers, Mass., and works in Boston. He spends 1 hr. 15 min. on the train going to his

work every day. It takes him the same time to get home. How much time does he spend on the train going to and coming from work each week? He works 6 da. a week.

5. How much time does Mr. Adams spend on the train in a year going to and from his work if he works 50 weeks a year?

6. Bob Brown lives on a farm. He helps his father do the chores every night. When Bob bought a watch, he liked to see how long his work took him. He found that he spent on the average 1 hr. 10 min. helping his father every night. How much time did he spend doing the chores each week (7 da.)?

7. In building a back-stop for their baseball field the Booster's Club of the Washington School found that 8 strips of chicken wire each 7 ft. 9 in. long were needed. What was the total length of the chicken wire needed?

8. In a certain lumber camp the cook estimates 3 pk. 3 qt. of potatoes will be used each day. How many should he buy for a month's supply (30 da.)?

9. Mr. Holmes ships daily to Omaha 8 gal. 3 qt. of milk. How much does he ship in a month (30 da.)?

10. Every time Mr. Wood changes the oil in his automobile, he puts in 7 qt. of fresh oil. One year he changed oil 12 times. How much fresh oil did he put in? (Express answer in reduced form.)

35. Rapid Drill. (ORAL WORK.) Work these examples without a pencil. In quotients give only the first figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	140 +146	$65\overline{)394}$	8+7+10	Why use Self-Help Index?	$58\overline{)527}$	28+6+16
2.	114 −98	*Reduce:* $\frac{15}{6}$	$96\overline{)868}$	8+18+6	112 −76	$98\overline{)590}$
3.	$54\overline{)378}$	$58\overline{)464}$	3+5+7	$\frac{1}{4} \div \frac{7}{8} = \frac{1}{4} \times \frac{?}{?}$	164 −89	69 ×8
4.	9+25+13	*Subtract:* .21 from 4.7	137 −98	$98\overline{)500}$	4+2+5	125 −87
5.	121 −67	$94\overline{)761}$	135 −47	107 −89	19+8+9	122 −45
6.	17×0	17+0	17−0	*What change needed?* 4 ft. 18 in. *Ans.*	$6 \div \frac{1}{3}$	146 −87
7.	$\frac{1}{5}$ of 20	3′ −2′ 8″	$\frac{1}{2}+\frac{1}{3}$	9 in. ×6	$4\frac{1}{8} = \frac{}{8}$	8 $-3\frac{2}{5}$
8.	16+3+3	148 −99	$\frac{2}{5} \times \frac{2}{3}$	8+6+6	*Read:* 4.78	$69\overline{)485}$

DIVISION OF DENOMINATE NUMBERS

36. Dividing Denominate Numbers. (Study Period.) Thus far our problems with hours and minutes, gallons and quarts, feet and inches, pounds and ounces, and tons and pounds have been addition, subtraction, or multiplication problems. Sometimes we need to be able to **divide** denominate numbers.

The next work shows **how to divide denominate numbers.**

Sample I. Johnny Wilson has to study 6 hr. 15 min. at home each week. If he studies an equal amount of time each school night, how much time will he study each night?

1 hr. 15 min. *Ans.* 5)6 hr. 15 min.

Think of the 1 hr. (left over from 6 hr. divided by 5) as 60 min. Then add: 60 min. + 15 min. = 75 min. Divide: 75 min. ÷ 5 = 15 min.

Sample II. Three boys put their money together and buy 10 ft. 4 in. of tape for their hockey sticks. They divide the tape equally. How much tape does each boy have?

3 ft. $5\frac{1}{3}$ in. *Ans.* 3)10 ft. 4 in.

Think of the 1 ft. (left over) as 12 in. 12 in. + 4 in. = 16 in. 16 in. ÷ 3 = $5\frac{1}{3}$ in.

Sample III. Mary, Virginia, Elsie, and Ruth were putting candy in small bags for a party. They divided the candy equally so that each one would do her share of the work. The candy weighed 5 lb. 12 oz. How much candy did each of the girls have to put in small bags?

1 lb. 7 oz. *Ans.* 4)5 lb. 12 oz.

1. What is left over in 5 lb. ÷ 4?
2. One pound equals how many ounces?
3. How is the 7 oz. in the answer gotten?
4. Is the hint on page 50 clear to you?

HINT ON THE "HARD SPOT" IN DIVISION

In dividing numbers such as $4\overline{)316}$ or $7\overline{)2249}$ the "carrying" is very easy. Thus:

$\begin{array}{r} 7 \\ 4\overline{)316} \end{array}$ 4 into 31 goes 7 times, with 3 to carry. You carry 3 to the 6 to make 36.

$\begin{array}{r} 3 \\ 7\overline{)2249} \end{array}$ 7 into 22 goes 3 times, with 1 to carry. You carry 1 to the 4 to make 14.

In dividing denominate numbers there is a "hard spot" in the carrying which you must watch out for. Thus:

$\begin{array}{r} 2 \text{ ft.} \\ 4\overline{)9 \text{ ft. } 10 \text{ in.}} \end{array}$ The **1** ft. left over becomes **12 in.** 12 in. + 10 in. = 22 in., to be divided by 4.

$\begin{array}{r} 1 \text{ lb.} \\ 6\overline{)8 \text{ lb. } 7 \text{ oz.}} \end{array}$ The 2 lb. left over becomes **32 oz.** 32 oz. + 7 oz. = 39 oz., to be divided by 6.

$\begin{array}{r} 3 \text{ bu.} \\ 3\overline{)11 \text{ bu. } 2 \text{ pk.}} \end{array}$ The 2 bu. left over becomes **8 pk.** 8 pk. + 2 pk. = 10 pk., to be divided by 3.

In "carrying" in division of denominate numbers, care must be taken to change numbers correctly.

PRACTICE ON THE "HARD SPOT" IN CARRYING

To what numbers will the remainders in the next divisions be changed?

1. $4\overline{)6 \text{ ft. } 3 \text{ in.}}$ (2 ft. remainder becomes 24 in. 24 in. + 3 in. = 27 in.)

2. $8\overline{)9 \text{ yd. } 2 \text{ ft.}}$
3. $6\overline{)9 \text{ lb. } 2 \text{ oz.}}$
4. $5\overline{)8 \text{ hr. } 30 \text{ min.}}$
5. $3\overline{)7 \text{ qt. } 2 \text{ pt.}}$
6. $4\overline{)4 \text{ ft. } 8 \text{ in.}}$
7. $2\overline{)3 \text{ gal. } 2 \text{ qt.}}$
8. $4\overline{)7 \text{ pk. } 4 \text{ qt.}}$
9. $8\overline{)10 \text{ sq. yd. } 2 \text{ sq. ft.}}$
10. $7\overline{)15 \text{ T. } 400 \text{ lb.}}$

➡It saves work to know the Table of Measures on page 456.

37. Practice in Dividing Denominate Numbers. (WRITTEN WORK.) If you have any trouble with the exercise below, restudy Article 36.

Divide each of the following by 5.

1. 6 ft. 10 in.	**3.** 1 pk. 2 qt.	**5.** 7 gal. 2 qt.
2. 8 lb. 2 oz.	**4.** 5 T.	**6.** 7 hr. 15 min.

Find the answers to the next 6 examples.

7. 6 gal. 1 qt. ÷ 3	**9.** 8 lb. ÷ 6	**11.** 6 hr. 21 min. ÷ 3
8. 4)9 ft. 5 in.	**10.** 8)9 pk. 4 qt.	**12.** 7 bu. 3 pk. ÷ 4

PROBLEM SCALE NO. II

38. Problems Using Denominate Numbers. (WRITTEN WORK.) You can earn 85 points by working every problem correctly. Find your score by adding the values printed at the right of each problem you worked correctly. Rate yourself by the standards given at the end of this article.

1. Mrs. Walker has 6 pk. 4 qt. of peaches. She plans to give one-fourth of them to a neighbor. How much does she plan to give to her neighbor? (7)

2. A salesman spent 3 hr. 35 min. selling goods to 7 customers. How much time did he average to a customer? (8)

3. Mr. Williams used 5 gal. 3 qt. of oil in 4 months. How much did he average a month? (8)

4. Sidney fed 4 gal. 3 qt. of milk to his young pigs in 6 days. How much did he feed them per day? (8)

5. Mary used 6 lb. 10 oz. of flour in making 4 cakes. She used the same amount for each cake. How much flour did she use in each cake? (8)

6. The creamery bought 54 gal. 2 qt. of milk in 5 days. What was the average amount bought per day? (8)

7. One year Mr. White used 7 T. 600 lb. of coal in his furnace. The second year he used 8 T. 1500 lb. The third year he used 6 T. 700 lb. How much coal did he use per year on the average? (9)

8. In learning to use the yardstick Alice measured the height of 4 of her schoolmates. Henry was 5 ft. 2 in. tall; Mildred, 4 ft. 9 in.; James, 4 ft. 11 in.; and Ruth, 4 ft. 7 in. What was their average height? (9)

9. The Robert Fulton School garden raised 6 pk. 5 qt. of radishes in the spring of 1927. Ten children had radish beds. How much did each child raise on the average? (10)

10. One week I studied: 1 hr. 45 min. on Monday night; 50 minutes on Tuesday night; 1 hr. 10 min. on Wednesday night; 45 min. on Thursday night; and 1 hr. on Friday night. What was the average length of my evening study periods that week? (10)

STANDARDS

Superior work:	16–85 points	Average work:	7–8 points
Good work:	9–15 points	Slow work:	0 points

SELF-TESTING DRILL NO. 2

39. Directions. Wait for the signal "Go!" Wherever possible save copying by placing your paper right below an example. Copy only when necessary. Number examples as you work them.

You will be allowed exactly 20 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order.

Try to beat your record on Drill 1. Rate yourself by the standards at the end of the drill.

1. 214×5

2. $12\overline{)264}$

3. $64\overline{)768}$

4. $\frac{4}{7} - \frac{3}{7} = ?$

5. $14409 - 5642$

6. $89176 - 24589$

7. $43.7 + 2.8 + 5.1 + 92.6$

8. $74\overline{)1184}$

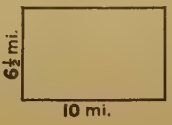
9. $2 \times \frac{1}{3} = ?$

10. 195×69

11. $\frac{1}{14} \times 3 = ?$

12. $169 + 598 + 798 + 529 + 699 + 538$

13. Find the area represented by this figure:



14. 864×628

15. $2 \div \frac{1}{2} = ?$

16. What will $3\frac{3}{5}$ yd. of cloth cost at $\$2\frac{3}{4}$ per yard?

Go on to the next page.

17. A man owned $\frac{9}{10}$ of an acre of land. He sold $\frac{2}{5}$ of an acre. How much did he have left?

18. *Write in numbers:* twelve thousand six and one tenth.

19.

$$\begin{array}{r} 3 \text{ ft. } 2 \text{ in.} \\ -1 \text{ ft. } 9 \text{ in.} \\ \hline \end{array}$$

20. *Write as one number:*

5 hundreds
6 tens
7 units
9 tenths

STANDARDS

Number correct	0–7	8–9	10	11	12	13	14	15	16	17–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

If your rating does not satisfy you, use the Self-Help Index on page 457, and practice on this drill in your spare time.

MULTIPLICATION OF DECIMALS

40. Problems Using Decimals. (STUDY PERIOD.) In this article are 3 problems which explain some new work that every sixth-grade pupil should understand. Following each problem are some questions. Try to study each problem so well that you can answer the questions correctly.

Problem 1. William's father has a farm which is in the form of a rectangle. It is 2.27 miles long and 1.3 miles wide.

1. How can we find the area of a rectangle?

2. If the unit of measure of the length and width of the farm is the mile, what is the unit of measure of the area?

3. Can the area of the farm in Problem 1 be **more than** 6 sq. mi.? (1.3 mi. is less than 2 mi; 2.27 mi. is less than 3 mi.)

4. Can the area be **less than** 2 sq. mi.? (1.3 mi. is more than 1 mi.; 2.27 mi. is more than 2 mi.)

Here is the way to do Problem 1 correctly.

2.27 mi.
$\times$1.3 mi.
681
227
2.951 sq. mi., *Ans.*

2.27 mi. $\times$1.3 mi. = ?

Look over the multiplication to see if it is correct.

Notice the decimal point in the answer.

5. Why would it be unreasonable for you to think that the answer might be 29.51 sq. mi. instead of 2.951 sq. mi.?

6. Why would it be unreasonable for you to think that the answer might be .2951 sq. mi. instead of 2.951 sq. mi.?

7. Why must the decimal point in the answer be where it is?

8. Count the decimal places in the product. Count and add the decimal places in the multiplier **and** the multiplicand. What fact about decimal places in multiplication is shown?

9. What new thing are you learning in this article?

Try Problem 2. **Remember the decimal point in the answer.**

Problem 2. Young chickens cost \$.78 each. I bought 6 of them. How much did they all cost?

1. How many decimal places are there in the multiplicand and the multiplier together?

2. How many decimal places are there in the answer?

3. What fact do you notice about the number of decimal places in the answer and the **sum** of the decimal places in the multiplicand and multiplier?

4.7
2.8
376
94
13.16

PROBLEM 3. Find the product of 2.8×4.7.

1. Can the answer be more than 15? Why?
2. Can the answer be less than 8? Why?
3. Is 13.16 a **reasonable** answer?
4. Is the decimal point in the product placed correctly?
5. How many decimal places are there in the product?
6. How many decimal places are there in the multiplier and the multiplicand together?

Multiplying by decimals is just like multiplying by whole numbers **except that we must know where to put the decimal point in the product.**

> *Rule:* **There are as many decimal places in the product as there are in the multiplier and multiplicand together.**

Study the two examples below. The rule is used for pointing off the products. Putting the decimal point in the right place is called **"pointing off."**

1. 4.27 (2 decimal places)
   6.34 (2 decimal places)
   1708 (4, sum of decimal places)
   1281
   2562
   27.0718 (4 decimal places)

2. 3.26 (2 decimal places)
   7.6 (1 decimal place)
   1956 (3, sum of decimal places)
   2282
   24.776 (3 decimal places in the product)

➡ Read the rule again. Did the rule work in the examples above? In what way is the multiplication of decimals like the multiplication of whole numbers? In what way is it different from the multiplication of whole numbers?

41. Practice on Placing the Point in Multiplication. (ORAL WORK.) The answers to the examples in this article are correct **except that the decimal points are missing.** Where there is no decimal point in the multiplicand or multiplier, we know the answer is a whole number. Numbers like 35 and 35. and 35.0 and 35.00 are the same. Tell where the decimal point in each product should be.

The first right answer is: "I point off two places **from the right.** The decimal point is between the 0 and 7. The product is one, one, zero, point, seven, two." Use the rule on page 56 if you need to.

EXERCISE

1.	34.6	**2.**	42.	**3.**	341.07	**4.**	43.9
	3.2		.69		.01		14.8
	11072		2898		34107		64972
5.	.97	**6.**	424	**7.**	.324	**8.**	4.541
	.6		730		2.7		24
	582		309520		8748		108984
9.	7232	**10.**	24.78	**11.**	7.64	**12.**	425
	.142		.379		32		73.2
	1026944		939162		24448		311100
13.	5.321	**14.**	436	**15.**	.726	**16.**	6.341
	6.409		593		2.8		6.43
	34102289		258548		20328		4077263
17.	2.37	**18.**	34.6	**19.**	5.319	**20.**	40.2
	.671		.952		904		.5
	159027		329392		4808376		2010

21.	9.24	22.	.672	23.	799.2	24.	8.26
	39		.398		.43		.7
	36036		267456		343656		5782
25.	942	26.	.345	27.	456	28.	136
	7.6		1.24		.331		.05
	71592		42780		150936		680

42. Thinking about the Multiplication of Decimals. (Silent Reading.) Study for 5 minutes so that you can give the correct answers promptly when the recitation begins.

1. In a certain problem the decimal part of the multiplicand is .7. The decimal part of the multiplier is .42. How many decimal places are there in the product?

2. In another problem the multiplicand is a whole number and the multiplier is 42.732. How many decimal places would you point off in the product?

3. If the multiplicand is .5 and the multiplier is 5.18, how many decimal places will there be in the product?

Think what ought to be on each blank in the next three problems. Do not mark in your book.

4. Multiplicand has tenths only. Multiplier is a whole number. The product has ..... decimal place(s).

5. Multiplier is 42.734. Multiplicand is 3.07. There are ..... decimal places in the product.

6. Product had four decimal places. Multiplier had one decimal place. There must have been ..... decimal places in the multiplicand.

7. Product had no decimal places. Multiplicand had no decimal places. Was the multiplier a whole number?

8. Tell, without multiplying, the number of decimal places in the product of each multiplication example below.

(*a*) 423.7 3241.	(*b*) 4237. .3241	(*c*) 42.37 32.41	(*d*) 423.7 .3241	(*e*) .4237 3.241
(*f*) 4237. 324.1	(*g*) 423.7 324.1	(*h*) 42.37 324.1	(*i*) 4.237 32.41	(*j*) 42.37 3241.
(*k*) 4237. 32.41	(*l*) 4.237 3.241	(*m*) 423.7 32.41	(*n*) .4237 324.1	(*o*) 423.7 3.241

Remember: **The number of decimal places to point off in the product is the number of places in the multiplicand added to the number of places in the multiplier. In the product begin counting decimal places from the right.**

43. Practice in Multiplication of Decimals. (WRITTEN WORK.) This exercise will give you good review in placing decimal points in answers.

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
8.74	187	49.5	16.3	8.60	820	.270
.5	.10	203	3.2	.78	.2	3
8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
690	1.93	.435	218	.206	73.4	.808
10	.89	.41	.146	.654	.76	.9
15.	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
87.9	986	.872	754	5.19	725	6.23
.342	.9	.78	.98	.657	36	90

PROBLEM SCALE NO. III

44. Problems Using Decimals. (WRITTEN WORK.) You can earn 85 points by working every problem correctly. Find your score by adding the numbers printed at the right of each problem you worked correctly.

Rate yourself by the standards given at the end of the article.

1. A lion in the circus eats 12.5 lb. of meat a day. How many pounds will he eat in 46 days? (4)

2. Tom makes $4.85 a week delivering papers. How much will he make in 4 months, counting 4 weeks to each month? (4)

3. A baker makes 1875 loaves of bread a day. How many pounds of salt will he use a day if he uses .008 lb. of salt for each loaf of bread? (5)

4. A hotel in Chicago serves 2500 cups of coffee every day. How many pounds of coffee are used each day if .03 of a pound is allowed per cup? (5)

5. A big restaurant figured that each customer used .114 lb. of sugar a meal. How much sugar will be needed at this rate to serve meals to 285 customers? (5)

6. Mr. Russell bought 9.4 T. of coal costing $9.75 a T. What should be the amount of Mr. Russell's bill? (5)

7. A rectangular room is 12.75 ft. long, and 10.5 ft. wide. How many square feet of floor space are there in the room? (6)

8. An average man eats .225 lb. of sugar per day. If sugar costs 9.5¢ a pound, how many cents' worth of sugar does he eat per day? (6)

9. A hotel bought coffee at 40¢ a pound. At this rate hat is the cost of the coffee in one cup if .03 lb. of coffee used per cup? (6)

10. A storekeeper bought 425 doz. marbles at 4.2¢ a doz. ow much did they cost him? (6)

11. A paper dollar bill weighs .003 lb. The paper of hich the bill is made costs $2.436 a lb. Find the cost the paper for the bill. (6)

12. Mary made in sewing class 6 dish towels for her other. The material cost $.235 a yard. Each towel was ft. long. How much did the material cost her? (6)

13. Mr. Smith, a baker, uses .75 lb. of flour in each loaf bread. He buys flour at 2.84¢ pound. How much does the our in each loaf of bread ost? (6)

14. Electricity costs 14.5¢ a ilowatt-hour. How much will rs. Jones's electric light bill for April if she burns 10.7 ilowatt-hours of electricity? (7)

15. In a city post office mps are canceled by a machine at the average rate of sec. to each letter. At this rate, how many minutes ll it take to cancel 2478 letters? (8)

STANDARDS

Superior work:	45–85 points	Slow work:	10–17 points
Good work:	31–44 points	Inferior work:	0– 9 points
Average work:	18–30 points		

SELF-TESTING DRILL NO. 3

45. Directions. Wait for the signal "Go!" Wherever possible save copying by placing your paper right below an example. Number examples as you work them.

Time allowed: 20 minutes. Work as fast as you can without mistakes. Do the examples and problems in order. Rate yourself by the standards at the end of the drill.

Is it fair to have Example 8 use decimals?

1. $51\overline{)1122}$

2.
526
540
124

3.
348
$\times 70$

4. $14.7 - 6.3 = ?$

5.
14332
-5729

6.
709
$\times 603$

7.
521
985
400

8.
228
239
548
139
747
529

9.
12.4
$\times .119$

10. $\frac{4}{7} \div 3 = ?$

11. A factory employing 64 workmen has a weekly pay roll of \$2048. What is the average pay of the workmen?

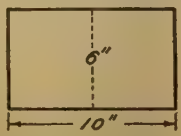
12. A field is $\frac{1}{2}$ mile long and $\frac{1}{3}$ mile wide. Express as a fraction of a mile the difference of its dimensions.

13. $\frac{1}{3} \div 2 = ?$

14. $\frac{9}{10} + \frac{1}{6} = ?$

15. *Add:*
3 qt. 1 pt.
2 qt.
1 qt. 1 pt.
3 qt. 1 pt.

16. Find the area of the rectangle.



17. $\frac{7}{8} - \frac{1}{16} = ?$

Go on to the next page.

18. Express $2\frac{1}{2}$ rods in feet and inches.

19. Find the volume of a box $3' \times 12'' \times 14''$.

20. What is the reciprocal of $\frac{5}{11}$?

STANDARDS

Number correct	0–5	6	7–8	9	10	11	12	13	14	15–19	20
Rating	0	1	2	3	4	5	6	7	8	9	10

Correct all errors at once. If you need practice, use the Self-Help Index on page 457 to find the kind of work you need.

MULTIPLYING BY 10, 100, AND 1000

46. Multiplying by 10. (Class Work.) There is a quick way of multiplying by 10. If we learn this quick way well, it will save time and trouble all through our arithmetic and through our mathematics in high school.

Study the example, $7.3 \times 10 = 73$.

Is 7.3×10 the same as 73.? Notice that only **the position of the decimal point** is changed when we multiply by 10.

$$\begin{array}{r} 7.3 \\ \times 10 \\ \hline 00 \\ 73 \\ \hline 73.0 \text{ or } 73. \end{array}$$

Look at this example: $1.85 \times 10 = 18.5$.

In both of these examples, can multiplication by 10 be thought of as: **moving the decimal point one place to the right?**

How would you multiply 7 by 10?

$7 \times 10 = 70$. Where is the decimal point in 7? Does 7 mean the same as 7.? Think 7×10 as follows: Move decimal point one place to the right. 7. becomes 7_. There is no number in units' place; so we use 0 to fill the vacant place and write 70.

Look at the example 68×10.

Is this answer the same as we would get by the following quick way? To multiply by 10, move decimal point one place to right (68 or 68. becomes 68_.). Put in zero after the 8 (68_. becomes 680.).

$$\begin{array}{r} 68 \\ \times 10 \\ \hline 680, \textit{Ans.} \end{array}$$

EXERCISE

1. Multiply each of these numbers by 10 **mentally:** 6, 10, 4.2, 73, 14.2, 3.05, 9, 18, 6.9, 94, 20, .671.

2. Give the following products mentally:

$2 \times 10 =$	$.17 \times 10 =$	$24 \times 10 =$	$21.5 \times 10 =$
$20 \times 10 =$	$1.7 \times 10 =$	$6.31 \times 10 =$	$1.78 \times 10 =$
$200 \times 10 =$	$17 \times 10 =$	$1 \times 10 =$	$17.23 \times 10 =$
$2000 \times 10 =$	$1700 \times 10 =$	$405 \times 10 =$	$82.96 \times 10 =$

Rule: **To multiply by 10, move the decimal point one place to the right. Insert zeros where necessary.**

Art. 47 gives you practice in using the rule.

47. Practice in Multiplying by 10. (Oral Work.) Do not get caught on any of the next examples. Multiply each of the following numbers by 10.

EXERCISE

(*a*) .6	(*f*) .124	(*k*) .1	(*p*) 9
(*b*) 6.01	(*g*) 124	(*l*) 4.00	(*q*) .9
(*c*) .06	(*h*) 12.4	(*m*) 930.	(*r*) .09
(*d*) 60	(*i*) 1240.	(*n*) 47.673	(*s*) 90
(*e*) 1.24	(*j*) .0124	(*o*) 62.	(*t*) 900

48. Multiplying by 100. (CLASS WORK.) Multiplying by 100 is exactly like multiplying by 10, except that the decimal point must be moved **two** places to the right. Thus:

(*a*) $6.55 \times 100 = 655.$ (*c*) $7 \text{ or } 7. \times 100 = 700$

(*b*) $1.396 \times 100 = 139.6$ (*d*) $87.8 \times 100 = 8780$

Why did we write two zeros after the 7 in Example (*c*)? Was the decimal point moved **two** places to the right in each of the above examples?

Work the following examples:

EXERCISE

1. $8 \times 100 =$
2. $24 \times 100 =$
3. $47.2 \times 100 =$
4. $32.7 \times 100 =$
5. $1.92 \times 100 =$
6. $87.62 \times 100 =$
7. $8.7 \times 100 =$
8. $.341 \times 100 =$
9. $2.438 \times 100 =$

49. Multiplying by 1000. (CLASS WORK.) If we will review our rules for multiplying by 10 and by 100, we will discover a general rule which works in such cases. Thus:

To multiply by 10, move the decimal point **one** place to the **right.**

To multiply by 100, move the decimal point **two** places to the **right.**

To multiply by 1000, move the decimal point ..... places to the **right.** Is three the correct word?

Some pupils remember to multiply by 10, 100, and 1000, by thinking like this: $\times 10$, one 0 in 10, move one place to right; $\times 100$, two 0's in 100, move two places to right; $\times 1000$, three 0's in 1000, move three places to right.

50. Practice in Multiplying by 10, 100, and 1000. (WRITTEN WORK.) Write only the answers of these examples. Be sure to put in a zero when you need to.

1. $.7 \times 10$	11. $.24 \times 10$	21. 76.2×10
2. $.7 \times 100$	12. 500×1000	22. 500×100
3. $.7 \times 1000$	13. 9.7×100	23. $.64 \times 100$
4. 42×100	14. 76.2×1000	24. $.64 \times 10$
5. 8×10	15. $.004 \times 1000$	25. 76.2×100
6. 8×100	16. 500×10	26. $.24 \times 1000$
7. 9.7×1000	17. $.004 \times 100$	27. 1.035×10
8. 1.035×1000	18. 9.7×10	28. 8×1000
9. 42×10	19. 1.035×100	29. $.004 \times 10$
10. $.24 \times 100$	20. 42×1000	30. $.64 \times 1000$

51. Problems in Multiplying by 10, 100, and 1000. (WRITTEN WORK.) Do these problems mentally. Write only the answers.

1. A man earns $4.50 a day and works 10 days. How much does he earn in all?

2. Fred Smith walks to and from school 5 days a week. His home is 1.4 miles from school. How far does he walk going to school and coming home each week?

3. How much do his school walks amount to in 10 weeks?

4. The round trip on a certain car line is 13.47 miles. How far are 10 round trips? 100 round trips? 1000 round trips?

5. At a Thanksgiving dinner there were 10 people. Each person ate $\frac{1}{4}$ of a pumpkin pie. How many pies were eaten? ($\frac{1}{4}$ = what decimal?)

6. If one twenty-five-cent piece weighs .07 lb., how much do one hundred twenty-five-cent pieces weigh?

7. If pencils cost 3¢ apiece, and each of 1000 children is given one, how many **dollars** will the 1000 pencils cost?

8. If an express train travels one mile in 1.32 minutes, how long will it take it to go 100 miles at the same rate?

9. In a certain city there are on the average 6 fire alarms a day. How many alarms are there in 100 days?

10. The cows on Mr. Black's farm give on the average 217.25 gal. of milk a month. How much will they give in 10 months?

52. Rapid Drill. (ORAL WORK.) No example in this drill should trouble a sixth-grade pupil. In division examples give only the first quotient figure.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	96 ×9	*Reduce:* 31″	29 ×8	.5 = what fraction?	89)391	18+4+7
2.	$\frac{2}{3}$ of 15	141 −67	.14 ×5	$3 \div 2\frac{1}{2} = 3 \times ?$	Why correct errors?	36 ×7
3.	How is acre abbreviated?	76)281	*Subtract:* .5 from 5	$\frac{1}{4}$ = what decimal?	200 −39	What units express area?

SELF-TESTING DRILL NO. 4

53. Directions. Wait for the signal "Go!" Wherever possible save copying by placing your paper right below an example. Number examples as you work them.

Time allowed: 20 minutes. Work as fast as you can without mistakes. Do the examples and problems in order. **Examples 2, 9, and 16 call for things you have learned this year.**

1. $55\overline{)1760}$

2. $47 \times 100 = ?$

3.
$$\begin{array}{r} 10565 \\ -3319 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 80656 \\ -39678 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 174 \\ 707 \\ 316 \\ 835 \\ 849 \\ 482 \\ \hline \end{array}$$

6. $5\frac{3}{8} + \frac{1}{2} = ?$

7. $72\overline{)664}$

8. $7\frac{1}{2} - 5\frac{1}{2} = ?$

9.
$$\begin{array}{r} .408 \\ \times 4.9 \\ \hline \end{array}$$

10. $71\overline{)6303}$

11. $3.66 + 6 + .25 + 1.75 = ?$

12. $2\frac{1}{6} \times 1\frac{1}{2} = ?$

13. $\frac{3}{4} \times \frac{3}{4} = ?$

14. $2\frac{1}{4} - \frac{3}{8} = ?$

15. $2.0075 - .385 = ?$

16. A pupil has 15 recitation periods per week, each 30 minutes long. What is the total time per week, in hours and minutes, of these recitation periods?

17.
$$\begin{array}{r} 675 \\ \times 540 \\ \hline \end{array}$$

18. $\frac{3}{4} \div \frac{1}{8} = ?$

19. $273 \div 102 = ?$

20. What is the reciprocal of $\frac{9}{13}$?

STANDARDS

Number correct	0–6	7–8	9–10	11	12	13	14	15	16	17–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

SUMMARY

54. General Review. (ORAL WORK.) Study the questions and examples in the exercise below for five minutes. If there are any questions or examples you cannot answer, do not be afraid to ask about them.

1. If you multiplied 7.324 by 100, which way would you move the decimal point?

2. How can you tell the number of decimal places there will be in the product of a multiplication example?

3. Why are there no decimal places in the product of 22×7?

4. If $447 \times 32 = 14304$, then $4.47 \times 3.2 =$ .....

5. If you find that you have trouble in working a problem, what are some good questions to ask yourself about that problem?

6. In adding $\frac{3}{7} + \frac{1}{6}$, why would 42 be used as the common denominator?

7. The example $\frac{2}{5} \div 4$ would be changed to what multiplication example?

8. How would you leave the answer $16\frac{7}{2}$?

9. What are the answers to these two examples?

$$\frac{1}{2} + \frac{1}{3} = ? \qquad \frac{1}{2} \times \frac{1}{3} = ?$$

10. Find the difference of $12\frac{1}{2}$ and $4\frac{3}{4}$.

11. What canceling can be done in this example?

$$\frac{2}{5} \times \frac{1}{2}\frac{5}{2} \times \frac{1}{2}\frac{1}{1}.$$

12. How would you write the 56 in. found by multiplying in this example? 7×4 ft. 8 in.

13. Will you use 11 qt. in this example? $4\overline{)10 \text{ gal. } 3 \text{ qt.}}$

14. *Find the products:*

(*a*) 24×10	(*d*) 3.1416×1000	(*g*) 1.59×100
(*b*) 9×1000	(*e*) 82.5×100	(*h*) $.0132 \times 10$
(*c*) 9.124×100	(*f*) $.124 \times 10$	(*i*) $.00169 \times 1000$

15. The rule for multiplying fractions is: Multiply the .....to get the..... of the answer. Then.....the denominators to get the .....of the answer.

16. Write on the board 246 cents in the way business men would write it.

17. How would this example be changed before working it? $4\frac{1}{2} \div 3$.

CHAPTER III

DIVIDING BY 10, 100, AND 1000. DRAWING TO SCALE. DIVISION OF DECIMALS. THE DECIMAL FORM OF REMAINDERS

MULTIPLYING AND DIVIDING BY 10, 100, 1000

55. The Decimal Point in Division. (STUDY PERIOD.) There is a short way of dividing by 10, 100, or 1000. It is worth while for you to learn it. Answer the seven questions that are given below. They are about the short way of dividing by 10, 100, or 1000.

QUESTIONS

1. When you divide by 10, **which way** does the decimal point move? $70. \div 10 = 7.0$ or 7 (7.0 and 7 are the same.)

2. **How many** places does the decimal point move?

3. When you divide by 100, which way does the decimal point move? $400 \div 100 = 4$ or 4.00 (400 and 400. are the same.)

4. How many places does the decimal point move?

5. If you divide by 1000, how many places is the decimal point moved? $5555.5 \div 1000 = 5.5555$

6. Which way does the decimal point move?

7. Make a rule for dividing by 10, 100, 1000, etc.

56. Dividing by Pointing. (WRITTEN WORK.) "Dividing by pointing" is a short way of writing "Dividing by moving the decimal point." Write only the answers to the examples below. Plan your work before you begin.

EXERCISE

1. *Divide these numbers by 10, then by 100, then by 1000.*

(*a*) .4	(*d*) 7.5	(*g*) 60.7	(*j*) 430	(*m*) 2.4
(*b*) 67	(*e*) 438	(*h*) 9	(*k*) 91	(*n*) .57
(*c*) 20	(*f*) $3.23	(*i*) 5.7	(*l*) 1	(*o*) 9.0

2. If dividing by 10, 100, or 1000 is hard for you, here is another list of numbers for extra practice:

(*a*) 347	(*c*) 70.08	(*e*) 439	(*g*) $762
(*b*) 892.5	(*d*) 16	(*f*) $25.11	(*h*) 14.3

57. Practice in Multiplying and Dividing by 10, 100, and 1000. (WRITTEN WORK.) Write only the answers to the examples below. Make your figures easy to read.

Spend a few minutes reviewing Art. 49, about multiplying, and Art. 55, about dividing, before beginning the written work.

1.	**5.**	**9.**	**13.**
$17 \times 10 = ?$	$4 \div 100 = ?$	$.02 \times 10 = ?$	$.72 \times 100 = ?$
2.	**6.**	**10.**	**14.**
$42 \div 1000 = ?$	$7 \div 1000 = ?$	$49.3 \div 100 = ?$	$.7 \div 100 = ?$
3.	**7.**	**11.**	**15.**
$9 \div 10 = ?$	$24 \times 100 = ?$	$17 \times 1000 = ?$	$.7 \times 100 = ?$
4.	**8.**	**12.**	**16.**
$1 \times 1000 = ?$	$.007 \times 1000 = ?$	$4.3 \div 10 = ?$	$.9 \times 100 = ?$

17.	22.	27.	32.
$80 \div 10 = ?$	$642 \div 100 = ?$	$32.4 \times 1000 = ?$	$.304 \times 100 = ?$
18.	**23.**	**28.**	**33.**
$.7 \div 10 = ?$	$.5 \times 100 = ?$	$.7 \times 10 = ?$	$87 \times 10 = ?$
19.	**24.**	**29.**	**34.**
$.46 \div 1000 = ?$	$.64 \times 100 = ?$	$10\overline{)720}$	$87 \div 10 = ?$
20.	**25.**	**30.**	**35.**
$.164 \times 10 = ?$	$617 \times 1000 = ?$	$1000\overline{)1000}$	$87 \times 100 = ?$
21.	**26.**	**31.**	**36.**
$1000\overline{)843}$	$100\overline{)4100}$	$.09 \times 1000 = ?$	$87 \div 100 = ?$

58. Problems Using Multiplication and Division by 10, 100, and 1000. (WRITTEN WORK.) You have learned how to multiply and divide by 10, 100, and 1000. Now, in solving the next problems, see if you can really use what you have learned.

1. The assembly hall of the Ellis School is 78 ft. by 100 ft. What is the area of the hall?

2. When the circus came to Centerville, the ice-cream man sold 1000 ice-cream cones at 5¢ a cone. How much money did he take in?

3. The population of Buffalo was about 550,000 in 1920. The population of New York City was about 5,500,000 at that time. New York City was about how many times as large as Buffalo?

4. Mr. Williams divided $5000 equally among his 10 children. How much did each receive?

5. Dr. Dean bought a house for $6250. He agreed to pay for it in 10 equal payments. One payment was to be made each year. What was the amount of each payment?

6. The Manville Real Estate Company bought 40 acres of land on the outskirts of a city. Five acres were used for streets. The remainder was divided into 100 house lots of equal size. How much land was there in each house lot?

7. The street cars in a certain eastern city carry 874,000 people every day. 1000 street cars are used. How many people are carried each day per car on the average?

8. The Quality Restaurant Company runs 10 restaurants in a city. The pay roll for each restaurant is $145 a week. What is the pay roll for all the restaurants?

9. In planning a hotel, it was decided to spend $87.50 in furnishing each guest room. There were 100 guest rooms in the hotel. How much did it cost to furnish them all?

10. The Boone School has 1000 desks. The total cost of these desks was $5650. What was the cost per desk?

11. A successful farmer in Kansas got 4250 bu. of corn from 100 acres. What was the average yield per acre?

12. A gas and electric company in a southern city supplies electricity to 1000 customers. The report of the company said: "Each customer uses 14.75 kilowatt-hours per month on the average." How many kilowatt-hours of electricity are used by all the customers per month?

DRAWING TO SCALE

59. The Arithmetic of Maps and Accurate Drawings. (Class Work.) Photographs and pictures such as we see in magazines and books are not only very interesting to look at, but they help us to get clear ideas of the topic we are studying. You will notice many excellent pictures in your textbooks in history and geography.

It is often necessary, however, to show with great accuracy the dimensions of certain figures. Thus it is not enough to show a picture. A drawing must be made so that a given distance on it represents an accurate measurement of the thing itself.

Notice the three drawings below. These drawings are accurate and are drawn to scale. The dimensions of the

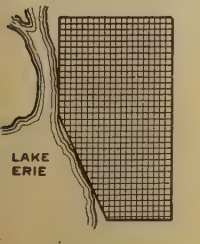


Fig. (*A*)

Fig. (*B*)

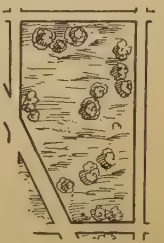


Fig. (*C*)

objects represented by Figs. (*A*), (*B*), and (*C*) can be computed by measuring the three figures and using the scales. In reading scales, the sign : stands for the preposition "to."

Figs. (*A*), (*B*), and (*C*) illustrate the fact that scale drawing may represent a large distance by a small one.

Thus, Fig. (*A*) is a rough outline of the thickly populated section of the city of Buffalo, N. Y.

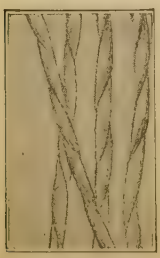
Fig. (*B*) uses a different scale. It represents the floor plan of a room in an office building.

Fig. (*C*) uses another scale. Fig. (*C*) represents a piece of land in a city residence district.

You cannot tell how large an object a drawing represents unless the object is drawn to scale and the scale is given. Maps, building plans, and machine designs are common examples of scale drawings that are made smaller than the objects themselves.

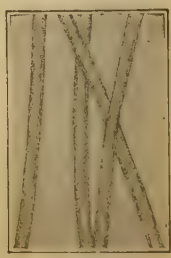
Scale drawing is sometimes used to make a drawing larger than the object itself.

The three pictures below show how cotton, wool, and silk fibers look when they are seen through a powerful microscope. The artist who drew these pictures used a microscope which enlarged the fibers nearly 200 times. Thus we



Cotton Fiber

Wool Fiber



Silk Fiber

are able to see, by drawings larger than the objects themselves, just what the objects look like. Drawings of this sort are most commonly found in science books and articles.

In Art. 60 you will have an opportunity to show that you understand the meaning of scale drawing.

60. A Project in Mensuration. (CLASS WORK.) Plan, by talking the matter over in class, to measure and make maps of some well-known piece of land near your school. Choose a piece of land that all your class have seen, such as your school grounds, a park, or a city block which is the shape of a rectangle. Write out your report so that it could be used in a school exhibit. Below are some suggestions.

1. How long is the piece of land?
2. How wide is it?
3. What is its area in sq. ft.?
4. What is its area in A.?
5. From someone who knows, find out what would be a reasonable price per acre for the land.
6. Draw a map of this piece of land like the one in Art. 61. Show on the map: (*a*) the scale used, (*b*) the length of the piece of land, (*c*) the width.

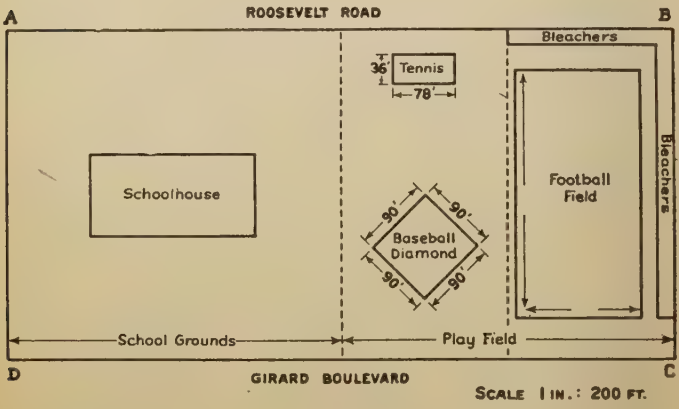
Draw also the proper location of the more important things on the land, such as houses, stores, churches, and water fountains. Draw these as nearly to scale as you can.

7. Have some pupils find out interesting bits of history about the land and the buildings upon it.
8. Have some pupils who can draw well make a bird's-eye view of the land, using colors if possible.
9. If any members of the class own cameras, have them take pictures of various parts of the land. Mark the pictures so that it will be easy to see what parts of the map they show. Think of other interesting things to do.

If you have forgotten how to do the arithmetic needed in this project, use the Self-Help Index, page 457. Look for "Measurement—finding areas of rectangles."

PROBLEM SCALE NO. IV

61. Problems about the School Grounds. (WRITTEN WORK.) You can earn 117 points by working every problem correctly. Find your score and rate yourself as you learned to do with Problem Scales I to III. In working these problems, you will need a ruler.



1. How far is it from Roosevelt Road to Girard Boulevard? (5)

2. Is the area of the football field about 6 acres, 2 acres, 1 acre, or $\frac{1}{2}$ acre? (6)

3. Is the baseball diamond about 1 acre, $\frac{1}{2}$ acre, $\frac{1}{5}$ acre, or $\frac{1}{10}$ acre? (6)

4. Is the area of the play field about 1 sq. mi., 10 acres, 4 acres, or 1 acre? (7)

5. What are the length and the width of the schoolhouse? (7)

6. If you walked around the whole property from *A* to *B* to *C* to *D*, and then to *A* again, would you walk about 2 mi., 1 mi., $\frac{1}{2}$ mi., or $\frac{1}{4}$ mi.? (7)

7. If you walked around the whole school property (that is, *A* to *B* to *C* to *D* to *A*) 4 times, about how many miles would you walk? (8)

8. What are the dimensions of the play field? (8)

9. What are the dimensions of the school grounds? (8)

10. What is the area of the baseball diamond in square feet? (8)

11. Mary Grinnell walks one-half mile to school. This is about how many times the perimeter of the school property? (8)

12. Henry said: "There are more than 7 acres but less than 8 acres in the whole school property." Billy said: "No, there are over 8 acres." Which boy was right? (9)

13. What is the area of the play field in sq. ft.? (10)

14. Counting 43,560 sq. ft. to the acre, how many acres are there in the play field? (10)

15. The school board paid 6¢ a sq. ft. for the land for the school grounds and play field. What was the total cost of the land? (10)

STANDARDS

Superior work:	20–117 points	Average work:	6–11 points
Good work:	12– 19 points	Slow work:	0– 5 points

➡If your work on problem solving needs improvement, find articles to review in the Self-Help Index, page 457.

62. Rapid Drill. (ORAL WORK.) In division examples give only the first quotient figure. These examples should not trouble a sixth-grade pupil.

	(A)	(B)	(C)	(D)	(E)
1.	68×7	$200 - 148$	$\$7500 \div 1000$	$9+7+8$	$72 \div 10$
2.	*Best form of answer?* $\frac{12}{8}$	$81\overline{)729}$	19×4	Subtract 1.6 from 5	*Read:* 25.67
3.	$36\overline{)264}$	36×10	$\frac{5}{6} = \frac{12}{}$	$\frac{1}{2} =$ *what decimal?*	*Reduce:* 2 yd. 7 ft.
4.	$9+9+7$	14×0	47×8	*Best form of answer?* $\frac{4}{2}$	62×1000
5.	$\frac{3}{4} = \frac{12}{}$	*Reduce:* 3′ 19″	What is scale drawing?	$68 - 29$	*Finish the answer:* $6.72 \times 3.6 = 24192$
6.	$50 \div 100$	$143 - 117$	$8+9+6$	$2\frac{3}{5} = \frac{}{5}$	45×100
7.	*Best form of answer?* $\frac{7}{3}$	$38\overline{)325}$	*Reduce:* 2 gal. 11 qt.	$47 - 28$	$\frac{7}{8} = \frac{}{24}$
8.	$9+8+9$	*Read:* XXIX	29×6	Meaning of 1″ : 5 mi.	How is multiplication checked?

SELF-TESTING DRILL NO. 5

63. Directions. Wait for the signal "Go!" Wherever possible, save copying by placing your paper right below any example. Number examples as you work them.

Time allowed: 20 minutes. Work as fast as you can without mistakes. Do the work in order. Rate yourself.

Does your Progress Chart show that you are improving?

1.
234
647
325
347
234
445

2. 186601 − 94901

3. 820 × 29

4. 567 × 67

5. $32\overline{)4000}$

6. $\frac{2}{7} \div \frac{3}{4} = ?$

7. $.95 \times 1000 = ?$

8. $\frac{1}{6} \times 2 = ?$

9. $876 \div 10 = ?$

10.
897
988
894
996
772
997

11. *Write with figures:* XXXVII

12. $6\frac{3}{4} - 3\frac{1}{8} = ?$

13. $419.3 - 47 = ?$

14. 2.73 × .35

15. $\frac{3}{9} \div 1\frac{1}{3} = ?$

16. What is the volume of Figure (*A*)?

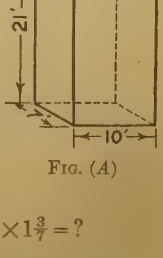


FIG. (*A*)

17. A girl has $7\frac{2}{3}$ yd. of cloth and buys $2\frac{2}{9}$ yd. more. How much cloth is there in the two pieces?

18. $\frac{2}{3} + \frac{5}{6} = ?$

19. $3\frac{8}{11} \times 1\frac{3}{7} = ?$

Go on to the next page.

20. A cook bought 4 bu. 2 pk. of vegetables, but used only 2 bu. 3 pk. How many were left?

STANDARDS

Number correct	0–6	7–8	9–10	11	12	13	14	15	16	17–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

If your rating does not satisfy you, use the Self-Help Index on page 457 and practice on this drill in your spare moments.

DIVIDING BY DECIMALS

64. Important Points about Dividing. (STUDY PERIOD.) This article teaches you **how to divide by decimals.** Make division of decimals easy for yourself by mastering the three parts of this article.

PART I

Examples in the division of decimals look like the ones below. Can you read these examples?

$3\overline{)\$18.75}$ $4.2\overline{)176.23}$ $17\overline{)32.173}$ $.37\overline{)4.76}$

PART II

Important! Dividing decimals is just like dividing whole numbers so far as the **figuring** is concerned. We must get a rule for the decimal point in the quotient.

Below and on the next page are three examples which show you how to take care of the pointing. They will help you understand the rule given a little later.

EXAMPLE 1. $3.9\overline{)24.96}$

First, move the decimal point of the divisor **one place to the right,** like this, $3.9\overline{)24.96}$ This changes 3.9 to the whole number 39.

Second, move the decimal point in the dividend one place to the right, because **when you move a decimal point in the divisor, you must always move the decimal point in the dividend,** also. Thus: $3.9\overline{)24.96}$

Third, the decimal point in the quotient is right above the **new** decimal point in the dividend. Thus: $3.9\overset{\cdot}{)}\overline{24.96}$

Now you can do the division as you always have, because the divisor is a whole number, and the decimal point is put in the quotient before you begin to divide. Is 6.4 the correct quotient?

EXAMPLE 2. $.39\overline{)2.496}$ becomes $.39\overset{\cdot}{)}\overline{2.496}$ or $39\overset{\cdot}{)}\overline{249.6}$

Think the work like this:

.39 is changed to a whole number by moving the decimal point **two** places to the right. In 2.496, also, the point must be moved **two** places to the right. Remember to do in the dividend exactly what you do in the divisor. The decimal point in the quotient is right over the new decimal point in the dividend. Is 6.4 the correct answer?

EXAMPLE 3. $.049\overline{)4.7}$

Notice the way in which this example is changed. It becomes $.049\overset{\cdot}{)}\overline{4.700}$ or $49\overset{\cdot}{)}\overline{4700}$. In .049 the decimal point is moved **three** places to the right. Why? So in 4.7 the point must be moved **three** places to the right. Why? Notice that we had to put in two zeros. The point in the quotient is **right above** the new point in the dividend.

Part III

Study these changes. They are all correct.

$4.9\overline{)472.5}$ becomes $49.\overline{)4725.}$ by thinking $4.9\overline{)472.5}$

$.32\overline{)24.7}$ becomes $32.\overline{)2470.}$ by thinking $.32\overline{)24.70}$

$23\overline{)2.436}$ remains $23\overline{)2.436.}$ There is no change, because the divisor has no decimal point to be moved. **But place the point in the quotient without fail.**

$.046\overline{)24.}$ becomes $.046\overline{)24.000}$ or $46\overline{)24000.}$ Annex as many 0's as are needed to fill out to the new decimal point.

65. Directions for Dividing Decimals. (Class Work.)

1. Move decimal point in divisor to the right as many places as are needed to make divisor a whole number.

2. Move decimal point in dividend to the right just as many places as you moved the decimal point in the divisor. Annex zeros when you need to.

3. Place decimal point in quotient exactly above the new decimal point in the dividend.

4. Then divide as you do with whole numbers.

5. If the divisor is a whole number to begin with, all you have to do is to place decimal point in quotient, and divide.

See if the above rules are used in the next examples.

(*a*) $24\overline{)362.4}$ Divisor a whole number. Decimal point in dividend not moved. Point in quotient put right above.

(*b*) $2.4\overline{)3.624}$ becomes $24\overline{)36.24}$. Notice where the point is in the quotient.

(*c*) $.24\overline{)362.4}$ becomes $.24\overline{)362.40}$. Notice zero needed.

See if these also are correct:

(*d*) $741\overline{).3456}$ (*f*) $.024\overline{)3.624}$ (*h*) $.741\overline{)3456.000}$

(*e*) $.24\overline{).3424}$ (*g*) $.24\overline{)36.24}$ (*i*) $74.1\overline{)34.56}$

In the examples below the quotient figures are correct, but the decimal points are left out. Tell where the decimal point should be and read each quotient correctly.

43	45885	1351
5. $7.4\overline{)3.182}$	6. $.932\overline{)427.64820}$	11. $.333\overline{)44.9883}$
54	410	7911
4. $8.62\overline{)46.548}$	7. $.922\overline{)378.02}$	12. $.75\overline{)5.93325}$
224	500	81010
3. $309\overline{)69.216}$	8. $.31\overline{)155}$	13. $0.8\overline{)64808}$
1982	043	0143
2. $.049\overline{).97118}$	9. $74\overline{)3.182}$	14. $65\overline{).9295}$
642934	642934	19
1. $.0005\overline{)3.214670}$	10. $.5\overline{)3.214670}$	15. $23.7\overline{)450.3}$

66. Easy Division of Decimals. (Oral Work.) Study the examples below and on the next page for three minutes. Be ready to answer any questions your teacher may ask about them.

(*a*) $7\overline{)56}$ — 8 — All whole numbers. No decimal points. *Ans.* 8 (or 8.).

(*b*) $7\overline{)5.6}$ — .8 — Think decimal point in quotient. *Ans.* .8 (eight tenths, or $\frac{8}{10}$)

(*c*) $7\overline{).56}$ Think decimal point in quotient. $7\overline{).56}$ (.08)
Ans. .08 (eight hundredths, or $\frac{8}{100}$)
Note the use of a 0 in the quotient.

Any vacant place **between point and first figure** in quotient is filled by zero.

(*d*) $.7\overline{)56}$ Think $.7\overline{)56.0}$ (80.) Divisor made a whole number.
Ans. 80 (or 80.)

(*e*) $.7\overline{)5.6}$ Think $.7\overline{)5.6}$ (8.)
Ans. 8 (or 8., or 8.0, or 8.00, etc.)

(*f*) $.07\overline{).56}$ Think $.07\overline{).56}$ (8.) Why move points 2 places?

Study the next examples until you can do them easily.
➧ Remember to fill with zeros any vacant places between the decimal point and the first quotient figures.

EXERCISE

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	$.6\overline{)3.6}$	$.5\overline{)75}$	$6\overline{).36}$	$8\overline{)72}$	$.6\overline{).36}$
2.	$5\overline{)7.5}$	$6\overline{)3.6}$	$.05\overline{).75}$	$.09\overline{)1.08}$	$.08\overline{).72}$
3.	$.06\overline{).36}$	$.9\overline{)10.8}$	$.7\overline{)6.3}$	$5\overline{)75}$	$.09\overline{).108}$
4.	$8\overline{).72}$	$9\overline{)10.8}$	$9\overline{)1.08}$	$8\overline{).72}$	$5\overline{).75}$
5.	$9\overline{)108}$	$8\overline{)7.2}$	$.5\overline{)7.5}$	$.9\overline{).0108}$	$.9\overline{).108}$
6.	$.8\overline{)7.2}$	$.5\overline{).75}$	$.09\overline{)10.8}$	$.9\overline{)1.08}$	$.6\overline{)36}$

67. Check for Division of Decimals. (CLASS WORK.) Check division of decimals just as you check division of whole numbers; but there is one thing of which you must be careful: **The remainder must be pointed off.**

Be sure to put a decimal point in the remainder directly below the decimal point in the dividend, inserting zeros in any vacant places. Then add the remainder to the product of the quotient and divisor. Study the samples below.

SAMPLE (*A*)

The Work	*The Check*
4.66	
79)368.31*	4.66
316	79
523	4194
474	3262
491	368.14
474	.17
.17	368.31*

SAMPLE (*B*)

The Work	*The Check*
5.07	
5.7)28.930*	5.07
285	5.7
430	3549
399	2535
.031	28.899
	.031
	28.930*

68. Practice in the Division of Decimals. (WRITTEN WORK.) Work the examples below. First, move the decimal points. Next, place the point for the quotient. Then divide as with whole numbers. **Check** each example.

1.	2.	3.	4.	5.
3.2)1.600	37).3219	93)595.2	.009)882.	46)32.70
6.	**7.**	**8.**	**9.**	**10.**
8.4)35.28	57)3648	8.9).2759	.61)298.9	6.4)2.480
11.	**12.**	**13.**	**14.**	**15.**
18).1710	50)29.00	7.4)39.22	.39)3.627	.092)6440

AN EXERCISE IN CRITICAL THINKING

69. A Yes-No Review. (ORAL WORK.) Read each of th statements below carefully before the oral recitation begins Be ready to tell whether a statement is true or whether i is false. Some of the statements are false.

1. A rating of 3 on a Drill is very satisfactory.

2. Division of decimals is exactly like division of whol numbers.

3. One should never use the Self-Help Index unles made to do so by his teacher.

4. The only way to tell how many decimal places ther should be in the product of a multiplication of decimal example is to guess at it or ask another pupil.

5. 7.89 is a hundred times larger than .0789.

6. Many mistakes in problems are caused by too hurrie reading.

7. The fraction $\frac{7}{3}$ is called a proper fraction.

8. 3 hr. 45 min. is called a denominate number.

9. $\frac{3}{4}$ has the same form as $\frac{9}{12}$ but a different value.

10. No pupil in this class can change the next fraction to decimal form without making at least two mistakes.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{5}$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{1}{8}$

11. To divide by 10 we shift the decimal point one plac to the left and fill in a vacant place with 5.

12. All answers to the examples below are correct.

(*a*) $6\overline{)36}$ 6 (*b*) $.6\overline{)36}$ 6 (*c*) $.6\overline{)3.6}$ 6 (*d*) $.06\overline{)36}$ 6

REVIEW OF FRACTIONS

70. Test in Subtraction of Fractions. (WRITTEN WORK.) Last year you worked hard to learn how to subtract fractions. Do you think that practice now would help you to **keep from forgetting** the subtraction of fractions?

If you can work 24 of these examples in 15 minutes correctly, you need not review subtraction of fractions; if you cannot, you should use the Self-Help Index on page 457.

☛ Only answers in simplest form count as correct.

Find the differences:

1. $\frac{5}{8} - \frac{1}{2}$

2. $\frac{7}{9} - \frac{3}{4}$

3. $1\frac{3}{5} - \frac{2}{3}$

4. $6\frac{9}{20}$ minus $2\frac{7}{20}$

5. $9\frac{3}{11} - \frac{3}{11}$

6. Ten and two-ninths minus one-sixth.

7. $\frac{7}{18} - \frac{1}{6}$

8. $4\frac{5}{11}$ minus 2

9. $\frac{12}{17} - \frac{10}{17}$

10. $1\frac{3}{4}$ minus $1\frac{2}{3}$

11. 3 minus $1\frac{2}{5}$

12. $7\frac{3}{8} - 6\frac{5}{24}$

13. $1 - \frac{1}{20}$

14. $5\frac{3}{20} - 2\frac{2}{5}$

15. Three and one-sixth minus three-fourths.

16. $7\frac{1}{6} - \frac{1}{2}$

17. $\frac{2}{3} - \frac{1}{2}$

18. $9\frac{3}{8} - \frac{3}{4}$

19. $7\frac{3}{5} - \frac{1}{4}$

20. Nine and one-sixth minus four and two-thirds.

21. $10\frac{5}{10} - \frac{1}{2}$

22. $5\frac{4}{7}$ minus 3

23. $6\frac{1}{2} - 6\frac{2}{5}$

24. $8\frac{9}{50} - \frac{29}{50}$

25. 9 minus $3\frac{4}{11}$

26. $\frac{7}{15}$ minus $\frac{4}{15}$

27. Eight and two-thirds minus four and one-half.

28. $7\frac{1}{14}$ minus $\frac{4}{7}$

29. $9\frac{5}{6}$ minus $3\frac{3}{4}$

30. 7 minus $\frac{3}{22}$

SELF-TESTING DRILL NO. 6

71. Directions. Time allowed: 20 minutes. Follow th directions given for Drills 1 to 5. Rate yourself. **Try t beat all your past records.**

1.
160
431
918
209
810
291

2.
8000
−476

3.
4.04
.75
8.39
.49

4.
$\frac{3}{7}$ times $\frac{4}{5}$ = ?

6. 963
×98

5.
64063
−15798

7.
$\frac{3}{2\,8} \div \frac{3}{2\,5}$ = ?

8.
43)55.9

9.
$10 - \frac{8}{9}$ = ?

10.
$\frac{2}{5}$ of $\frac{1}{6}$ =

11. If 10 caps cost a dealer $8.90, what was the averag cost per cap?

12. A class of 15 pupils has a savings-bank account whic totals $470. What is the average share per pupil?

13.
$\frac{4}{7} + 6\frac{2}{5}$ = ?

14.
$8\frac{1\,1}{3\,0} - \frac{6}{3\,0}$ = ?

15. *Subtract:*
$8\frac{1}{4}$
$2\frac{1}{2}$

16.
712.5
×.2284

17.
3.56)104.23

18.
.159
×.751

19.
.734 ÷ 100 =

20. Which of these two numbers is the larger?
(*a*) 18.039 (*b*) Eighteen and thirty-nine hundredths

STANDARDS

Number correct	0–5	6–7	8	9–10	11	12	13	14	15	16–19	20
Rating	0	1	2	3	4	5	6	7	8	9	10

REMAINDERS AS DECIMALS

72. The Decimal Form of Remainder. (STUDY PERIOD.) Study the three samples in this lesson to find **a new way of writing the remainder in a division example.**

SAMPLE 1.

$$90\overline{)207}\quad 2\tfrac{27}{90}$$
$$180$$
$$27$$

The quotient can be written $2\frac{27}{90} = 2\frac{3}{10}$. Here the remainder is in **fraction form.**

Below is the way to get the remainder **in decimal form.**

Is 207 the same as 207.0? By writing 207 as 207.0 and putting the decimal point in the quotient above the decimal point in the dividend, we can keep on dividing.

This gives the remainder **in decimal form** instead of in fraction form.

$$90\overline{)207.0}\quad 2.3$$
$$180$$
$$270$$
$$270$$

SAMPLE 2.

$$4\overline{)33}\quad 8\tfrac{1}{4}$$
$$32$$
$$1$$

The quotient could be written $8\frac{1}{4}$.

.33. is the same as 33.0 or 33.00. 8.25 was obtained by putting decimal points in dividend and in quotient. Zeros were then annexed after the decimal point in the dividend, and the division was continued.

Does annexing zeros **after** the decimal point change the value of a number?

$$4\overline{)33.00}\quad 8.25$$
$$32$$
$$10$$
$$8$$
$$20$$
$$20$$

SAMPLE 3. 92 will not go even 1 time in 61. Study what is done in the work at the right.

$92\overline{)61}$

We have found the quotient to 3 decimal places. We show that there is still a small remainder by writing a plus sign (+) after the 3 in the quotient, as shown at the right.

```
      .663+
92)61.000
   55 2
   -----
    5 80
    5 52
    ----
     280
     276
     ---
       4
```

73. Rule for Decimals in Remainders. (SILENT STUDY.) Annexing zeros after the decimal point in the dividend like this, $13\overline{)82}$ or $\overset{\bullet}{)}\overline{82.0}$ $\overset{\bullet}{)}\overline{82.00}$ or $\overset{\bullet}{)}\overline{82.000}$, **does not change the value of the dividen**

> *Rule:* **If you see that the quotient in a long division example will have a remainder, you can place a decimal point after the dividend and annex zeros. Then proceed as for any other division of decimals.**

In dividends which **already** have a decimal figure, y can annex zeros without changing the value. Thus 8.2 $8.20 = 8.200 = 8.2000$, because zeros annexed **after the de** mal point **do not change the value of a number.**

74. Practice in Division with Decimal Remainde (WRITTEN WORK.) Find the quotient to **two decimal place** Then use the plus sign to indicate remainders.

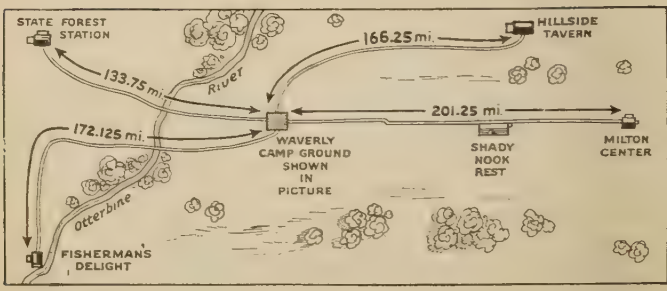
1. $8\overline{)33}$	4. $98\overline{)4.9}$	7. $65\overline{)403}$	10. $42\overline{)3.78}$	13. $60\overline{)28}$
2. $29\overline{)84}$	5. $54\overline{)189}$	8. $99\overline{)62}$	11. $32\overline{)200}$	14. $.16\overline{)9}$
3. $23\overline{)86}$	6. $7\overline{)41}$	9. $3.9\overline{)12}$	12. $.18\overline{)4.97}$	15. $58\overline{)9.}$

PROBLEM WORK

75. Problems Using Decimals. (WRITTEN WORK.) The people seen in the picture are summer tourists. They are touring by auto and sleep each night in "camp grounds" which are provided by many cities and towns for this purpose. Many tourists plan their trips so that they can reach some good camp ground each evening.

After supper the tourists get acquainted with each other and take great pleasure in exchanging ideas and opinions on all sorts of subjects. Probably the favorite topic of discussion is, "How many miles do you average per day?" "Politics" and "business" are also favorite subjects for friendly debate.

The men in the picture are figuring up their relative speeds of travel for the day. On the next two pages are some problems about these men and their travels. In working the problems, you will need to use the picture at the top of the next page.



PROBLEMS

1. Mr. Simmons is the man dressed in decidedly "western" fashion. He is a cattle raiser in Montana and is touring in order "to see what the rest of our country looks like." He was on the road for 6.5 hours in coming in from State Forest Station. What speed did he average per hour?

2. The man with glasses is Mr. Bell. He is an "office man" for a well known investment house in New York. Mr. Simmons thinks he is pretty far **east,** and Mr. Bell thinks he is pretty far **west,** because the camp ground is not far from the Mississippi River. Mr. Bell was on the road for 7 hours in making the trip from Hillside Tavern. What speed did Mr. Bell average per hour?

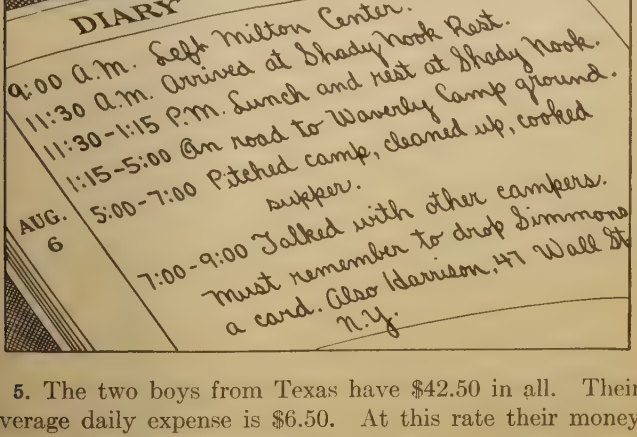
3. The two boys are students in a college in Texas. They are traveling in their own private car, which you can see in the picture. They bought it last spring for $75 and fixed it up themselves.

They started from Fisherman's Delight at 7 in the morning but had "just a little trouble with the car" so they spent only 4.5 hours in actual travel. What was their average speed per hour?

These boys think they are pretty far north because the camp ground is not far from Saint Louis.

4. Mr. Washburn is the man sitting on the new camp stool. This is his first touring trip, and he is keeping a very careful diary. He is going west and "take in" Yellowstone Park and the Grand Canyon. A page from Mr. Washburn's diary is shown below.

Can you figure his average speed per hour for the time he was actually traveling?


DIARY
AUG. 6
9:00 A.M. Left Milton Center.
11:30 A.M. Arrived at Shady Nook Rest.
11:30-1:15 P.M. Lunch and rest at Shady Nook.
1:15-5:00 On road to Waverly Camp ground.
5:00-7:00 Pitched camp, cleaned up, cooked supper.
7:00-9:00 Talked with other campers. Must remember to drop Simmons a card. Also Harrison, 47 Wall St. N.Y.

5. The two boys from Texas have $42.50 in all. Their average daily expense is $6.50. At this rate their money will last for how many days?

6. Mr. Bell found that he could get more miles per gallon of gasoline while touring than when driving in the city. He said that he used 14 gal. in traveling 220.5 mi. How many miles per gallon did he average?

7. Mr. Simmons was telling Mr. Bell about the cattle business. He said he had recently shipped 170 head of cattle, totaling 163,829 lb. What was the average weight of the cattle?

DECIMAL POINTS IN QUOTIENTS

76. Placing the Decimal Point. (ORAL WORK.) Here are division examples with **only** the number part of the quotients already written. The decimal points have **not** been changed, and the decimal points in the quotients are **left out.** Study each example. Tell where decimal points and zeros are needed. Then read the right quotient.

The answer to Example 1 is "six, point, one" (thus, 6.1).

EXERCISE

1. $.98\overline{)5.978}$ → 61
2. $78\overline{)5226}$ → 67
3. $7.7\overline{)354.2}$ → 46
4. $86\overline{)62.78}$ → 73
5. $.48\overline{)3552}$ → 74
6. $.039\overline{)2.145}$ → 55
7. $52\overline{)24.44}$ → 47
8. $.032\overline{)19.2}$ → 6
9. $56\overline{)4.928}$ → 88
10. $37\overline{)347.8}$ → 94
11. $1.6\overline{)1216}$ → 76
12. $.64\overline{)62.72}$ → 98
13. $8.3\overline{)4.316}$ → 52
14. $19\overline{)760}$ → 40
15. $.53\overline{)1.484}$ → 28
16. $6\overline{).594}$ → 99
17. $.97\overline{)455.9}$ → 47
18. $2.4\overline{)21.12}$ → 88
19. $.054\overline{)1782}$ → 33
20. $.041\overline{)18.45}$ → 45
21. $.079\overline{)51.35}$ → 65
22. $.49\overline{)17.64}$ → 36
23. $9\overline{).3159}$ → 351
24. $.152\overline{)3.952}$ → 26

PROBLEM SCALE NO. V

77. Using Decimals. (WRITTEN WORK.) You can earn 65 points by working every problem correctly.

Find remainders to two decimal places where necessary. Name answers carefully.

1. How long will 25.5 T. of coal last if 1.7 T. are used each week? (4)

2. A train went from A to B, a distance of 25.74 mi., in 39 min. Find the average speed per min. (5)

3. How many soldiers can be given bacon if there are 450 lb. of bacon, and .25 lb. is allowed per soldier? (6)

4. Mr. Reynolds sold a load of milk for $40.96. The rate was $.032 a lb. How many pounds were in the load? (6)

5. In testing an automobile, the machine was driven 17.6 mi. in 14 min. What was the rate of speed per min.? (6)

6. A farmer offered to sell his cows for $6308. He owned 56 cows. What was the average price per head? (6)

7. John wants to buy a $12.00 radio. If he saves $1.75 a week, how long will it take him to save the $12.00? (7)

8. Mr. Smith offers his farm for sale at $1575. The farm contains 84.3 acres. What is his price per acre? (8)

9. The People's Laundry charged Mrs. Robbins $1.70 for washing 22.5 lb. of clothes. What was the cost per lb.? (8)

10. The weather report showed that 2.76 in. of rain fell in 3.25 hr. Find the average rainfall in inches per hour. (9)

STANDARDS

Superior work:	25–65 points	Slow work:	4–5 points
Good work:	13–24 points	Inferior work:	0 points
Average work:	6–12 points		

78. Rapid Drill. (ORAL WORK.) This oral practice will help you in the Self-Testing Drills if you do the work carefully and correctly. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$\begin{array}{r} 164 \\ -86 \\ \hline \end{array}$	*Best form of answer?* 2 bu. 15 pk.	$\begin{array}{r} 68 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 135 \\ -78 \\ \hline \end{array}$	$\begin{array}{r} 119 \\ -75 \\ \hline \end{array}$	8+18+7
2.	$\begin{array}{r} 147 \\ -68 \\ \hline \end{array}$	$\begin{array}{r} 123 \\ -28 \\ \hline \end{array}$	$\begin{array}{r} 123 \\ -67 \\ \hline \end{array}$	$\begin{array}{r} 147 \\ -69 \\ \hline \end{array}$	.75 = what fraction?	19+3+8
3.	$87\overline{)844}$	$\begin{array}{r} 134 \\ -97 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 34 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 123 \\ -89 \\ \hline \end{array}$	8+26+5
4.	$\begin{array}{r} 142 \\ -46 \\ \hline \end{array}$	7+19+6	$\begin{array}{r} 164 \\ -78 \\ \hline \end{array}$	19+5+4	7+15+9	$\begin{array}{r} 157 \\ -98 \\ \hline \end{array}$
5.	$\begin{array}{r} 8.9 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 121 \\ -35 \\ \hline \end{array}$	Does 4 = 4.00?	$\begin{array}{r} 175 \\ -86 \\ \hline \end{array}$	$5\overline{)305}$	$\begin{array}{r} 132 \\ -35 \\ \hline \end{array}$
6.	28+7+7	*Best form of answer?* $2\frac{1}{6}$	$\frac{1}{4}$ = what decimal?	6000 ÷ 1000	$\frac{3}{4}+\frac{3}{8}$	18 ÷ 10
7.	$\frac{1}{5}$ of 31	*Finish the answer:* $.4\overline{)18}$ .72	$\begin{array}{r} 6 \\ -4\frac{5}{6} \\ \hline \end{array}$	13+5+6	20 ÷ .5	18 ÷ $\frac{1}{2}$
8.	$1 \div 3\frac{1}{3} =$ $1 \times \ldots$	147.5 ÷100	$\begin{array}{r} 98 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ \times .9 \\ \hline \end{array}$	Does .5 = 0.5?	$68\overline{)500}$

SELF-TESTING DRILL NO. 7

79. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

Is it fair to expect you to work Example 15 correctly?

1. $6\overline{)43}$

2. *Subtract:*

$$\begin{array}{r} \$164.68 \\ 72.49 \\ \hline \end{array}$$

3. $32\overline{)2080}$

4. *Subtract:*

$$\begin{array}{r} 87336 \\ 45147 \\ \hline \end{array}$$

5. *Add:*

$$\begin{array}{r} 416 \\ 623 \\ 448 \\ 52 \\ 9 \\ 790 \\ 35 \\ \hline \end{array}$$

6. *Multiply:*

$$\begin{array}{r} 9 \\ 2\frac{2}{3} \\ \hline \end{array}$$

7.

$$\begin{array}{r} 3\frac{1}{2} \\ 16\frac{3}{4} \\ 6\frac{5}{8} \\ \hline \end{array}$$

8. *Multiply:*

$$\begin{array}{r} 41.24 \\ 4.3 \\ \hline \end{array}$$

9. $4\frac{1}{2} \times 2\frac{1}{3} = ?$

10. $\frac{3}{5} \times 175 = ?$

11. A man divided his farm of 712 acres equally among his 3 sons. How many acres did each receive?

12. $3\frac{1}{4}$ of $2\frac{2}{3} = ?$

13. $\frac{2}{3} \div \frac{4}{7} = ?$

14. $\$2.50 \div 17 = ?$

15. *Carry remainder to two decimal places:* $117 \div 36 = ?$

16. $\frac{1}{2} + \frac{3}{7} + \frac{2}{14} + \frac{1}{7} = ?$

17. $4\frac{1}{5} \div 2\frac{1}{3} = ?$

18. *Subtract:*

$$\begin{array}{r} 8 \text{ lb. } 6 \text{ oz.} \\ 3 \text{ lb. } 12 \text{ oz.} \\ \hline \end{array}$$

19. $7.6\overline{)7.323}$

20. How far would two strips of carpet, each 3 yd. 2 ft. 4 in. long, reach if placed end to end?

STANDARDS

Number correct	0–6	7–8	9–10	11	12	13	14	15	16	17–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Are you correcting errors at once and using the Self-Help Index?

SUMMARY

80. General Review. (Class Work.) Study for ten minutes before the oral work begins.

1. Which of the following two numbers is the larger, 2.98 or 3?

2. How many decimal places will there be in the product of this example? $9.27 \times 62.18 = ?$

3. Does $2 = 2.00$?

4. In an example in division of decimals, what part is first changed to a whole number?

5. How do you check answers in long division?

6. Is this example worked correctly?→

$$2.14\overline{)8.0464}\quad 3.76$$

7. Give the numbers of the pages on which "division of decimals" is explained.

8. On what page is there an explanation of how to write remainders in decimal form?

9. Find the product: $27.4 \times 8.09 = ?$

10. Find the difference: $9.472 - 8.547 = ?$

11. Find the sum: $9.85 + 42.8 + 6. + 60.53.$

12. Find the quotient: $83\overline{).2324}$

13. Find the quotient: $65\overline{)31.85}$

14. Find the quotient: $.47\overline{).3901}$

15. Find the quotient: $.56\overline{)2632.}$

16. Carry the remainder to 2 decimal places: $34\overline{)828}$

17. $2.78 \times 100 = ?$

18. $936 \div 10 = ?$

19. Suppose you wanted to review the subtraction of fractions. What articles would you use for this review?

20. What does the word "perimeter" mean?

21. Explain the following:
(*a*) Scale 1 in.: 5 mi. (*b*) The object was enlarged 100 times in the drawing.

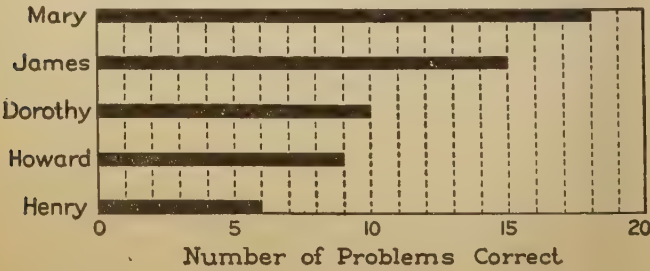
CHAPTER IV

GRAPHS. BANKING PRACTICES AND FORMS. ALIQUOT PARTS. BUSINESS ARITHMETIC

GRAPHS

81. A New Way to Record Facts. (A STUDY PERIOD.) The picture below represents the number of problems in Article 82 solved correctly by five pupils.

Diagrams or drawings like this one are called **graphs.** Graphs are used to "picture" facts. You will find graphs in many geography and history textbooks and in newspapers and magazines.



Answer these questions from the diagram (graph):

1. How many problems did James do correctly?
2. Who worked $\frac{1}{2}$ as many problems as Mary?
3. Who worked $\frac{2}{3}$ as many as James?

4. How many more problems did James work than Henry?

5. There are 20 problems in Art. 82. What fraction of the total number did Mary work correctly?

6. What pupil did .75 of the 20 problems?

7. What pupil did .5 of the exercise?

8. What is the name of the drawing on page 102 which shows the number of problems each of the five pupils worked?

9. Draw a graph, similar to the one on page 102, which represents the following facts:

Mildred did correctly 20 examples, Joseph 17, John 12, and Alice 10.

Bring to class tomorrow some graphs you have found in newspapers, magazines, or books.

PROBLEM SCALE NO. VI

82. General Problems. (WRITTEN WORK.) You can earn 85 points by working every problem correctly. Find your score and rate yourself according to the directions given for Scales I to III.

1. A baseball squad of 12 players made a trip. The round-trip railroad fare was $2.00 for each member of the squad. What was the total railroad fare for the squad? (2)

2. Mary earns 15¢ a week helping her mother. How much will she earn in 8 weeks? (2)

3. A man had $4.50. He spent $1.75. How much money did he have left? (3)

4. Mr. Brown's telephone costs him $4.60 a month. What does the telephone cost him per week? (Count 4 wk. to the month.) (3)

5. George worked for 5 hr. at 20¢ an hour. How much did he save from what he earned if he spent 30¢ of it and saved the rest? (3)

6. A rural mail carrier rides 30 miles every day over his mail route. How far does he ride per week? (No mail on Sundays.) (3)

7. As a birthday present, Henry Adams gave $\frac{1}{3}$ of his marbles to his chum, Walter. How many marbles did Walter get if Henry had 216 marbles before he gave any away? (3)

8. Mr. Jones pays $28.50 a month rent. How much rent does he pay a year? (4)

9. A certain wagon full of coal weighs 4856 lb. The wagon when empty weighs 1038 lb. How much does the coal weigh? (4)

10. A wholesale grocer has on hand 24 barrels of Red River flour. The average amount of flour in each barrel is 196 lb. How many pounds of flour are there in the twenty-four barrels? (4)

11. Mr. Smith bought a car for $660. He used it all summer, and then sold it for $\frac{2}{3}$ of what he paid for it. For how much did he sell it? (4)

12. Margaret went shopping and spent the following amounts of money: $1.25 for a present, 65¢ for candy, and 30¢ for some tooth powder. To pay for the articles, she handed the clerk a $5 bill. How much change should she have received from the bill? (4)

13. The following 4 loads of coal were delivered to a school: The first contained 6.7 T., the second, 9.25 T., the third, 8.0 T., and the fourth, 10.57 T. Find the total weight of the coal. (5)

14. Mary opened her bank and found in it:

2 one-dollar bills	3 50-cent pieces
6 dimes	5 25-cent pieces
3 nickels	

How much did she have in all? (5)

15. How far will an automobile go in $3\frac{1}{3}$ hr. if its average speed is 27 mi. an hr.? (5)

16. Sidney walks at the rate of 4 mi. an hour. How long will it take him to walk 7 mi.? (5)

17. Mrs. Jones had 10 lb. of sugar on hand. She used just $3\frac{1}{3}$ lb. How much had she left? (6)

18. There are 28 pupils in Helen's class. If only 21 are present, what fraction of the class is absent? (6)

19. A gallon is equal to 231 cu. in. How many gallons can be put into a tank $6'' \times 7'' \times 22''$? (6)

20. If Mr. Jones uses $\frac{2}{3}$ of a ton of coal per week, how long will $6\frac{1}{2}$ tons last him? (8)

STANDARDS

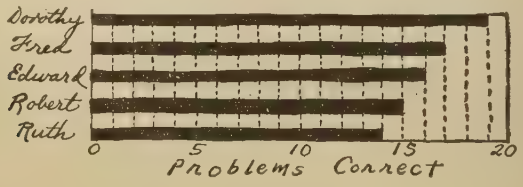
Superior work:	67–85 points	Slow work:	35 41 points
Good work:	56–66 points	Inferior work:	0–34 points
Average work:	42–55 points		

➡*Note:* Remember the number of problems you worked correctly on your first trial. You will use this number in Art. 83.

MAKING AND READING GRAPHS

83. Drawing Graphs. (WRITTEN WORK.) Study the entire article before you begin work. It will give you practice in drawing **graphs.** Below is a graph that was drawn by a sixth-grade boy; it is here shown reduced in size to fit your book. Make yours much larger. The picture is a graph of the following facts:

On Art. 82 Dorothy did 19 problems correctly, Fred 17, Edward 16, Robert (the boy who drew the graph) 15, and Ruth 14.



PROBLEMS

1. Draw a graph which will represent these facts:

(*a*) Pupil *A* did 20 correctly.

(*b*) Pupil *B* did 15 correctly.

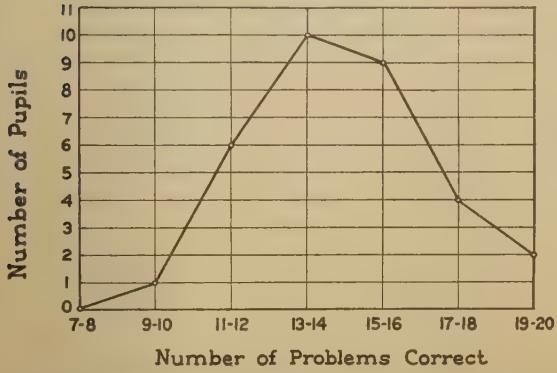
(*c*) Pupil *C* did 10 correctly.

(*d*) Pupil *D* did 5 correctly.

(*e*) Your own score on Art. 82. (Put your bar in the proper place. Count only the problems you did correctly.)

2. Graphs can be made to represent many different things. There are also many kinds of graphs. You should learn how to read them. On the next page is a graph showing certain facts about a sixth-grade class which worked Art. 82. These facts are given at the top of the next page.

2 pupils solved 19 or 20 problems correctly.
4 pupils solved 17 or 18 problems correctly.
9 pupils solved 15 or 16 problems correctly.
10 pupils solved 13 or 14 problems correctly.
6 pupils solved 11 or 12 problems correctly.
1 pupil solved 9 or 10 problems correctly.



3. Make a graph of the record of your class the first time it tried Art. 82.

First, make a list on the blackboard, like the one that is shown at the top of this page. Use the record of your class on Art. 82.

Second, make a graph of the facts for your class.

Third, draw on your graph a dotted line to represent the work of the class reported in the graph in Problem 2.

Did your class do better than the class whose record is given in Problem 2? How would you find this out from studying the two graphs?

When two sets of facts are represented by two lines on a graph, we can compare the facts very easily.

SELF-TESTING DRILL NO. 8

84. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. **Remember to reduce fractions in answers.**

1.	2.	3.	4.	5.
956	$4\frac{1}{2}+7\frac{1}{2}=?$	38.92	$\frac{1}{6}\times\frac{3}{4}=?$	4067
$\times 521$		-12.68		-3878

6.	7.	8.	9.	10.
871	657	$1\frac{1}{24}\div 1\frac{1}{4}=?$	628	$\frac{3}{4}+\frac{5}{6}=?$
$\times 718$	464		773	
	676		956	
	551		197	
	983		578	
	390		968	

11. A factory owner bought 20 acres of land for which he paid $8250. What was the average cost per acre?

12.	13.	14.	15.
1 ft. 3 in.	$1\frac{1}{26}\div\frac{9}{13}=?$	$2.14\times.037=?$	3.27
$\times 7$			$\times 1376$

16. $.051\overline{)4284}$

17. A pole $9\frac{1}{2}$ ft. long is how much longer than one $7\frac{3}{5}$ ft. long?

18. $.4932\div 100=?$

19. According to this graph, what fraction of the total income is spent for rent and food together?

20. In the graph for Problem 19, how much more was spent for education than was saved?

STANDARDS

Number correct	0 4	5	6–7	8	9	10–11	12	13	14	15 19	20
Rating	0	1	2	3	4	5	6	7	8	9	10

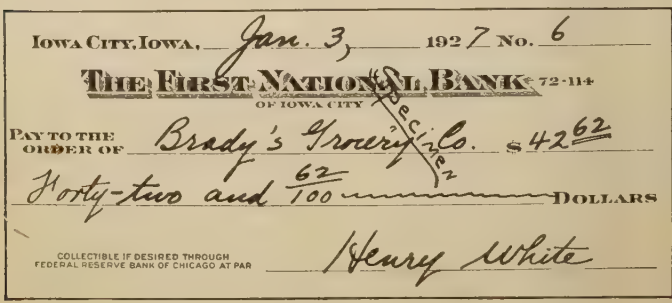
BANK ACCOUNTS, CHECKS, AND BANK FORMS

85. Paying Bills by Check. (Study Period.) Many people do not pay their bills with cash; that is, with actual money. Nor do they carry large amounts of money in their pockets. Instead, when they receive money as wages, salaries, and for bills due them, they put that money (**deposit** it) in a **checking account** in a **bank.** When they pay a bill, instead of giving the actual money, they give a **check** on their account at the bank.

A check is a written statement telling a bank to pay a certain amount of money to a certain person. The bank pays the money from the account of the one who wrote the check. When you open a checking account at a bank, the bank supplies you with a book of blank checks for your use.

Illustration: Mr. Henry White puts the money he receives for working into his checking account in the First National Bank, Iowa City, Iowa. On January 3, 1927, he

paid his grocery bill of $42.62 by check. Brady's Grocery Co. then took the check to the bank and received the money. The bank "canceled" the check (stamped "Paid" on the check), showing that the $42.62 was paid, and returned the canceled check to Mr. White. Here is a picture of the check Mr. White gave Brady's. Study it.


IOWA CITY, IOWA, Jan. 3, 1927 No. 6

THE FIRST NATIONAL BANK OF IOWA CITY 72-114

PAY TO THE ORDER OF Brady's Grocery Co. $ 42 62

Forty-two and 62/100 DOLLARS

COLLECTIBLE IF DESIRED THROUGH FEDERAL RESERVE BANK OF CHICAGO AT PAR

Henry White

Received

Questions about Mr. White's Check

1. Why did he pay Brady's Grocery Co. $42.62?

2. When the bank paid Brady's Grocery $42.62, from whose account did the bank take the $42.62?

3. Why did the bank pay $42.62 to Brady's?

4. When Mr. White paid his bill to Brady's, he said: "Here is my check for $42.62 on the First National Bank." Why has Mr. White the right to ask the bank to cash his check? (To "cash" a check means to pay in actual money the amount written on the check.)

5. Would Mr. White's check be good if he did not have at least $42.62 in his account at the bank?

6. Why should one keep a written account of all the money he deposits in a bank and of the amounts of money he checks out in payment of bills?

7. Why was the $42 written out in words on the check as well as in figures?

8. What do you think the "No. 6" on the check means?

9. If this check were lost, could it be cashed by anyone for $42.62 in actual money?

10. Could Mr. White make out a check to himself in order to get some actual money?

EXERCISE

Write some make-believe checks. Suppose that you have $155 on deposit in the Third National Bank. Suppose that you owe the bills listed below. Make out a check in payment of each. Use today's date on your check.

(*a*) Brigg's Bicycle Shop, $16.75.
(*b*) Dr. F. B. Whinery, $8.50.
(*c*) Stewart Shoe Store, $5.00.
(*d*) Smith's Music House, $12.00.

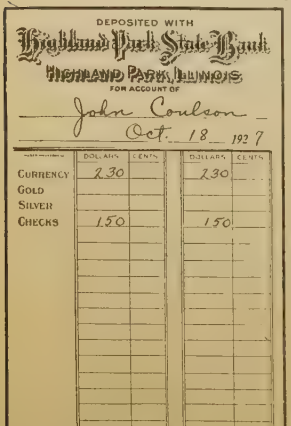
Figure what your **balance** at the bank (the amount of money you still have) is after you have paid these bills by check.

USING A BANK

86. Keeping Your Account at the Bank. (STUDY PERIOD.) Of course, you cannot write a real check on a bank unless you have a checking account at that bank. That would be breaking the law. In this article you are to study five important things about using a bank.

First Fact about Banking: **Get a receipt for every amount you deposit.** This receipt may be one of two things: (1) the man who receives your money ("the receiving teller") will put down (enter) the amount you have deposited and the date in a **deposit book,** or **pass book,** which the bank furnishes when you open your account. On the next page is a picture of a deposit book.

(2) If you do not have your book with you, you may make out two **deposit slips.** One of these the receiving teller will "receipt" (write his name or initials on) and give to you to keep. At some later time you can take your deposit book and the slip to the bank, and the teller will enter the deposit in your book.


DEPOSITED WITH
Highland Park State Bank
HIGHLAND PARK, ILLINOIS
FOR ACCOUNT OF
John Coulson
Oct. 18 1927

	DOLLARS	CENTS	DOLLARS	CENTS
CURRENCY	230		230	
GOLD				
SILVER				
CHECKS	150		150	
	Total $		380	

A Deposit Slip

QUESTIONS

1. Why should you always get a receipt for money you have deposited in the bank?

2. Who deposited the money shown on the deposit slip?

3. How much did he deposit?

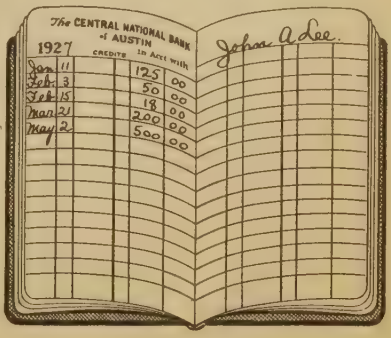
4. In what bank was the money deposited?

5. Why do you think it wise to keep your canceled checks?

6. Have you ever watched someone make a deposit at the bank?

Below is a picture of a deposit book. You can see that it shows each amount Mr. Lee has deposited, together with the date of each deposit.

SECOND FACT ABOUT BANKING: **Ask for a monthly statement of deposits and canceled checks.** The bank keeps an account of every deposit you make and of every check you "write against" your account as your checks come in to be cashed. Usually you are given at the end of the month a **statement,** like the one shown on page 114, of how your account stands at the bank.



This statement shows every amount you have deposited, and the date of every deposit. It also shows the amounts that are taken day by day from your account to pay the checks which you have written against your account at the bank.

When you give a check to pay a bill, the check is sent to your bank and **canceled,** i.e., the bank pays the money and takes it from your account. The canceled checks are saved at the bank. At the end of the month all your canceled checks are given back to you. You can then examine them to see if the correct amounts have been deducted for the canceled checks that are returned. Your canceled checks are valuable records. They prove that you have paid certain amounts of money.

Here is a picture of a monthly statement:

IN ACCOUNT WITH

THIRD NATIONAL BANK

SCRANTON, PA.

Statement of your account to Feb. 1, 1928

Name William D. Bryden

Address 1026 Clay Avenue

Scranton, Pa.

Old Balance	Date	Checks in Detail	Date	Deposits	Date	Balance
143.52	Balance brought forward: 143.52					
	Jan. 5	26.00	Jan. 5	285.26	Jan. 5	360.78
		42.00				
	Jan. 17	15.30			Jan. 17	207.90
		95.00				
		42.58	Jan. 19	7.25	Jan. 19	215.15
	Jan. 27	6.20	Jan. 27	59.30	Jan. 27	256.50
		11.75				
	Jan. 28	10.00			Jan. 28	246.50
					Jan. 31	246.50

EXERCISE

1. Is each balance correct in the monthly statement above? How will you find out whether each is correct?

2. Prepare a statement using these facts:

(*a*) Make the monthly statement just as if it were from the Davis National Bank, Akron, Ohio.

(*b*) The month is to be December, 1927.

(*c*) Use your own name.

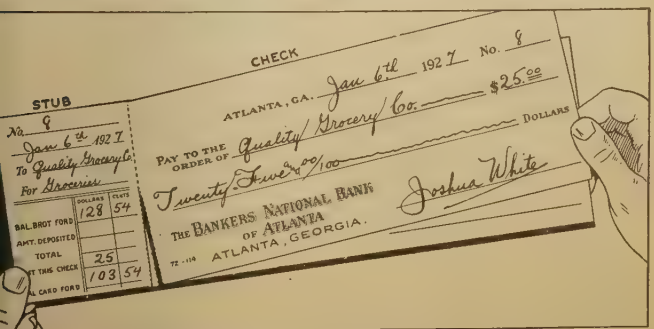
(*d*) Deposits show: Balance from Nov., 1927 $178.50
Deposited Dec. 8 100.00
Deposited Dec. 22 100.00

(*e*) Canceled checks show:

Dec. 1—$50.00 Dec. 17—$ 5.25 Dec. 21 $20.00
Dec. 1— 32.00 Dec. 21— 49.75

(*f*) Find the last balance and write it in the proper place.

THIRD FACT ABOUT BANKING: **It is unlawful to overdraw your account.** "Overdraw" means to write a check for more money than you have in the bank. The next paragraph describes a method of keeping an account of your balance at the bank so that you will not overdraw.



THE STUB AND THE CHECK BOOK

Check books are made in such a way that when you use a check, you tear it out of a check book. For each check there is a space (called the "stub") in the book to record the amount of the check, the date, to whom made out, etc. Notice the facts asked for on the stub. Every check has

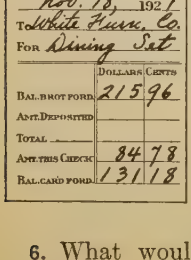
a stub. The facts called for by the stub should be writte in before you tear out the check to give it to anyone i payment.

QUESTIONS

1. Do you see how keeping every stub correctly wi show you what your bank balance is after you have writte each check?

2. Why are checks and stubs numbered?

3. When you deposit money in your checking account, d you add it to or subtract it from the balance on the stub Why?



4. When you write a check, do yo add the amount to or subtract it from th last balance on the stub? Why?

5. Suppose the stub for Check No. 1 were as shown in the picture. Make check and stub. Number them 18.

Use Check 18 to pay Raymond Jone $15.75 for repairing the roof of your house Show what ought to appear on Stub 18

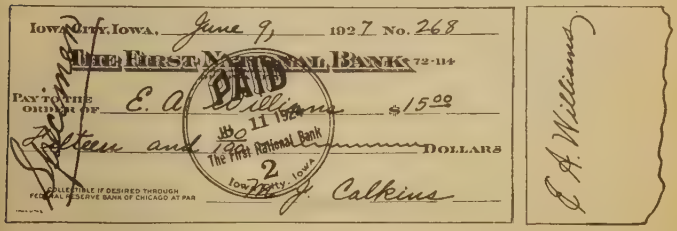
6. What would be the balance brought forward (*Ba Bro't For'd*) on Stub 19? Make Stub 19. Suppose befor you use Check 19, you deposit $100.00 in your checkin account. What would be written on Stub 19?

7. Show Stub 19, if Check 19 was for $12.00 for a con tribution to the Red Cross.

8. What does "overdrawing" your account mean?

9. Why must one be careful never to overdraw hi checking account?

FOURTH FACT ABOUT BANKING: **When a check is cashed, the person who receives the money must write his name on the back of the check.** This is called "indorsing" the check. Below is a picture showing both sides of the same canceled (paid) check:


IOWA CITY, IOWA. June 9, 1927 No. 268
THE FIRST NATIONAL BANK 72-114
PAY TO THE ORDER OF E. A. Williams $15.00
Fifteen and DOLLARS
COLLECTIBLE IF DESIRED THROUGH FEDERAL RESERVE BANK OF CHICAGO AT PAR
PAID
JUN 11 1927
The First National Bank
Iowa City, Iowa
2
Mary J. Calkins
E. A. Williams

The signature on the back of this check shows that Mr. Williams was the one who cashed it, thus recording the fact that he has received the sum of money written on the check. The canceled check becomes a **receipt,** because it shows that the bank has paid the money from what you have on deposit. Thus, your canceled checks are a valuable record of money you have paid out.

All your canceled checks will have on the back the signature of the person to whom you paid the check. The bank will pay the money from your account, on your check, only when the name of the person receiving the money is written on the back of your check, to show that he has received the money.

QUESTIONS

1. Suppose Mrs. E. A. White paid you with a check for $5.00. What must you do to the check before the bank will pay you the $5.00 from Mrs. White's account?

2. When the monthly statement of an account is given to a depositor together with all his canceled checks, what will the depositor find on the back of each check?

3. If you should indorse a check and then lose it, what might happen?

4. If a check is given you, why should you wait to indorse it until immediately before cashing it?

5. Why is it wise to keep all canceled checks?

Ask your father or mother to show you a real check book with stub records.

87. Review Questions on Banking Practices. (CLASS WORK.) Answer these questions in class after you have studied them by yourself.

1. What is a "stub"? What are stubs for?

2. What is found on a monthly statement?

3. When you write out a check and give it to someone, what becomes of it before you get it back from the bank?

4. Why must checks always be indorsed before they are cashed?

5. Why should you be careful to number your checks and your stubs?

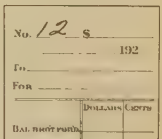
6. Why is it unlawful and unfair to overdraw your account at the bank?

7. How do you deposit money at the bank?

Prepare checks and stubs. Pretend they are Nos. 11 and 12 in your check book. Show what would be on your Checks 11 and 12 and on the stub of each of these checks if the facts given in Questions 8 and 9 were true.

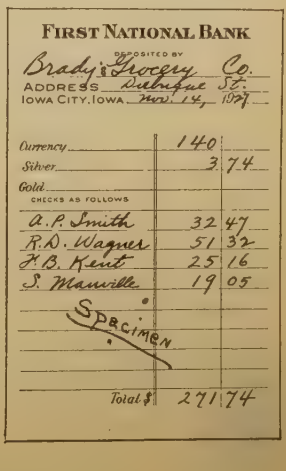
8. Today (What date is it? Use it.) your balance is $144.50. Write Check No. 11 to pay $10.00 to the Community Hospital Fund. Record facts on Stub No. 11, and record *Bal. Bro't For'd* on Stub 12.

9. Later in the day you deposit $50.00 in your checking account. That $50.00 would be recorded on Stub No. 12. Copy this stub and show everything that would be on it after carrying forward the balance and depositing the $50.



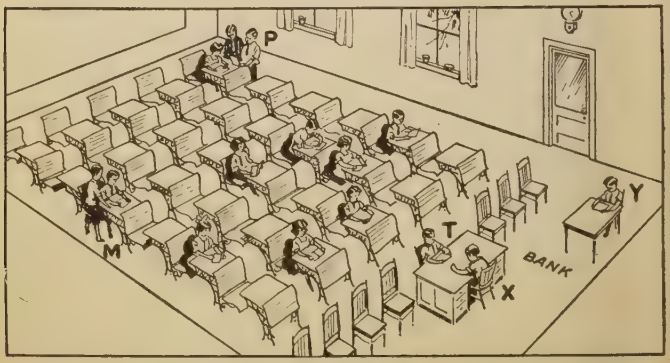
10. Brady's Grocery Company made the following deposits on Nov. 14, 1927, in the First National Bank, Iowa City, Iowa. See if the deposit slip is correct.

Currency (paper money)	$140.00
Silver	3.74
Check (A. P. Smith)	32.47
Check (R. D. Wagner)	51.32
Check (F. B. Kent)	25.16
Check (S. Manville)	19.05
	$271.74



11. If you have time, it is good fun to play bank in your own school room. Choose some pupils to run the bank, and others to be grocers, carpenters, etc. Pay bills to each other. Play that each pupil has $300 on deposit. The picture on page 120 shows a class "playing" bank.

88. Running a Bank. (A Class Project.) Make plans to have a play bank. Choose a committee to run the bank. Decide what each pupil must do. Appoint two boys to



P. HOSPITAL FINANCE COMMITTEE AT WORK. *M.* BRADY'S GROCERY STORE. *T.* PERSON DOING BUSINESS AT THE BANK. *X.* THE BANK TELLER. *Y.* A BANK EMPLOYEE KEEPING THE BOOKS OF THE BANK. DEPOSITORS GOING OVER THEIR BANK ACCOUNTS

run a grocery store, two girls to run a bake-shop, have a Finance Committee for the Red Cross, and have several pupils be private citizens.

When you begin to play, allow the grocery store to have a bank account of $500. Make believe that the citizens owe bills, and receive money for work. Allow each citizen to have at least $100.00 on deposit.

Talk over your plans in class. Make, or get from a bank, make-believe blank checks, blank statements, deposit slips, etc. Do your work carefully. Be sure that every one's account is balanced correctly before you complete the project.

☛ Do not spend more than two class periods on Art. 88.

89. Rapid Drill. (ORAL WORK.) Do these examples without your pencil. In quotients give first figure only.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$67\overline{)391}$	$\begin{array}{r}172\\-85\\\hline\end{array}$	$6+24+8$	$\begin{array}{r}178\\-99\\\hline\end{array}$	$64\overline{)449}$	$\begin{array}{r}59\\\times 6\\\hline\end{array}$
2.	$\begin{array}{r}123\\-85\\\hline\end{array}$	*Best form of this answer?* $6\frac{1}{9}^{2}$	$\begin{array}{r}154\\-76\\\hline\end{array}$	$97\overline{)585}$	$\begin{array}{r}89\\\times .7\\\hline\end{array}$	$54\overline{)528}$
3.	$96\overline{)849}$	$29+7+9$	Meaning of "indorse a check."	$7+26+8$	$86\overline{)775}$	$\begin{array}{r}164\\-95\\\hline\end{array}$
4.	$78\overline{)607}$	$\begin{array}{r}96\\\times 8\\\hline\end{array}$	$95\overline{)667}$	$\begin{array}{r}123\\-96\\\hline\end{array}$	$7+18+7$	$97\overline{)389}$
5.	$\begin{array}{r}120\\-57\\\hline\end{array}$	$\begin{array}{r}12.1\\-7.6\\\hline\end{array}$	$\begin{array}{r}174\\-78\\\hline\end{array}$	$8+38+8$	$\begin{array}{r}132\\-67\\\hline\end{array}$	$\begin{array}{r}143\\-84\\\hline\end{array}$
6.	$37\overline{)298}$	$8\frac{2}{5}=\frac{}{5}$	$\begin{array}{r}130\\-38\\\hline\end{array}$	$97\overline{)875}$	$\begin{array}{r}130\\-49\\\hline\end{array}$	$68\overline{)3.40}$

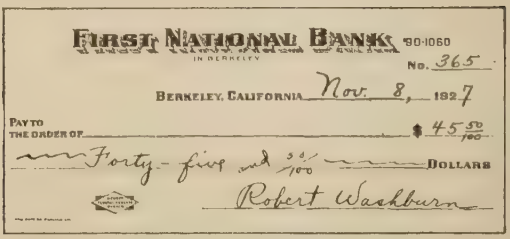
SELF-TESTING DRILL NO. 9

90. Directions. Time allowed: 20 minutes. Rate yourself. **Are you keeping your Progress Chart accurately?**

1. $\begin{array}{r}17945\\-8698\\\hline\end{array}$ 2. $\begin{array}{r}648\\\times 56\\\hline\end{array}$ 3. $16\overline{)192}$ 4. $\begin{array}{r}.04\\\times 404\\\hline\end{array}$ 5. $84\overline{)6943}$

Go on to the next page.

6. What is missing from this check?


FIRST NATIONAL BANK
IN BERKELEY
90-1060
No. 365
BERKELEY, CALIFORNIA Nov. 8, 1927
PAY TO THE ORDER OF $ 45 50/100
Forty-five and 50/100 DOLLARS
Robert Washburn

7. $5.5+63.4+77+.8=?$

8. Write $7\frac{1}{9}$ as an improper fraction.

9. A rectangular lot is 12 rods by 7 rods. Find its area.

10. $.603 \times .5465$

11. $5\frac{2}{7}-\frac{4}{7}=?$

12. $\frac{5}{9}$ of $\frac{3}{10}=?$

13. 5367×97

14. $\frac{7}{12}+\frac{1}{4}=?$

15.

July 1, 1927, Deposit $91.42
July 8, 1927, Checks cashed 14.19
Balance, July 8 .....

16. $.042\overline{)97.86}$

17. $\frac{9}{22}\div 6\frac{3}{4}=?$

18.
$14\frac{1}{3}$
$10\frac{1}{2}$
$8\frac{5}{6}$

19. $2\frac{2}{21}\times\frac{9}{22}=?$

20. $9\frac{1}{6}-4\frac{5}{9}=?$

STANDARDS

Number correct	0–4	5	6	7–8	9	10–11	12	13	14	15–16	17–19	20
Rating....	0	1	2	3	4	5	6	7	8	9	10	

Remember to use the Self-Help Index (page 457) whenever independent study is needed. **Are you correcting mistakes?**

ALIQUOT PARTS

91. Aliquot Parts of 100. (Class Work.) To know certain facts perfectly saves time in the end. The following list of facts should be remembered, so that when you have to use them, you will know them without having to work them out. Study silently on this article before the oral work begins. **Aliquot** is a rather uncommon word. Look it up in the dictionary.

If any fact in this list is not clear to you, work it out for yourself to be sure that it is correct, and then learn it.

Thus, the first fact is that $33\frac{1}{3} = \frac{1}{3}$ of 100. See if that statement is really true.

$\frac{1}{3}$ of $100 = \frac{1}{3} \times 100 = \frac{100}{3}$ or $3\overline{)100} = 33\frac{1}{3}$.

Test out any other statement you want to.

$$
\begin{array}{r}
33\frac{1}{3} \\
3\overline{)100} \\
\underline{9} \\
10 \\
\underline{9} \\
1
\end{array}
$$

TABLE OF ALIQUOT PARTS

$33\frac{1}{3} = \frac{1}{3}$ of 100
$66\frac{2}{3} = \frac{2}{3}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100
$25 = \frac{2}{8}$ or $\frac{1}{4}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100
$50 = \frac{4}{8}$ or $\frac{1}{2}$ of 100
$62\frac{1}{2} = \frac{5}{8}$ of 100
$75 = \frac{6}{8}$ or $\frac{3}{4}$ of 100
$87\frac{1}{2} = \frac{7}{8}$ of 100
$25 = \frac{1}{4}$ of 100
$50 = \frac{2}{4}$ or $\frac{1}{2}$ of 100
$75 = \frac{3}{4}$ of 100
$16\frac{2}{3} = \frac{1}{6}$ of 100
$33\frac{1}{3} = \frac{2}{6}$ or $\frac{1}{3}$ of 100
$50 = \frac{3}{6}$ or $\frac{1}{2}$ of 100
$66\frac{2}{3} = \frac{4}{6}$ or $\frac{2}{3}$ of 100
$83\frac{1}{3} = \frac{5}{6}$ of 100
$20 = \frac{2}{10}$ or $\frac{1}{5}$ of 100
$40 = \frac{4}{10}$ or $\frac{2}{5}$ of 100
$60 = \frac{6}{10}$ or $\frac{3}{5}$ of 100
$80 = \frac{8}{10}$ or $\frac{4}{5}$ of 100
$100 = \frac{100}{100}$ or $\frac{10}{10}$ or $\frac{1}{1}$ or the whole of 100.

➡It will save you both time and trouble if you memorize the table of aliquot parts. Think of these aliquot parts as time-savers.

ORAL DRILL

1. Tell quickly what fraction of 100 each of the following is: 50, $33\frac{1}{3}$, $12\frac{1}{2}$, 60, 75, $66\frac{2}{3}$, $87\frac{1}{2}$, 20.

2. Tell quickly what fraction of 100 each of the following is: 25, $37\frac{1}{2}$, $16\frac{2}{3}$, 80, $62\frac{1}{2}$, $83\frac{1}{3}$, 100, 40.

3. Give quickly the missing number for each of the following: (It is unfair to mark in your book.)

$\frac{3}{4}$ of 100 is .....
$\frac{1}{2}$ of 100 is .....
$\frac{5}{6}$ of 100 is .....
$\frac{4}{5}$ of 100 is .....
$\frac{2}{3}$ of 100 is .....
$\frac{5}{8}$ of 100 is .....
$\frac{1}{5}$ of 100 is .....
$\frac{3}{8}$ of 100 is .....

4. Give quickly the missing number:

$\frac{1}{3}$ of 100 is .....
$\frac{2}{8}$ of 100 is .....
$\frac{4}{5}$ of 100 is .....
$\frac{3}{8}$ of 100 is .....
$\frac{3}{6}$ of 100 is .....
$\frac{7}{8}$ of 100 is .....

5. Give quickly the missing number:

$\frac{1}{4}$ of 100 is .....
$\frac{1}{8}$ of 100 is .....
$\frac{1}{6}$ of 100 is .....
$\frac{7}{8}$ of 100 is .....
$\frac{2}{5}$ of 100 is .....
$\frac{1}{3}$ of 100 is .....
$\frac{3}{5}$ of 100 is .....
$\frac{1}{1}$ of the whole of 100 is ....

6. Give quickly the missing number:

50 is $\frac{1}{2}$ of 100.
$33\frac{1}{3}$ is.....of 100.
25 is.....of 100.
$12\frac{1}{2}$ is.....of 100.
$66\frac{2}{3}$ is.....of 100.
10 is.....of 100.
100 is.....of 100.
$87\frac{1}{2}$ is.....of 100.

REVIEW OF FRACTIONS

92. Test in Multiplication of Fractions. (WRITTEN WORK.) Find the products in the examples below. Be sure to leave your answers in simplest form. Time allowed: 15 minutes.

EXERCISE

1. Four and two-fifths times two and three-elevenths.
2. $7 \times \frac{1}{2}$
3. $\frac{2}{3} \times \frac{1}{3}$
4. $\frac{3}{4} \times \frac{2}{7}$
5. $\frac{4}{5} \times \frac{5}{8}$
6. $2\frac{1}{4} \times \frac{16}{27}$
7. $4\frac{1}{2} \times \$3.50$
8. $\frac{7}{9}$ of $\frac{3}{14}$
9. $\frac{1}{2} \times 2\frac{1}{5}$
10. $\frac{5}{12}$ of 4
11. $5\frac{1}{7} \times 1\frac{9}{12}$
12. $5 \times \frac{4}{21}$
13. $8\frac{1}{3} \times \frac{2}{7}$
14. $\frac{5}{11} \times \frac{2}{3}$
15. $\frac{9}{16} \times 8$
16. $\frac{4}{15} \times \frac{3}{16}$
17. $\frac{3}{8} \times \frac{4}{9} \times 1\frac{1}{5}$
18. $1\frac{1}{4} \times \$1.60$
19. $\frac{1}{2} \times \frac{1}{5} \times 7$
20. $\frac{5}{12}$ of $\frac{3}{10} \times \frac{4}{5}$
21. $3\frac{1}{3} \times \frac{1}{4} \times 1\frac{1}{2}$
22. $\frac{1}{2}$ of $\frac{3}{4} \times \frac{2}{3}$
23. $1\frac{4}{5} \times \frac{1}{4} \times 15$
24. $1\frac{2}{9} \times \frac{3}{5} \times 1\frac{2}{3}$
25. $1\frac{1}{7} \times \frac{1}{6} \times 5\frac{1}{4}$
26. $\frac{1}{2}$ of $\frac{8}{15}$ of $\frac{5}{16}$
27. $7 \times 2\frac{2}{3} \times \frac{3}{10}$
28. $\frac{7}{12} \times \frac{2}{7}$
29. One-half times three twenty-fifths.
30. One-third of one-fourth times five.

An Important Note. If you found the Test in Multiplication of Fractions hard, decide for yourself to do some review work. If you worked less than 16 examples correctly, find references to articles on Multiplication of Fractions in the Self-Help Index and study those articles carefully.

The successful man is the one who decides to improve without being made to do so.

PROBLEMS IN BUSINESS ARITHMETIC

93. The Use of Sales Slips in Stores. (WRITTEN WORK.) Most of us have been in department stores with our parents when they were buying goods of some kind. When the salesman takes our purchase to wrap it up, he makes out a slip of paper which tells what we bought and how much we are to pay for the purchase. These little slips of paper are called "sales slips." Here is a picture of a sales slip.

IN CASE OF ERROR PLEASE RETURN THIS BILL

E. A. HOLDEN

DRY GOODS AND MEN'S FURNISHINGS

7030-32 North Clark Street

Everything Ready to Wear for Men, Women and Children
WE SELL FOR CASH ONLY. ONE PRICE TO EVERYBODY.

9/8/ 193 7.

Name Raymond Ward

Dept. Amt Rec'd 9.95 Sold by 6

6	Yds.		50
1	Shirt		2 70
1 pr.	Shoes		4 95
2	neckties		1 80
			9 95

A salesman or salesgirl has to do arithmetic all day long. Money has to be changed with almost every purchase. Prices may have to be added to find how much the customer owes. Fractions and decimals are constantly used.

The hardest arithmetic in a department store is probably the multiplication used to find out what to charge a customer in a case like the following:

Mrs. Manville buys $3\frac{1}{2}$ yards of cloth at 75¢ a yard. The salesgirl has to multiply 75¢ by $3\frac{1}{2}$ to find how much to charge for the purchase.

On page 127 you will find a list of sample problems copied from sales slips. These problems represent real purchases. They are the arithmetic work done in one day by a salesgirl in a certain department store. See if you could do the arithmetic that would have to be done for each purchase.

Before you leave an answer be sure that it is reasonable. Some of these problems can be worked in two ways.

Thus, in Problem 1: $3\frac{3}{4} \times 89¢$ (multiplication of fractions). $3.75 \times \$.89$ (multiplication of decimals).

In Problem 3: $1\frac{1}{2} \times 23¢$ (multiplication of fractions). $1.5 \times \$.23$ (multiplication of decimals).

The sign @ means "at."

Find the cost of each of these:

1. $3\frac{3}{4}$ yd. gingham @ 89¢ a yd.
2. $\frac{3}{4}$ yd. belting @ 25¢ a yd.
3. $1\frac{1}{2}$ yd. elastic @ 23¢ a yd.
4. $3\frac{1}{3}$ yd. ribbon @ 69¢ a yd.
5. $13\frac{2}{3}$ yd. cretonne @ 59¢ a yd.
6. 23 yd. trimming @ 29¢ a yd.
7. $7\frac{1}{2}$ yd. voile @ 29¢ a yd.
8. $1\frac{1}{4}$ yd. ribbon @ 69¢ a yd.
9. $1\frac{3}{4}$ yd. ribbon @ 39¢ a yd.
10. $8\frac{1}{2}$ yd. crepe @ 29¢ a yd.
11. $4\frac{3}{4}$ yd. lace @ \$1.29 a yd.
12. $\frac{3}{4}$ yd. belting @ 35¢ a yd.
13. $1\frac{1}{2}$ yd. percale @ 19¢ a yd.
14. $2\frac{5}{6}$ yd. percale @ $13\frac{1}{2}$¢ a yd.
15. 14 yd. unbleached muslin @ 19¢ a yd.
16. 10 yd. tape @ $2\frac{1}{2}$¢ a yd.
17. $17\frac{5}{6}$ yd. bleached muslin @ 24¢ a yd.
18. $3\frac{2}{3}$ yd. percale @ $17\frac{1}{2}$¢ a yd.
19. $1\frac{5}{8}$ yd. ribbon @ 52¢ a yd.

THE SAVINGS BANK

94. Another Kind of Bank Account. (STUDY PERIOD.) Study this article by yourself before the oral recitation begins. If there is anything that you do not understand, be ready to ask good questions during the oral work.

You have already studied about depositing money which you expect to use by drawing checks to pay bills.

When you have money which you wish to **save,** it is wise to put it in a savings bank. This is a good way to save, for two reasons: (1) the money will be very safe there, and (2) the bank will pay you for the use of your money; that is, the bank will pay you **interest on your account.**

The savings bank pays you interest on your account, not as a favor to you, but because it is good business. Money is valuable, and people pay for the use of money just as they pay for the use of a house or the use of a telephone. If you put your money in a savings bank, your money will work for you, because it will earn interest. And you will be surprised to see how rapidly your money will increase.

A good habit for you to develop is that of earning and **saving regularly** even a small amount of money. As Benjamin Franklin said, "A penny saved is a penny earned."

On the next page are two advertisements of savings banks. Study them.

CITIZENS SAVINGS BANK
We pay 3% interest. We welcome small accounts.
Keep your savings growing.

FRANKLIN SAVINGS BANK
You can start an account with One Dollar.
4% paid on deposits.

QUESTIONS

1. What is meant by the statement: "Money in a savings bank will work for you"?

2. If you own land and allow another man to use it, he pays you for the use of the land. Suppose you own money and put it in a savings bank. Will the bank pay you for the use of your money?

3. What do we call the money which a savings bank pays for the use of our money?

4. Could you start a savings account if you had only $1.00 to deposit?

5. Do you know where there is a savings bank?

6. How much interest will that savings bank pay?

7. Do you know anyone who has a savings account?

8. Have you started a savings account yet?

9. In the advertisements of a savings bank what do you think 3% and 4% mean? If you do not know, remember this question until you come to Art. 99.

10. Send a committee of your class to a savings bank to find out how a savings account is started, what records are kept, and how much interest is paid.

Have the committee make a report to the class. After the report, ask the committee questions about savings accounts.

PROBLEM SCALE NO. VII

95. Problems Using Fundamental Processes. (WRITTEN WORK.) You can earn 187 points by working every problem correctly. Find your score and rate yourself as directed for Scales I to III.

Notice that some of the problems have more than one part. Number and letter your answers plainly.

1. If a dress pattern cost 40 cents, the material $14.00, and the lace for trimming $2.00, what was the total cost of the dress? (3)

2. If one bar of chocolate costs 27¢, what do one dozen bars cost at the same rate? (4)

3. If a pair of hose costs 98¢, what will be the cost of 6 pairs of the same hose? (4)

4. The satin required to make a certain dress costs $13.00, the lace $3.00, the lining and bindings $1.25. The dress ready-made costs $47.00. How much would you save by getting the material and making it yourself? (5)

5. A certain window and its frame are $7\frac{1}{2}$ feet high. How many feet of drapery will be required for a strip down each side of the window and frame? (5)

6. The price of a certain magazine is 10 cents. The subscription by the year (12 issues) is $1.00. How much

cheaper is it to subscribe for a year than to buy single copies for a year? (5)

7. Pineapple costs 35 cents a can when bought by the can, but you can buy a crate of one dozen cans for $3.00. How much would you save per dozen cans if you bought by the dozen? (5)

8. If today is Saturday, Nov. 24th, what will be the date two weeks from today? (5)

9. A woman has an allowance of $65 a month and spends for foodstuffs the amount shown at the right. Adding $5 spent for extras, how much can she save a month for herself? (6)

Meat	$10.00
Groceries	33.00
Milk	2.50
Bread	2.50
	$48.00

10. Suppose it is now Nov. 24th. How many days are left until Christmas? (6)

11. If two boxes of jello cost 25 cents, what is the cost per dozen boxes at the same rate? (6)

12. If the cost in serving a Thanksgiving dinner averages 40¢ per plate, how much can be made by serving 100 persons at $1.00 per plate? (6)

13. What is the saving on a case of 24 cans of peaches which have been reduced from 30¢ to 23¢ a can? (6)

14. If a family uses on an average three loaves of bread in two days, what does bread cost them per month (30 da.) at twelve cents a loaf? (6)

15. There are $6\frac{3}{4}$ pounds of candy to be divided equally among 4 children. How much candy should each child receive? (6)

16. The following is a recipe for a date-fruit cake, which will serve ten people. Find amounts needed to make a cake which will serve 15 people.

(*a*) 1 tsp. baking powder	(6)	(*g*) 2 cups raisins	(7)
(*b*) 2 packages dates	(6)	(*h*) $\frac{1}{4}$ tsp. salt	(7)
(*c*) 4 cups sugar	(6)	(*i*) $\frac{1}{2}$ tsp. soda	(7)
(*d*) 2 tsp. cinnamon	(6)	(*j*) $1\frac{1}{2}$ cups butter	(7)
(*e*) 2 cups milk	(6)	(*k*) $6\frac{1}{2}$ cups flour	(7)
(*f*) 6 eggs	(7)		

17. If forty pieces of candy require 2 cups of sugar worth five cents per cup, $\frac{1}{2}$ bar of chocolate @ 25¢ a bar, and one egg when eggs are worth 35¢ a doz., what is the average cost per piece of candy? (8)

18. Find the cost of a velvet dress requiring:

$3\frac{1}{2}$ yd. of chiffon velvet	@ $7.40 a yd.	
$\frac{1}{2}$ yd. of China silk	@ $1.50 a yd.	
$1\frac{3}{4}$ yd. of chiffon	@ $2.00 a yd.	
$\frac{3}{4}$ yd. of gold cloth	@ $7.50 a yd.	
Pattern, etc.	$1.37	(9)

19. How much material 24 in. wide will be required to cut twelve quilt blocks each 12 in. square? (10)

20. A label on a can of paint reads: "This can of paint will cover 8 sq. yd. of surface." How many cans must I buy to paint the four walls of a room which is 6′ wide, 10′ long, and 8′ high, allowing for two windows, each 8 ft. $\times$ 3 ft.? (10)

STANDARDS

Superior work:	94–187 points	Slow work:	16–29 points
Good work:	50– 93 points	Inferior work:	0–15 points
Average work:	30– 49 points		

SOME CAUSES OF ERRORS

96. Types of Errors Often Made. (Class Discussion.) Surely no pupil ever went through the sixth grade without making a mistake in arithmetic. But all of us want to make as few mistakes as possible. We can **decrease** the number of our mistakes by studying our own weaknesses carefully and then practicing to overcome those weaknesses. You are frequently referred to the Self-Help Index so that you can correct a weakness by study and practice just as soon as you realize your need.

To correct a weakness most effectively you must of course **know what it is.** Pupils rarely improve in general. They improve **little by little** by first taking care of one particular weakness, and then another, and so on.

The explanations below will be interesting to you because they point out common weaknesses of sixth-grade pupils.

ADDITION ERRORS

Error 1. **Mistakes in combinations.** The most frequent error in addition is getting the wrong sum for a number combination, such as $2+2=5$, $9+7=17$, etc. These combinations are often "hidden" in a column as shown below.

$8+9$ is easily seen, but $17+8$, and $25+7$ are hidden in the column. There is little excuse for any sixth-grade pupil making mistakes in addition combinations. If some combinations are always bothering you, would it not be a good plan to conquer them once for all without delay? If you will use your Self-Help Index carefully, you will find some articles which list the harder addition combinations. Study these if you are bothered by addition.

7
8
9
8

Error 2. **Trouble with carrying.** Sometimes we don't carry at all, sometimes we carry 1, sometimes 2, sometimes 3, and so on. Many pupils find that carrying is their greatest weakness in addition. Study these samples of errors in carrying:

(*A*)
$$\begin{array}{r} 43 \\ 68 \\ 22 \\ \hline 123 \end{array}$$

Error: *Forgot to carry.*

(*B*)
$$\begin{array}{r} 79 \\ 28 \\ 3 \\ 20 \\ \hline 120 \end{array}$$

Error: *Carried 1 instead of 2.*

If carrying troubles you, now is a good time to "cure" this weakness. Use the Self-Help Index to guide you.

Error 3. **Losing one's place in the column.** The third error which many pupils make is a very natural one. It is not easy to add up a column and keep your place so that you will add all the numbers but not add any one number twice. Notice two errors shown below.

(*A*)
$$\begin{array}{r} 3 \\ 7 \\ 8 \\ 9 \\ 4 \\ \hline 38 \end{array}$$

Error: *Added the 7 twice.*

(*B*)
$$\begin{array}{r} 3 \\ 7 \\ 8 \\ 9 \\ 4 \\ \hline 24 \end{array}$$

Error: *Skipped the 7 entirely.*

☛ Here is some good advice: When you make a mistake in addition, find out what kind of a mistake it is and then overcome that weakness by careful practice.

SUBTRACTION ERRORS

Error 1. **Wrong answers to the subtraction combinations.** Would you excuse a sixth-grade pupil who thought that $16-7=8$ or that $11-4=6$? As you work subtraction, if you notice that certain combinations trouble you, that you get them wrong or have to think a long time to get the correct answer, make a list of those combinations and practice on them until they become as easy as $4-2=2$. The Self-Help Index will show you where you can find worthwhile practice on the subtraction combinations.

Error 2. The next most serious "hard-spot" in subtraction is forgetting that a number has been changed to make the subtraction easier. Study the errors shown below.

(*A*) 342
−183
169

Error: *Forgot that a number in the* {4, 8} *column was changed.*

(*B*) 4000
−2673
1427

Error: *Remembered the change in* {0, 7} *column but forgot it in the* {0, 6} *column.*

Any pupil who has the right to be in the sixth grade ought to get the subtraction examples in the Self-Testing Drills correct about every time. If subtraction is bothering you, take the responsibility yourself of correcting this weakness through independent study and practice.

Careful workers who check for accuracy often discover mistakes before an example is counted wrong.

SELF-TESTING DRILL NO. 10

97. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Should sixth-grade pupils be expected to work Example 12 correctly?**

1. What number is $\frac{3}{4}$ of 100?

2. $\frac{5}{21}+\frac{4}{21}+\frac{2}{21}=?$

3. $644.9-239.9=?$

4. $496.2+66.3=?$

5.
885
946
767
730
698
679

6.
143597
−79619

7.
689
×34

8. $\frac{15}{16}\div 3=?$

9.
569
758
487
575
558
495

10.
$7\frac{1}{2}$
$4\frac{5}{6}$
$\frac{1}{4}$

11. $\frac{1}{2}\div\frac{1}{2}=?$

12. $.11\overline{)605}$

13.
143
×269

14.
78.9
×90

15. The scale of a map is $\frac{1}{4}''$ to a mile. How long must a line be on this map to represent 8 miles?

16. As shown by the diagrams at the right, what fraction of the money earned was saved?

17. $4\frac{5}{6}-\frac{1}{2}=?$

18. $1\frac{1}{9}\times 5\frac{1}{4}=?$

19.
1 bu. 3 pk.
4 bu. 1 pk.
1 bu. 1 pk.
3 pk.

20. A boy who weighed 100 lb. before an illness, lost $12\frac{1}{2}$ lb. What fractional part of his weight did he lose?

STANDARDS

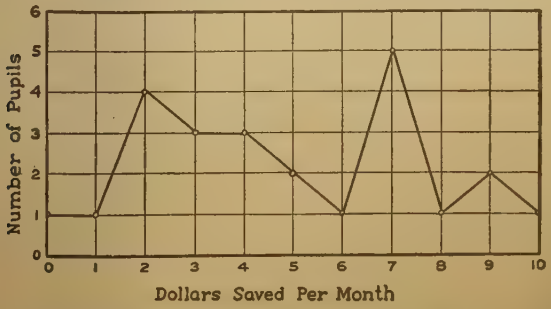
Number correct	0–5	6–7	8–9	10–11	12	13	14	15	16	17–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Are you correcting errors and using the Self-Help Index? Do you practice on the drills on which you make low ratings?

SUMMARY

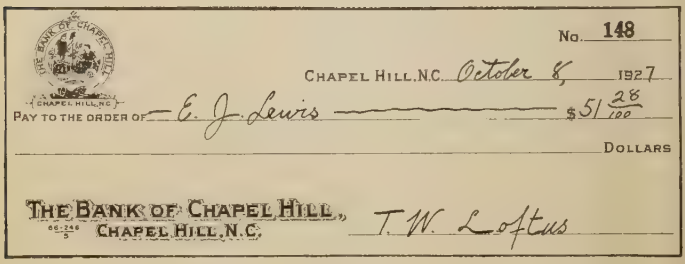
98. General Review. (WRITTEN WORK.) If you have forgotten how to do any of the work below, you should restudy it before beginning Chapter V.

1. Answer questions (*a*), (*b*), and (*c*) below from this graph, which represents the amounts of money saved in one month by pupils of a sixth-grade class in Pennsylvania.

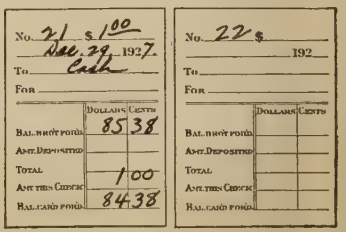


(*a*) How many pupils in this class saved \$2 in a month?
(*b*) How many pupils in this class saved \$10 in a month?
(*c*) How many pupils in this class saved \$5 in a month?

2. Copy the check shown below and add anything that is missing.


No. 148
CHAPEL HILL, N.C. October 8, 1927
PAY TO THE ORDER OF E. J. Lewis $51 28/100
DOLLARS
THE BANK OF CHAPEL HILL
CHAPEL HILL, N.C.
T. W. Loftus

3. Copy Stub 22 and show what ought to be on it, given these facts: Check 22 was made out to The Huntington Stove Company for a gas stove; amount, $55.75; paid Jan. 4, 1928; but before Check 22 was made out, $125.00 was deposited in the bank.



4. Write on your paper what should be on the blanks below.

(*a*) $\frac{1}{4}$ of 100 is.....
(*b*) $\frac{3}{8}$ of 100 is.....
(*c*) $\frac{2}{3}$ of 100 is.....
(*d*) $\frac{3}{4}$ of 100 is.....
(*e*) $\frac{2}{5}$ of 100 is.....
(*f*) $\frac{1}{6}$ of 100 is.....

5. Edward Smith deposited $100 in a savings bank on January 3, 1927. On July 1 he drew out $101.75. His money had earned $1.75. What is the name given to money earned in this way?

6. Draw a graph which represents these facts: Walter brought to school 12 samples of graphs, Emily brought 8, Paul 6, Margaret 3. (See Art. 83.)

7. Write on your paper what should be on the blanks below.

(*a*) $66\frac{2}{3}$ = ...of 100. (*d*) $16\frac{2}{3}$ = ...of 100.
(*b*) 75 = ...of 100. (*e*) $37\frac{1}{2}$ = ... of 100.
(*c*) 20 = ...of 100. (*f*) 1 = ...of 100.

8. The quotient of this example is wrong, but the 72 is correct. Study the example and then write the correct quotient.

$$3.42\overline{)24624}\qquad 72$$

9. Check your answer to 8 above.

10. One of the words below is misspelled. Write it correctly.

multiplicand *decimal* *diviser* *interest*

CHAPTER V

INTRODUCTION TO PERCENTAGE. INSTRUCTION IN PROBLEM SOLVING

LEARNING HOW TO WORK PERCENTAGE

99. Per Cent (%) Means Hundredths. (STUDY PERIOD.) The fraction $\frac{25}{100}$ may be written in three different ways as shown below.

$\frac{25}{100}$ → fraction way.
.25 → decimal way.
25% → per cent way.

Any fraction whose denominator is $\frac{}{100}$ can be written a a fraction, as a decimal, or as a per cent (%).

Per cent (%) means **hundredths,** or $\frac{}{100}$.

4% or 4 per cent means $\frac{4}{100}$ or .04.

10% or 10 per cent means $\frac{10}{100}$ or .10 or .1 or $\frac{1}{10}$.

$33\frac{1}{3}$% or $33\frac{1}{3}$ per cent means $\frac{33\frac{1}{3}}{100}$ or $.33\frac{1}{3}$ or $\frac{1}{3}$.

$12\frac{1}{2}$% or $12\frac{1}{2}$ per cent means $\frac{12\frac{1}{2}}{100}$ or $.12\frac{1}{2}$ or .125 or $\frac{1}{8}$.

25% or 25 per cent means $\frac{25}{100}$ or .25 or $\frac{1}{4}$.

50% or 50 per cent means $\frac{50}{100}$ or .50 or .5 or $\frac{1}{2}$.

75% or 75 per cent means $\frac{75}{100}$ or .75 or $\frac{3}{4}$.

$66\frac{2}{3}$% or $66\frac{2}{3}$ per cent means $.66\frac{2}{3}$ or $\frac{2}{3}$.

$16\frac{2}{3}$% or $16\frac{2}{3}$ per cent means $.16\frac{2}{3}$ or $\frac{1}{6}$.

➡This new way of using hundredths is called **percentage** All percentages are decimals, or fractions with the denominator $\frac{}{100}$, written another way.

What do you think each of the following means?

1. It will save you trouble in the 7th and 8th grades if you master the first work in **percentage** very thoroughly, because about 75% of 7th- and 8th-grade arithmetic work calls for the use of percentage.

2. A successful business man said, "80% of success is due to hard work and absolute honesty."

3. A wise doctor said, "To be healthy spend $33\frac{1}{3}$% of every 24 hours in good, sound sleep."

4. To save 50% of your dentist's bill, clean your teeth carefully each day.

5. About 45% of the people in the United States live on farms.

6. The population of New York increases 10% yearly.

7. About 65% of the surface of the earth is ocean or lake.

8. John got 100% of his problems correct, Mary got 70% correct, and Henry got 50% correct.

☛ 100% of anything means "all of it;" 200% of anything means "twice it;" 300% means "3 times it," 400% means "4 times it," etc. What do 500%, 700%, and 1000% of anything mean?

9. The Savings Bank pays 4% interest on deposits.

10. The People's Gas Company allows a 2% reduction upon monthly bills paid before the 10th of the month.

➡ **Per cent is the business way of saying hundredths. One hundred is the denominator that occurs most frequently in the business uses of arithmetic. "Of" means "times" in percentage.**

100. Practice with Per Cent (%). (STUDY PERIOD.) Study Parts (*A*), (*B*), and (*C*) of this article without help. Get clearly in mind the answer to every question. If there is anything that you do not understand, write out good questions to ask when the recitation begins.

PART (*A*). Turn back to Art. 91. Read the Table of Aliquot Parts, putting in **per cents.**

Read "$33\frac{1}{3} = \frac{1}{3}$ of 100" as "$33\frac{1}{3}\% = \frac{1}{3}$ of 100%."

PART (*B*). In studying the examples in the exercise below, work some of them with your pencil to make sure that your answers are correct. **Each example is written in several ways.** Read Art. 99 again if you wish.

Multiplying by a decimal gives the same answer as multiplying by the common fraction that is equal to the decimal. This is shown at the right.

$$\begin{array}{r} 84 \\ \times .25 \\ \hline 420 \\ 168 \\ \hline 21.00 \end{array}$$

$\frac{1}{4}$ of $84 = \frac{1}{4} \times 84 = \frac{84}{4}$

$4\overline{)84}$ or $4\overline{)84}$

1.

$.25 \times 84 = ?$
$\frac{25}{100}$ of $84 = ?$
$\frac{1}{4}$ of $84 = ?$
25% of $84 = ?$

2.

50% of $60 = ?$
$\frac{1}{2} \times 60 = ?$
$.50$ of $60 = ?$
$\frac{50}{100} \times 60 = ?$

3.

$.75$ of $4 = ?$
$\frac{75}{100}$ of $4 = ?$
$\frac{3}{4} \times 4 = ?$
75% of $4 = ?$

4.

$.36 \times 75 = ?$
$\frac{36}{100}$ of $75 = ?$
36% of $75 = ?$

5.

$.15 \times 200 = ?$
$\frac{15}{100} \times 200 = ?$
15% of $200 = ?$

6.

$\frac{1}{3}$ of $45 = ?$
$.33\frac{1}{3}$ of $45 = ?$
$33\frac{1}{3}\%$ of $45 = ?$

PART (*C*). Answer the questions below.

I. Does $\frac{1}{4}$ of any number = 100% of it, 50% of it, 25% of it, or 4% of it?

II. Does the whole of any number = 100% of it, 50% of it, 25% of it, or 4% of it?

III. Does $\frac{1}{3}$ of any number = 100% of it, 50% of it, 25% of it, or $33\frac{1}{3}$% of it?

IV. Does .04 of any number = 100% of it, 50% of it, 25% of it, 4% of it, or 40% of it?

V. Does $12\frac{1}{2}$% of any number = $\frac{1}{2}$ of it, $\frac{1}{4}$ of it, $\frac{1}{8}$ of it, or $\frac{1}{2}$ of it?

VI. Does $16\frac{2}{3}$% of any number = $\frac{5}{6}$ of it, $\frac{1}{3}$ of it, $\frac{1}{6}$ of it, or $\frac{1}{16}$ of it?

VII. Does $66\frac{2}{3}$% of any number = $\frac{6}{100}$ of it, $\frac{2}{3}$ of it, .6 of it, or .06 of it?

VIII. Does 25% of any number = $\frac{1}{25}$ of it, $\frac{4}{25}$ of it, of it, or $\frac{1}{4}$ of it?

IX. Does 6% of 100 = 60, $\frac{1}{6}$, 6, or $\frac{6}{100}$?

X. Does 6% of 200 = 12, 24, 8, or 16?

XI. Does 100% of 750 = 75000, 7500, 750, or 75?

XII. Does 1% of 625 = 6, $62\frac{1}{2}$, .625, or 6.25?

XIII. Is 100%, $\frac{1}{4}$%, 25%, or 50% of the rectangle shaded?

XIV. Is 3%, 13%, $33\frac{1}{3}$%, or $66\frac{2}{3}$% of the circle shaded?

Notice that the examples above are numbered I, II, etc., to XIV. What are such numbers called?

101. Using Percentage. (A READING LESSON.) The following clippings were taken from magazines and newspapers. You will notice that per cent (%) is found in your reading as well as in arithmetic. Study these and then answer the questions on the next page.

I

This year's sales at this writing show an increase of 26% over those of the corresponding period of 1926

II

AN ELECTRIC
FIRELESS COOKER
Saves 50% to 75% Fuel Cost

III

Tests made six months afterwards showed a great improvement in reading ability. All had improved markedly. The October average on the XX Reading Test was 54% correct. The March average was 82%.

IV

MONEY to Loan at

5½%

SEVERAL big funds have been placed with us for investment in high grade 5½% first mortgages. Loans made with these funds only on well-located Chicago apartments, business properties, or high-class residences.

V

Trade Review says: "Wage advances continue under discussion and a 4 cent hourly rate for common labor is mentioned, representing an increase of about 11 per cent. Chicago mills for the third consecutive week have broken the record."

VI

Steel Operations Set Record.

Another new high record for steel operations is reported this week. The Steel Corporation is running close to 93 per cent of capacity, the independents at 88 per cent, and the average for the whole industry is above 90 per cent.

"Still higher production of iron and steel is promised."

VII

Each year about 1,550,000 tons of copper are produced. Of this

The U. S. produces		62%
Japan	"	6%
Chile	"	5%
Canada	"	4%

VIII

From a School Principal's Report: "Children absent 25% of the time very often fail of promotion."

QUESTIONS

CLIPPING I. An increase of 26% equals roughly what fractional increase?

CLIPPING II. If 50% of the cost were saved, how much would you save if the fuel cost, without a fireless cooker, were $42? 50% means $\frac{50}{100}$ or $\frac{1}{2}$. $\frac{1}{2}$ of $42 is how much?

CLIPPING III. Suppose there were 100 questions. 54% correct means how many questions correctly answered?

CLIPPING IV. What does "Money to Loan at $5\frac{1}{2}\%$" mean? Suppose I borrow $100 for a year and pay $5\frac{1}{2}\%$ for the use of it. How much will I pay for the money? 1% of $100 is $1.00.

CLIPPING V. "An increase of about 11 per cent." If a young man earns $100 a month and gets an 11% increase, how much will his monthly pay then be?

CLIPPING VI. If 93 tons of steel rails are made per day in a factory which is working at 93% capacity, how many tons of rails would be made if the factory was working at 100%, or full capacity? ("Capacity" here means the largest amount the factory can produce.)

CLIPPING VII. How many tons of copper does the United States produce for every 100 tons produced in all? Does the United States produce about 5 times, 10 times, or 20 times as much copper as Japan?

CLIPPING VIII. If there are 20 school days in a month, how many days would a child be absent who was away 25% of the time?

➡ For tomorrow bring to class 4 examples of the use of per cent (%) in newspapers, magazines, or in some book. Be able to tell what the %, or per cent, means in each of your examples.

102. Practice on Percentage Relations. (CLASS WORK.) Read the following examples carefully. Notice that the three examples in each group deal with the same numbers.

(*A*)

1. 10% of 80 = ?
2. 8 is what % of 80?
3. If 10% of N = 8, 100% of N = what?

(*B*)

1. 30 is what % of 60?
2. 50% of 60 = ?
3. If 50% of a number = 30, the number (or 100% of it) = what?

(*C*)

1. If 25% of a number equals 12, the number itself is .....
2. Find 25% of 48.
3. 12 is what % of 48?

(*D*)

1. What is 75% of 36?
2. What % of 36 is 27?
3. When 27 = 75% of a number, 100% of it = ...

(*E*)

1. 15 is what % of 45?
2. When 15 = $33\frac{1}{3}$% of N, N itself equals .....
3. What is $33\frac{1}{3}$% of 45?

(*F*)

1. What number is 200% of 40?
2. 80 is what % of 40?
3. 80 equals 200% of what number?

(*G*)

1. 20 is what % of 30?
2. $66\frac{2}{3}$% of a number = 20. What is 100% of the number?
3. $66\frac{2}{3}$% of 30 = ?

(*H*)

1. If 100% = 300, 1% = what?
2. 3 is what % of 300?
3. If 3 = 1% of N, N = ...

(*I*)

1. 300% of 6 = ...
2. 18 is what % of 6?
3. If 18 = 300% of N, N must equal .....

103. Problems Using Percentage. (WRITTEN WORK.) Talk these problems over in class and then work them alone. In working, write down any questions that you want to ask about the problems after the study period.

1. Montana produces 17% of all the flaxseed raised in the United States. Of every 100 bu. of flaxseed raised in the United States, how many are raised in Montana?

2. Out of every hundred dollars earned, John saves $15. What % of his earnings does he save?

NOTE: 15 is how many hundredths of 100? What per cent?

3. Henry Wilson borrowed $100 for a year. The bank which loaned the money charged him 6% for the money for the year. When Mr. Wilson paid back the loan, he paid $106 instead of the $100 which he borrowed. Why was this?

4. Suppose there are 860 pupils in the Davis School. 85% of them are under 15 years of age. How many are under 15 years of age? (.85×860)

5. James earned $2.00 in one week. What per cent of the $2.00 did he earn on Saturday when he was paid $1.00 for that day's work?

NOTE: $1.00 is what fraction of $2.00? What per cent?

6. At a certain "marked-down" sale, goods were advertised to sell for 75% of their original price. Give the "marked-down" prices of these goods, if the regular prices were as follows: (REMEMBER: 75% of means .75× or $\frac{3}{4}$ times.)

Goods *A*: regular price $1.00.
Goods *B*: regular price $4.00.
Goods *C*: regular price 36¢.
Goods *D*: regular price $100.
Goods *E*: regular price $200.
Goods *F*: regular price $2.40.

M H. K. Stevenson

450 N. River St.

IN ACCOUNT WITH

McNAMARA FURNITURE CO.

VICKSBURG, MISSISSIPPI

FURNITURE—MUSIC—RUGS

ARTISTIC FURNITURE AT A PRICE YOU CAN AFFORD TO PAY

1927		
June 2	3 wicker chairs @ 12.50	$ 37.50
	1 wicker table	18.25
	1 rug, 2 x 4	7.80
June 6	1 victrola	150.00
	4 victrola records	3.25
		$216.80

10% OFF IF PAID WITHIN 10 DAYS OF DELIVERY

7. Answer the following questions about bills like the one above. Study the bill as often as you wish.

(*a*) What does "10% off" mean?

(*b*) A bill came to $100 and was paid within 10 days after the goods were received. How much had to be paid?

(*c*) If a bill came to $65 and was paid 20 days after the goods were received, how much would need to be paid?

(*d*) If a bill came to $14 and was paid 6 days after delivery of the goods, how much would need to be paid?

(*e*) If a bill came to $216.80 and was paid 7 days after delivery, how much would need to be paid?

8. What per cent of the games it played did the Pershing School team win, if it played 10 games, won 7, and lost 3? (7 is what fraction of 10? What per cent?)

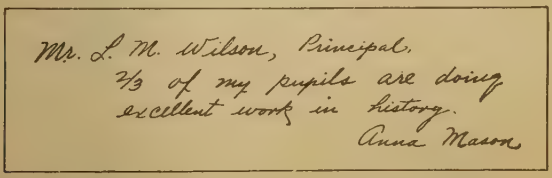
9. A school paper said: "Our football team had a poor season. It played 8 games, winning only 25% of them." How many games were won?

NOTE: Do newspapers report the per cent of games won by a baseball team in per cent form or in decimal form?

10. There are 40 boys in all in Troop A. $12\frac{1}{2}\%$ of the boy scouts of Troop A are in the Howard School sixth grade. How many are Howard sixth-graders?

11. In raising money for a hospital fund of \$4240, the committee reported in the evening paper that 25% of the money for the fund was raised the first day. (*a*) How much money was raised the first day? (*b*) What per cent remained to be raised? (*c*) How much money remained to be raised?

12. Answer the questions below. Study the teacher's note. There are 36 pupils in Miss Mason's class.



(*a*) How many were doing excellent work in history?

(*b*) What per cent of the class was doing excellent work?

13. Of the accidents on the streets of a certain city, $33\frac{1}{3}$ per cent are due to carelessness in crossing from one sidewalk to another. There were 156 accidents one year. How many were due to carelessness in crossing?

14. In a northern state 50% of all the water which fell last year fell in the form of snow. 28.6 in. of water fell in all. How much fell in the form of snow?

15. Three partners, Jones, McCarthy, and White, divided the profits of their business as follows: Jones received 25%, McCarthy 25%, and White the remainder. What per cent of the profits did White get? If the profits for one year's business were $24,000, what was the amount of McCarthy's share?

16. There are 332 seats at the ball park. 75% of these were filled at the first game of the season. How many of the seats were filled?

17. The School League made $188 at its Christmas Fair. The League voted to use 75% of this money to buy a picture for the assembly room. How much money did the League vote to use for the picture?

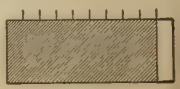
104. Thinking in Per Cent Language. (STUDY PERIOD.) If there is any question on this page and the next that you do not understand, you may study Arts. 99 and 100 for help. Do not leave this article until you are sure that everything in it is very clear to you.

QUESTIONS

1. $\frac{3}{4}$ of anything is what per cent of it?

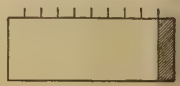
2. $12\frac{1}{2}\%$ is what decimal? What fraction?

3. What per cent of the first rectangle is shaded?



4. How would you find 26% of 657? (26% **of means** .26×)

5. What per cent of the second rectangle is shaded?



6. What per cents are equivalent to these fractions:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{8}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$?

7. What per cents are equivalent to these decimals:
.125 .5 .25 .75 .95 1.00?

8. What number is 200% of 34?

(200% of = twice or 2×. 300% of = 3×. 400% of = 4×. 250% of = 2.5× or $2\frac{1}{2}$×.)

9. $66\frac{2}{3}$% is equal to what fraction?

10. If 89% of the pupils in a school are present, what per cent of them are absent?

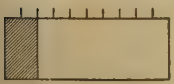
11. 8% of a number is how many times as large as 4% of the same number?

12. If you wrote 77% as a fraction, what would the denominator of the fraction be?

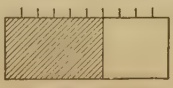
13. Is 6% or 60% or 600% of the figure at the left shaded?

14. .06×\$125 equals \$7.50. \$7.50 is what per cent of \$125?

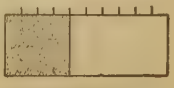
15. 30 is what per cent of 40? (Think: $\frac{30}{40}$ equals what?)



RECTANGLE (*A*)



RECTANGLE (*B*)



RECTANGLE (*C*)

16. 20%, or $\frac{1}{5}$, of Rectangle (*A*) is shaded. Draw a rectangle and shade 40% of it. Draw another and shade 75% of it.

17. What per cent of the area of Rectangle (*B*) is shaded?

18. What per cent of Rectangle (*C*) is dotted? What fraction of Rectangle (*C*) is blank?

19. Draw a rectangle which you let represent 100%. Then draw a second rectangle which is 250% as large as your first rectangle.

➡ If there are any of these 19 examples that you do not **really understand,** ask your teacher about them **now.**

105. Rapid Drill. (ORAL WORK.) Many pupils do not need to use their pencils in this work. Can you work all the examples mentally? In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$147 - 49$	$87\overline{)575}$	$164 - 68$	$96\overline{)849}$	$9+9+17$	$5.8\overline{)46.4}$
2.	$85\overline{)838}$	$59\overline{)179}$	$173 - 85$	$69\overline{)414}$	$7+35+6$	$185 - 89$
3.	$\frac{1}{4}$ = what %?	$67\overline{)655}$	$8+15+5$	.75 = what fraction?	50% of 44 = ?	$133 - 76$
4.	$144 - 96$	$27+7+9$	$133 - 87$	$154 - 59$	$17+6+4$	$18+9+8$
5.	98×7	$95\overline{)747}$	$25+8+9$	$110 - 56$	69×9	$1.35 - .97$
6.	$15+7+7$	$116 - 97$	2.7×8	$111 - 46$	$68\overline{)340}$	$16\frac{2}{3}\%$ = what fraction?

SELF-TESTING DRILL NO. 11

106. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Only lazy pupils do not bother to correct mistakes.**

1. $17\overline{)306}$

2. $57488 - 26959$

3. $\$77.13 - \$36.82 = ?$

4. $1.2\overline{)4.92}$

5. What fractional part of the rectangle at the right is shaded?

6.
488
4978
687
6785
979

7.
.5609
×68

8. $93\overline{)1832}$

9. $3\frac{1}{2} \times 1\frac{1}{2} = ?$

10.
.0491
.8672
.3568
.9828

11. $\frac{3}{5} + \frac{1}{20} + \frac{3}{10} = ?$

12. $7\frac{1}{3} - 3\frac{2}{3} = ?$

13. Find 17% of 342. (Hint: $.17 \times 342 = ?$)

14. $2 \times \frac{1}{3} \times \frac{1}{4} = ?$

15. What is 25% of \$8.44?

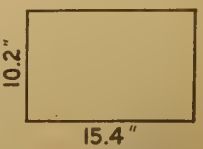
16. $8\frac{2}{15} - 1\frac{3}{5} = ?$

17.
1.05
×295

18. $\frac{5}{12} \div 1\frac{7}{8} = ?$

19. A pupil working the example $17\frac{1}{3} - 9\frac{3}{4}$ got the answer 9. What is the correct answer?

20. Find the area of the rectangle at the right.



STANDARDS

Number correct..	0–3	4–5	6–7	8–9	10	11	12	13–14	15	16–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

PERCENTAGE, CONTINUED

107. Correcting a Sixth-Grade Paper. (SILENT STUDY.) The answers shown on the dotted lines in the following exercise were given by a sixth-grade pupil. **Some of them are correct and some of them are wrong.** You are to study the exercise to find out which answers are right and which are wrong. Be ready also to give the correct answers to any examples you think are wrong.

EXERCISE

1. The word per cent means *hundredths or* $\frac{\%}{100}$.
2. 23% of anything means $\frac{23}{100}$ of it.
3. 10% or $\frac{1}{10}$ of 567 is *5670*.
4. $33\frac{1}{3}\%$ of 33 is *11*.
5. 20% of 200 is *40*
6. If 15 equals 10% of a number, 150 equals *ten times 15 or 100% of the number.*
7. The fraction that equals $33\frac{1}{3}\%$ is $\frac{1}{3}$.
8. The per cent which equals .39 is *3900%*.
9. 50% of 94 is *47*.
10. If 25% or $\frac{1}{4}$ of a number equals 55, then *225* is 100% of the number.
11. $133\frac{1}{3}\%$ is equal to the improper fraction $\frac{4}{3}$.
12. 150% written in fraction form is $\frac{3}{2}$.
13. 60 is the *correct* answer for 30% of 200.
14. $2\frac{1}{2}\times$ a number = *250%* of it.
15. 300 is *200%* of 150.

108. Making True Statements about Percentage. (WRITTEN WORK.) In each sentence below something is needed to complete the statement. Write on your paper what belongs on each blank.

EXERCISE

1. Both $\frac{54}{100}$ and .54 could be written as.....
2. 75 is $\frac{1}{2}$ or..... per cent of 150.
3. 300% of 14 is.....
4. The fraction $\frac{3}{4}$ equals.....per cent.
5. 19% written in decimal form is .....
6. $\frac{1}{5}$ equals.....per cent.
7. 40 is.....per cent of 20.
8. $133\frac{1}{3}$% of 100 is.....
9. 4% written in decimal form is.....
10. 37% of 478 is found by multiplying 478 by.....
11. 18% is.....times as large as 6%.
12. 4 times a number equals .....% of the number.
13. If 44 is 10% or $\frac{1}{10}$ of a number, then 440 is .....% of it.
14. .78 of any number is.....% of it.
15. 66 increased by 50% of itself, becomes.....
16. $\frac{2}{3}$ =.....per cent.

109. A "Hard Spot" in Percentage. (CLASS WORK.) For some pupils per cents larger than 100% seem harder than those which are less than 100%. Thus, 25% is easy. We know that 25% is the same as the decimal .25 or the

fraction $\frac{1}{4}$. But 125% seems to bother many pupils because it is more than 100%.

Per cents greater than 100% are used frequently, and sixth-grade pupils should be sure that they understand them clearly.

100% of a number equals the number. 100% of a number $= \frac{100}{100}$ of it or $1 \times$ it.

100% of 7 is 7. 100% of 7 is really the same as 1×7 which is 7.

125% of a number equals $\frac{125}{100}$, or 1.25, or $1\frac{1}{4}$, or $\frac{5}{4}\times$ the number.

125% of 240 = 300, because

$$\begin{array}{r} 240 \\ \times 1.25 \\ \hline 1200 \\ 480 \\ 240 \\ \hline 300.00 \end{array}$$

or $\cancel{240} \times \frac{5}{\cancel{4}} = 300$, 1

$133\frac{1}{3}\%$ of a number $= \frac{4}{3}$ of the number.

$133\frac{1}{3}\% = 100\% + 33\frac{1}{3}\%$.

100% = the whole or $\frac{3}{3}$ of a number
$33\frac{1}{3}\% =$ ⟶ $\frac{1}{3}$ of a number
$133\frac{1}{3}\% =$ ⟶ $\frac{4}{3}$ of a number

QUESTIONS

1. Can you explain why 150% is 1.50 or 1.5 in decimal language and is $\frac{3}{2}$ in fraction language?

2. Can you show how 175% could be expressed as the decimal 1.75 or the fraction $\frac{7}{4}$?

3. 200% of a number is 2 times the number. Can you show how 250% of a number is (in decimal language) $2.5\times$ the number? Show how it is (in fraction language) $\frac{5}{2}\times$ the number.

FINDING A PER CENT OF ANY NUMBER

110. Two Methods of Finding a Per Cent of Any Number. (STUDY PERIOD.) There are always two methods of finding a given per cent of a number. Study these two methods carefully, because in some cases it saves time to use the first method, and in other cases it saves time to use the second.

METHOD I. **Multiply by a decimal.** Find 25% of 324.

Think: 25% **of** means 25%×, and 25%=.25

Thus: 25% of 324 is the same as

$$\begin{array}{r} 324 \\ .25 \\ \hline 1620 \\ 648 \\ \hline 81.00 \end{array}$$ or 81, *Ans.*

METHOD II. **Multiply by a fraction.** Find 25% of 324.

Think: 25% **of** means $\frac{1}{4}$ of; and $\frac{1}{4}$ **of** means $\frac{1}{4}\times$.

Thus: 25% of 324 is the same as $\frac{1}{\cancel{4}}\times\cancel{324}^{81}=81$, *Ans.*
$\quad\quad\quad

111. Practice in Finding a Per Cent of a Number. (WRITTEN WORK.) For each example below, you are shown both methods of finding a per cent of a number. Either method is correct. Do not work the examples **now,** but tell which method you think is better for each example.

Example	*Method I*		*Method II*
1. Find 25% of 324.	$\begin{array}{r} 324 \\ \times .25 \\ \hline \end{array}$	or	$\frac{1}{4} \times 324 =$
2. Find $16\frac{2}{3}\%$ of 126.	$\begin{array}{r} 126 \\ \times .16\frac{2}{3} \\ \hline \end{array}$	or	$\frac{1}{6} \times 126 =$
3. Find 36% of 294.	$\begin{array}{r} 294 \\ \times .36 \\ \hline \end{array}$	or	$\frac{36}{100} \times$ or $\frac{9}{25} \times 294 =$
4. Find 7% of 531.	$\begin{array}{r} 531 \\ \times .07 \\ \hline \end{array}$	or	$\frac{7}{100} \times 531 =$
5. Find 120% of 475.	$\begin{array}{r} 475 \\ \times 1.20 \\ \hline \end{array}$	or	$\frac{120}{100} \times$ or $\frac{6}{5} \times 475 =$
6. Find 14% of 47.	$\begin{array}{r} 47 \\ \times .14 \\ \hline \end{array}$	or	$\frac{14}{100} \times$ or $\frac{7}{50} \times 47 =$
7. Find 60% of 84.	$\begin{array}{r} 84 \\ \times .60 \\ \hline \end{array}$	or	$\frac{60}{100} \times$ or $\frac{3}{5} \times 84 =$
8. Find $33\frac{1}{3}\%$ of 927.	$\begin{array}{r} 927 \\ \times .33\frac{1}{3} \\ \hline \end{array}$	or	$\frac{1}{3} \times 927 =$
9. Find 5% of 750.	$\begin{array}{r} 750 \\ \times .05 \\ \hline \end{array}$	or	$\frac{5}{100} \times$ or $\frac{1}{20} \times 750 =$

10. Find $12\frac{1}{2}\%$ of 1236. $1236 \times .125$ or $\frac{1}{8} \times 1236 =$

11. Find 265% of 629. 629×2.65 or $\frac{265}{100} \times$ or $\frac{53}{20} \times 629 =$

12. Find $62\frac{1}{2}\%$ of 360. $360 \times .625$ or $\frac{5}{8} \times 360 =$

13. Find 125% of 484. 484×1.25 or $\frac{5}{4} \times 484 =$

Work Examples 1-13. Use the method you think is the better one. Ask yourself: **Is my answer a reasonable one?**

MULTIPLYING BY LARGE MIXED NUMBERS

112. Multiplying by a Large Mixed Number. (Study Period.) You have learned to work an example like $2\frac{1}{2} \times 3\frac{2}{3}$ by first changing the mixed numbers to improper fractions. It is correct to change $2\frac{1}{2}$ to $\frac{5}{2}$, and $3\frac{2}{3}$ to $\frac{11}{3}$.

In working percentage examples you sometimes have to multiply by a **large** mixed number, as in: What is $18\frac{1}{6}\%$ of 642? Since $18\frac{1}{6}\%$, or $.18\frac{1}{6}$, does not equal any easy fraction, it is better to do the work like this: $.18\frac{1}{6} \times 642$

The work could be: $\frac{1.09}{6} \times 642$, because $.18\frac{1}{6} = \frac{1.09}{6}$; but it is easier to multiply by a **large** mixed number, thus:

642
$\times .18\frac{1}{6}$
107
5136
642
116.63

$\frac{1}{6} \times 642 = 107$

The 107 is added with the 5136 and the 642 because $\frac{1}{6}$ is a part of the multiplier, $18\frac{1}{6}$ hundredths.

Notice the decimal point: $18\frac{1}{6}\% = .18\frac{1}{6}$.

Study the four samples on the next page.

Sample 1

$$\begin{array}{r} 295 \\ \times 21\tfrac{3}{5} \\ \hline 177 \\ 295 \\ 590 \\ \hline 6372 \end{array}$$

First, find $\frac{3}{5}$ of 295.

$\frac{3}{5}$ of $295 = 177$

Notice that the first **7** comes directly under the **1**.

Sample 2

$$\begin{array}{r} 497 \\ \times .30\tfrac{4}{7} \\ \hline 284 \\ 000 \\ 1491 \\ \hline 151.94 \end{array}$$

First, find $\frac{4}{7}$ of 497.

$\frac{4}{7}$ of $497 = 284$

Notice that the 4 of the 284 is directly under the 0 of the $30\frac{4}{7}$.

Sample 3

$$\begin{array}{r} 921 \\ \times .16\tfrac{1}{8} \\ \hline 115\tfrac{1}{8} \\ 5526 \\ 921 \\ \hline 148.51\tfrac{1}{8} \end{array}$$

$921 \times \frac{1}{8} = 115\frac{1}{8}$

Sample 4

$$\begin{array}{r} 153 \\ \times 21\tfrac{1}{3} \\ \hline 51 \\ 153 \\ 306 \\ \hline 3264 \end{array}$$

Does $\frac{1}{3}$ of $153 = 51$?

When you have **large** mixed numbers, it is easier to work the examples as in the samples above.

EXERCISE

1.	2.	3.	4.	5.
$429 \times 17\frac{1}{3}$	$654 \times 18\frac{1}{2}$	$1205 \times 20\frac{2}{5} =$	$2415 \times .19\frac{1}{7}$	Find $26\frac{3}{4}\%$ of \$2416.

Notice how the work is done when the fraction is in the multiplicand.

$$\begin{array}{r} 248\frac{1}{4} \\ 96 \\ \hline 24 \\ 1488 \\ 2232 \\ \hline 23832 \end{array}$$

$96 \times \frac{1}{4} = 24$

$$\begin{array}{r} 2.56\frac{1}{3} \\ 33 \\ \hline 11 \\ 768 \\ 768 \\ \hline 84.59 \end{array}$$

$33 \times \frac{1}{3} = 11$

Notice the decimal point.

Find the products:

1.	2.	3.	4.	5.
$272\frac{2}{5}$	$98\frac{1}{7}$	$8.47\frac{4}{7}$	936	$151\frac{1}{2}$
25	.14	.49	$14\frac{2}{3}$	25

When $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{8}$ occurs in a large mixed number, you can easily change the fraction to a decimal. Many people always do this. Thus:

(*a*) 1784 $\times 19\frac{1}{2}$	becomes	1784 $\times 19.5$	(*d*) $846\frac{1}{2}$ $\times 24\frac{1}{5}$	becomes	846.5 $\times 24.2$
(*b*) 924 $\times 26\frac{1}{5}$	becomes	924 $\times 26.2$	(*e*) 946 $\times 12\frac{1}{8}$	becomes ($\frac{1}{8}=.12\frac{1}{2}=.125$)	946 $\times 12.125$
(*c*) $562\frac{4}{5}$ $\times 19$	becomes	562.8 $\times 19$	(*f*) $64\frac{3}{4}$ $\times 17\frac{3}{8}$	becomes	64.75 $\times 17.375$

113. Practice in Finding Large Per Cents. (WRITTEN WORK.) In working these examples, notice what answers are the same, and be able to tell why they are the same.

1. $1\frac{1}{4}$ times 440	10. $1\frac{1}{6}$ times 432	19. $1\frac{1}{8}\times 336$
2. 1.25 times 440	11. $1.16\frac{2}{3}\times 432$	20. $1.12\frac{1}{2}$ times 336
3. 125% of 440	12. $116\frac{2}{3}$% of 432	21. $112\frac{1}{2}$% of 336
4. $1\frac{3}{8}$ times 328	13. $1\frac{7}{8}$ times 240	22. $1\frac{5}{6}$ times 138
5. $1.37\frac{1}{2}$ times 328	14. $1.87\frac{1}{2}$ times 240	23. $1.83\frac{1}{3}$ times 138
6. $137\frac{1}{2}$% of 328	15. $187\frac{1}{2}$% of 240	24. $183\frac{1}{3}$% of 138
7. $1\frac{1}{3}\times 213$	16. $1\frac{3}{4}$ times 144	25. $2\frac{1}{2}\times 668$
8. $1.33\frac{1}{3}$ times 213	17. 1.75×144	26. 2.5×668
9. $133\frac{1}{3}$% of 213	18. 175% of 144	27. 250% of 668

114. Practice in Using Judgment. (Class Work.) Study the answers given with each example in the exercise below. Select the best answer in each case.

EXERCISE

1. Which fraction below is the same as $433\frac{1}{3}\%$?

(*a*) $\frac{4\,3}{3}$ (*b*) $4\frac{2}{3}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{1\,3}{3}$

2. Which of the examples below is the best to use for solving $37\frac{4}{5}\times 108\frac{9}{10}$?

(*a*) $\frac{1\,8\,9}{5}\times\frac{1\,0\,8\,9}{1\,0}$ (*b*) $37\frac{4}{5}\times 108\frac{9}{10}$ (*c*) 108.9×37.8 (*d*) 37.8×108.9

3. 26% of anything is about:

(*a*) $\frac{1}{4}$ of it (*b*) 4 times it (*c*) 26 times it (*d*) 2.6 times it

4. 350% of 9 is about:

(*a*) 3150 (*b*) 31 (*c*) $\frac{3}{10}$ of 9, or 2.7 (*d*) $9\overline{)350}$, or about 40

5. The quotient for $28\overline{)2.24}$ is:

(*a*) 80 (*b*) 8 (*c*) .8 (*d*) .08

6. The answers to which examples below will be a quarter of a dollar?

(*a*) $\frac{1}{6}$ of \$24
(*b*) 500% of a nickel
(*c*) 50% of $12\frac{1}{2}\text{¢}$
(*d*) $12\frac{1}{2}\%$ of \$2

7. $7\frac{1}{2}\%$ changed to the form of a decimal is:

(*a*) .75 (*b*) 7.5 (*c*) .075 (*d*) .0075

SELF-TESTING DRILL NO. 12

115. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Is it fair to expect you to work a percentage problem like Problem 15?**

1. $1526 - 809$

2. $7\frac{3}{8} - \frac{1}{2} = ?$

3. $241.07 + 2.342 = ?$

4. $128341 - 62389$

5. $678\overline{)25087}$

6. $\frac{4}{5}$ times $\frac{1}{2}$ times $\frac{5}{6} = ?$

7. $59\overline{)3068}$

8. $10 - \frac{1}{6} = ?$

9. 156×370

10. *Find the quotient:* $\frac{2}{5}$ divided by $\frac{3}{10}$.

11. $\frac{5}{8}$, $\frac{1}{2}$, $\frac{1}{8}$

12. $1\frac{1}{2}\frac{1}{7} \times 1\frac{2}{7} \times \frac{1}{1}\frac{1}{2} = ?$

13. Find $33\frac{1}{3}\%$ of 639.

14. Find the total population of 5 towns whose populations are 891, 783, 448, 833, and 699 persons.

15. A farmer had 96 sheep. He sold $12\frac{1}{2}\%$ of them. How many did he sell?

16. $37\overline{).953}$

17. $197 \div .16 = ?$

18. $.3673 \times .1294$

19. Find 150% of 60

20. Find 8% of $217.25.

STANDARDS

Number correct	0–2	3–4	5–6	7	8	9	10–11	12	13	14–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

PERCENTAGE, CONTINUED

116. Practice in Finding Per Cents. (WRITTEN WORK.) Read both parts I and II of this article before you begin your written work.

I. In each example in the exercise below tell whether you would think the per cent as a **fraction** or as a **decimal.** (See Art. 110.)

EXERCISE

Find:

1. 25% of 100
2. $16\frac{2}{3}$% of 66
3. 31% of 875
4. 8% of 934
5. 75% of 444
6. 93% of 145
7. 5% of 112
8. $66\frac{2}{3}$% of \$3.36
9. 50% of 1236
10. $12\frac{1}{2}$% of 824
11. $62\frac{1}{2}$% of 32
12. 20% of 375
13. $37\frac{1}{2}$% of 928
14. 76% of 509
15. $33\frac{1}{3}$% of 129
16. 60% of 700
17. $87\frac{1}{2}$% of 96
18. 40% of 25
19. 125% of 184
20. $83\frac{1}{3}$% of 126
21. 4% of 919
22. 80% of 300
23. $37\frac{1}{2}$% of 824
24. $62\frac{1}{2}$% of 720

II. Work the above examples, using the way you think is easier in each case.

Remember that:

$50\% = \frac{1}{2}$	$62\frac{1}{2}\% = \frac{5}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$	$40\% = \frac{2}{5}$
$25\% = \frac{1}{4}$	$87\frac{1}{2}\% = \frac{7}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$	$60\% = \frac{3}{5}$
$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$	$10\% = \frac{1}{10}$	$80\% = \frac{4}{5}$
$37\frac{1}{2}\% = \frac{3}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$	$20\% = \frac{1}{5}$	$75\% = \frac{3}{4}$

INSTRUCTION IN PROBLEM SOLVING

117. Hints on Problem Solving. (STUDY PERIOD.) Read the three problems. Study the problems and the four questions that follow each problem.

PROBLEM 1

Suppose I borrow \$500 from you and agree to pay you 7% of the \$500 for the use of your money for one year. How much will I have to pay you at the end of the year?

QUESTION 1. Just what does the problem ask for?

Ans. I want to find out how much I will owe you at the end of the year. I will owe the \$500 which I borrowed and 7% of the \$500 more.

QUESTION 2. What would be a reasonable answer?

Ans. I know the answer will be a little more than \$500.

QUESTION 3. What is the thing to do to get the answer?

Ans. Find out what 7% of \$500 equals, and add that amount to the \$500 I borrowed from you.

Solution: 7% of \$500 is

$$\begin{array}{r} \$500 \\ \times .07 \\ \hline \end{array}$$

\$35.00, the amount I will pay you for the use of your money for a year. Thus, the total amount I will owe you is \$500 plus \$35, or \$535, *Ans.*

QUESTION 4. Is \$535 a reasonable answer?

PROBLEM 2

A baseball team played 125 games during the season. The team won 68% of its games. How many games did this team win?

QUESTION 1. Just what does the problem ask for?

Ans. We want to find how many games the team won during the season.

QUESTION 2. What would be a reasonable answer?

Ans. The number of games won would be less than 125 for that was all that were played and only 68% were won. The answer will be more than $\frac{1}{2}$ of 125, because 68% is more than 50% or $\frac{1}{2}$. A good guess would be about 80 games won.

QUESTION 3. What is the thing to do in order to get the answer?

Ans. Find 68% of 125.

Solution: 68% of 125 is $125 \times .68$ or

$$\begin{array}{r} 125 \\ \times .68 \\ \hline 1000 \\ 750 \\ \hline 85.00 \end{array}$$ *Ans.*

The team won 85 games.

QUESTION 4. Is this a reasonable answer?

PROBLEM 3

If Mr. Brown spends 60% of what he earns for living expenses, 12% for insurance, 5% for gifts, and saves the rest, how much will he save? He earns $2500 a year.

QUESTION 1. Just what does the problem ask for?

Think carefully before you answer.

QUESTION 2. What would be a reasonable answer?

Mr. Brown saves what is left after 60% and 12% and 5% have been spent. He spends 77% in all; so he must save 23% of his $2500. Why?

23% of $2500 is almost $\frac{1}{4}$ of $2500; so a reasonable answer will be about $600.

QUESTION 3. What must we do to get the **exact** answer?

Solution: He saves 23%, as we can see without using a pencil. We must now find how much 23% of $2500 is.

23% of $2500 is .23 × $2500 or →

$ 2500
× .23
7500
5000
$575.00 *Ans.*

Mr. Brown saves $575.

QUESTION 4. Is the answer a reasonable one?

When you are working problems, it is not necessary to ask yourself exactly the same questions as the following ones. It is helpful, however, to read the problem over carefully and ask yourself questions which will guide you in thinking clearly and working accurately.

1. What is asked for?

2. What would be a reasonable answer? (This keeps you from making foolish mistakes.)

3. What must I do to get the answer?

Then after you have worked the problem, ask yourself:

4. Is the answer a reasonable one?

5. Have I checked my work?

A suggestion for ambitious pupils. Many pupils like to learn more about the things mentioned in problems. Thus, in Problem 3 on page 166 some pupils will want to know the meaning of "living expenses," "insurance," etc.

To know all about the meanings of such things makes problem work more interesting. Use the dictionary and other reference books to find interesting facts concerning things you read about in arithmetic problems.

SELF-TESTING DRILL NO. 13

118. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Be careful or you will make mistakes in Example 6.**

1. $\frac{7}{12}+\frac{1}{12}+\frac{4}{12}=?$

2.
$$\begin{array}{r} 78945 \\ -23774 \\ \hline \end{array}$$

3. $82\overline{)3854}$

4.
$$\begin{array}{r} 679 \\ \times 35 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 70.3 \\ \times .27 \\ \hline \end{array}$$

6.
$$\begin{array}{r} \$6{,}979{,}687{,}956.99 \\ +\$7{,}439{,}295{,}687.46 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 834 \\ \times 982 \\ \hline \end{array}$$

8. $8\frac{5}{7}-\frac{1}{2}=?$

9. $\frac{7}{8}\div\frac{3}{4}=?$

10. $4.233-.51=?$

11.
$$\begin{array}{r} 7.5 \\ \times 3\frac{2}{5} \\ \hline \end{array}$$

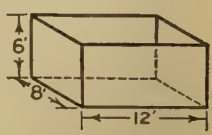
12. $1\frac{1}{2}\times\frac{1}{8}\times 2\frac{1}{2}=?$

13.
$$\begin{array}{r} 63. \\ -4.97 \\ \hline \end{array}$$

14.
$$\begin{array}{r} \frac{5}{6} \\ \frac{1}{2} \\ \frac{3}{5} \\ \hline \end{array}$$

15. $4\frac{1}{2}\div 1\frac{1}{3}=?$

16. Find the volume of this box: →



17. If a board 3 ft. 1 in. long is cut into four equal lengths, how long will each part be if no allowance is made for waste?

18. What is $13\frac{1}{2}\%$ of 231?

19. What is 48% of \$17.84?

20. A man owed a bill for \$219.54, on which he was offered a reduction of 2% for cash. How much would he save by paying cash?

STANDARDS

Number correct.	0–4	5–6	7	8–9	10	11	12	13	14–15	16–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

ESTIMATING ANSWERS

119. Two Good Habits in Problem Solving. (STUDY PERIOD.) A good problem-solver often does these two things in working problems:

1. He gets in mind what a reasonable answer would be, before working the problem.

2. He compares his estimate with the answer he works out.

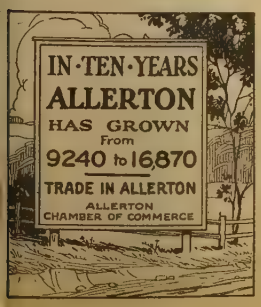
It is not very difficult to form this habit of "getting in mind a reasonable answer before starting to work," if you practice doing it every time you solve a problem. A good student is one who not only works hard **but who also uses good habits.**

In the problems on this and the next page practice the habit of getting in mind what a reasonable answer would be. Write your estimates down and compare them in a class discussion after the study period.

PROBLEMS

Do not work these problems **now. Estimate** the answers.

1. In a long-distance walking contest a man averaged 32 miles a day for 15 days. How far did he walk in the 15 days?



2. A city grew in population from 9240 people to 16,870 in a period of ten years. What was the average yearly increase in population?

3. A farm of 640 A. was divided into 1280 building lots. What was the average area of a lot?

4. During the last 5 years Mr. Jones sold goods worth the following amounts: \$4252, \$5346, \$4962, \$6501, and \$8272. How much did he sell in the five-year period?

5. Mrs. Wagner canned $\frac{3}{4}$ of her tomatoes. She had $2\frac{1}{4}$ bu. How many did she can?

6. Frank Smith receives as his salary 15% of the amount of his sales. In August he sold goods to the amount of \$1450. What was his salary that month?

7. Water runs over a certain dam at the rate of 1000 gal. a second. How much goes over in an hour?

After estimating your answers and comparing your estimates in class, work the seven problems.

How good were your estimates?

PROBLEM SCALE NO. VIII

120. Problems Using Percentage. (WRITTEN WORK.) You can earn 100 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

Notice that some of the problems have more than one part. Number and letter your answers plainly.

1. In the sixth grade of the Washington School a record was kept of the per cents of problems solved correctly. One boy, Arthur Black, tried 560 problems during the year. His record was 85% correct. How many problems did he do correctly? (4)

2. In the Washington School there are 800 pupils. The distribution of these pupils by grade in per cents is given below. Find the number of pupils in each grade.

(*a*) Grade 1, 16% (6)
(*b*) Grade 2, 14% (6)
(*c*) Grade 3, $12\frac{1}{2}$% (6)
(*d*) Grade 4, 10% (6)
(*e*) Grade 5, 12% (6)
(*f*) Grade 6, 11% (6)
(*g*) Grade 7, 9% (6)
(*h*) Grade 8, 8% (6)
(*i*) Grade 9, $7\frac{1}{2}$% (6)

3. Some stores will take a little off a bill if payment is made at the time of or very soon after the purchase of goods. The Davis Dry Goods Store will take off 2% if the bill is paid when the goods are bought.

If Mrs. Smith buys \$17.50 worth of goods at the Davis Dry Goods Store,

(*a*) How much will she save if she pays cash? (6)
(*b*) How much will she pay if she pays cash? (6)

4. When a little is taken off the purchase price for cash payment, we say that the bill is "discounted for cash." The per cent of discount is not the same in all stores. Find the discounts for the following bills:

(*a*) Bill \$6.00, discount for cash 20% (6)
(*b*) Bill \$15.80, discount for cash 15% (6)
(*c*) Bill \$359.00, discount for cash $12\frac{1}{2}$% (6)
(*d*) Bill \$74.75, discount for cash 5% (6)
(*e*) Bill \$20.50, discount for cash 4% (6)

STANDARDS

Superior work:	85–100 points	Slow work:	5–11 points
Good work:	47– 84 points	Inferior work:	0– 4 points
Average work:	12– 46 points		

SUMMARY

121. General Review. (Class Work.) This article reviews the more important topics of the first half of grade six arithmetic.

1. (*a*) 9 ft. 8 in. $\div 4 =$ (*b*) 3×7 hr. 25 min. $=$

2. There is **one mistake** in the list below. Find it.

(*a*) $\frac{1}{3}$ of $45 = \frac{1}{3} \times 45$ or $33\frac{1}{3}\%$ of 45.

(*b*) "50% of" is the same as ".50 $\times$."

(*c*) $12\frac{1}{2}\%$ of 32 is 4.

(*d*) 150% of anything $= 1\frac{1}{2} \times$ or $1.5 \times$ the thing.

(*e*) $7\frac{1}{2}\% = .075$. $8\frac{1}{4}\% = .00825$.

(*f*) Ten per cent means $\frac{1}{10}$.

(*g*) $1\frac{1}{4}\%$ of $N = .0125 \times N$.

3. If 50% of the pupils in a certain school are girls and there are 140 girls in the school, how many pupils are there in all?

4. 60 is what per cent of 30? Of 120?

5. Explain multiplier, multiplicand, and reciprocal.

6. $1944 \div 324 = 6$. Find the quotients of the following

(*a*) $324\overline{)19.44}$ (*c*) $32.4\overline{)1944}$ (*e*) $32.4\overline{)19.44}$

(*b*) $32.4\overline{)194.4}$ (*d*) $.324\overline{)1944}$ (*f*) $3.24\overline{)19.44}$

7. Find (*a*) 36% of 294; (*b*) $66\frac{2}{3}\%$ of 156; (*c*) 25% of 1400. (See Art. 110.)

8. (*a*) $41\frac{3}{8} \times 328 = ?$ (*b*) $24\frac{1}{5} \times 846\frac{1}{2} = ?$ (*c*) $248\frac{1}{2}$
$\times 96$
(See Art. 112.)

9. Give fractions that are equal to:

50% 10% $12\frac{1}{2}$% $33\frac{1}{3}$% 75% $37\frac{1}{2}$% $16\frac{2}{3}$% $87\frac{1}{2}$%

10. What are some good questions to ask yourself about a problem which troubles you?

11. How would you check a subtraction example?

12. By what method would you check a division example?

13. How would you check the answer in addition? In multiplication?

14. In multiplying decimals how do you find the number of decimal places in the product?

15. What does a check stub show?

A TEST IN FRACTIONS

A sixth-grade pupil has two things to think about in arithmetic: one is to learn well the new work; the other is **to keep from forgetting** what has already been learned.

If you can get 15 of the examples below correctly in 15 minutes **without any help,** you are not forgetting how to use fractions. If you cannot get 15 examples correct in 15 minutes, use the Self-Help Index on page 457 at once.

Written Work. **Leave answers in simplest form.**

1. $\frac{1}{8}+\frac{1}{8}$
2. $\frac{11}{12}\div\frac{5}{6}$
3. $\frac{1}{4}-\frac{1}{8}$
4. $\frac{1}{6}\times\frac{5}{9}$
5. $\frac{1}{8}$ of 8
6. $\frac{4}{7}$ of $1\frac{2}{3}$
7. $4\frac{2}{3}\div 9$
8. $4\frac{7}{8}-3\frac{1}{2}$
9. $\frac{7}{12}\div 1\frac{5}{9}$
10. $\frac{4}{9}\times\frac{1}{5}$
11. $4\frac{1}{4}\times 2\frac{2}{3}$
12. $\frac{2}{3}+\frac{5}{6}+\frac{2}{9}$
13. $23\frac{1}{8}-2\frac{1}{10}$
14. $1\frac{5}{6}\times 2\frac{1}{4}$
15. $13\frac{4}{8}$ $\quad\frac{\frac{6}{12}}{\frac{1}{2}}$
16. $6\frac{3}{5}\div 4\frac{1}{2}$
17. $5\frac{3}{7}\times 6$
18. $14-8\frac{3}{8}$
19. $14\frac{1}{3}-8\frac{3}{5}$

PART TWO

CHAPTER VI

REVIEW TESTS ON ESSENTIALS

TESTING YOUR ABILITY IN ARITHMETIC

122. Are You Ready for the Last Half of Sixth-Grade Arithmetic? (CLASS WORK.) If you are beginning the last half of your sixth-grade arithmetic in September, you will be given the opportunity to find just how well prepared you are to go on with the work. During the long summer vacation you may have forgotten many things that you will need to know if you are to complete successfully the arithmetic work of this grade.

Now is a good time to "check up" on yourself. In this chapter are a number of tests like the ones you took when you began sixth-grade arithmetic. The standards following each test will show you, after you take the test, just how well you understand and can do the particular kind of arithmetic work covered by the test. Take a minute or so to look at the tests. Notice the standards particularly.

Of course, it won't help much just to know that you are weak in certain parts of arithmetic. Taking a test is only part of the job. You must, if needed, do something to strengthen the "weak spots" that the test may show. The **Self-Help Study and Practice Lessons** on pages 359-440 will give you just the study and practice you may need to get yourself in "good trim" for the work of the next few

NOTE TO THE TEACHER: With classes reaching Chapter VI in mid-year, the chapter may be omitted.

months. Spend a minute or so looking at these Self-Help Lessons. Read Article 235, page 359, carefully.

Your work on some of the tests or on all of them may show that you do not need restudy. If that is the case, turn back to pages 4-5 and 6-7, where you will find suggestions regarding how to spend spare time.

Remember that the work covered by these tests will be needed in doing the rest of the arithmetic of this grade. You will make a much better record in the last half of sixth-grade arithmetic if you are honest with yourself in taking these tests and in using the Self-Help Lessons.

BASIC TESTS IN WHOLE NUMBERS

123. How Well Can You Add? (WRITTEN WORK.) Have pencil and paper ready. When your teacher gives the signal, start with Example 1 and work the examples in order. Do not copy the examples unless necessary. Just place the edge of your paper below the example and write your answer.

When time is up, your teacher will read the correct answers, and you will put a "C" after each correct answer on your paper. Then count your correct answers and from the standards at the end of the test find how well you have done. **Time allowed:** 6 minutes.

BASIC ADDITION TEST

1.	2.	3.	4.	5.
Find the sum of	89	63	50	$7.48
twenty-seven and	49	87	6	6.77
seventy-six.		50	44	6.07
			97	
			58	

Go on to the next page.

6.	7.	8.	9.	10.
198	47	$89.32	5327	978
483	24	35.56	1364	2322
678	95	25.87	2990	1104
638	58	51.99	2438	89
	60			6618
	19			6340
	58			

11. *Copy and add:* $4.90+$.28+$9.81+$.96+$9.30+$1.19.

12. *Copy and add:* 703+90+677+580+563.

STANDARDS

1-7 examples correct→poor sixth-grade work
8-10 examples correct→fair sixth-grade work
11 examples correct→good sixth-grade work
12 examples correct→excellent sixth-grade work

Does your score on the Addition Test show that you need restudy before you can safely go on with sixth-grade arithmetic? The Self-Help Lessons on addition are on pages 360-363. Be honest with yourself. Use them if you need to.

124. Testing Your Ability to Subtract. (WRITTEN WORK.) On the next page is a subtraction test. Have pencil and paper ready before the signal to begin is given. You don't need to copy examples unless told to do so. **Time allowed:** 6 minutes.

BASIC SUBTRACTION TEST

1.	2.	3.	4.	5.
139	64	889	1562	$8.49
74	25	657	907	7.68

6.	7.	8.	9.	10.
30418	42121	$7000	$118.63	553975
23809	5491	4926	75.68	96068

11.	12.	13.	14.	15.
128022	825566	131367	$40.00	112472
41686	66417	39358	25.15	73795

16.	17.	18.	19.	20.
134717	107263	309110	70098	47377
84852	90479	27823	30183	35546

21. *Copy and subtract:* 824934 − 6997.

22. What must be added to $89.00 to make $497.50?

STANDARDS

1-11 examples correct→poor sixth-grade work
12-16 examples correct→fair sixth-grade work
17-20 examples correct→good sixth-grade work
21-22 examples correct→excellent sixth-grade work

If you made a low score on the Subtraction Test, it would be wise for you to use the Self-Help Lessons on pages 364-368.

125. Testing Your Ability to Multiply. (WRITTEN WORK.) You will be given **15 minutes** in which to do the 15 examples in this test. Wait for the signal to begin. Copy examples only when told to do so. Do the work in order.

BASIC MULTIPLICATION TEST

1.	2.	3.	4.	5.
534	508	3009	629	905
2	70	9	54	38

6.	7.	8.	9.	10.
907	768	4379	862	8435
106	398	4007	2067	5986

11.	12.	13.
7196	91058	4002
6895	214	3009

14. *Copy and multiply:* 37×1679

15. *Copy and multiply:* 234×3478

STANDARDS

1-5 examples correct→poor sixth-grade work
6-9 examples correct→fair sixth-grade work
10-13 examples correct→good sixth-grade work
14-15 examples correct→excellent sixth-grade work

What did your score and the standards tell you about your ability to multiply whole numbers? Restudy and practice now will make this year's arithmetic easier. Pages 369-375 give the Self-Help work on multiplication of whole numbers.

126. How Accurately Can You Divide Whole Numbers? (WRITTEN WORK.) You will have to copy all these examples. Be sure you copy correctly. Wait for the signal to begin. **Time allowed:** 20 minutes.

BASIC LONG-DIVISION TEST

1.	2.	3.	4.
$98\overline{)71148}$	$63\overline{)39375}$	$45\overline{)15899}$	$67\overline{)8457}$
5.	**6.**	**7.**	**8.**
$73\overline{)15045}$	$89\overline{)534267}$	$67\overline{)351093}$	$75\overline{)180000}$
9.	**10.**	**11.**	**12.**
$28\overline{)9579}$	$35\overline{)2642}$	$41\overline{)29531}$	$560\overline{)37537}$

STANDARDS

1-5 examples correct→poor sixth-grade work
6-8 examples correct→fair sixth-grade work
9-11 examples correct→good sixth-grade work
12 examples correct→excellent sixth-grade work

Long division is pretty hard for many pupils. How well did you do on this test? You can't afford to "pass up" this opportunity to strengthen yourself in long division if your score was low. The Self-Help Lessons on division are on pages 376-387.

You have now completed your testing and review work on whole numbers. The next few pages will give you the opportunity to find how well you can work with fractions.

BASIC TESTS IN FRACTIONS

127. Can You Add and Subtract Fractions Easily and Correctly? (WRITTEN WORK.) Copy carefully, and watch the signs. **Time allowed:** 12 minutes. **Answers are correct only when expressed in lowest terms.**

BASIC TEST IN ADDITION AND SUBTRACTION OF FRACTIONS

1.	2.	3.	4.
$\frac{2}{3}+\frac{1}{4}=?$	$\frac{2}{3}-\frac{1}{4}=?$	$4\frac{1}{4}+3\frac{5}{6}=?$	$4\frac{1}{4}-3\frac{5}{6}=?$
5. $11\frac{5}{6}$ 7 $20\frac{3}{8}$ $8\frac{1}{2}$	6. *Subtract:* 10 $9\frac{2}{3}$	7. $5\frac{1}{2}$ $3\frac{4}{9}$ $\frac{7}{12}$ $2\frac{1}{2}$	8. $18\frac{4}{5}$ -6
9. $3\frac{5}{6}$ $5\frac{4}{9}$ 8 $\frac{1}{3}$	10. $11\frac{1}{3}$ $-2\frac{3}{5}$	11. $20\frac{3}{8}$ $8\frac{1}{2}$ $7\frac{2}{3}$ $13\frac{1}{4}$	12. *Subtract:* $20\frac{3}{8}$ $16\frac{5}{6}$

STANDARDS

1-4 examples correct→poor sixth-grade work
5-7 examples correct→fair sixth-grade work
8-11 examples correct→good sixth-grade work
12 examples correct→excellent sixth-grade work

If you did poor work on Examples 1, 3, 5, 7, 9 and 11, use the Self-Help Lessons on pages 391-398. For Examples 2, 4, 6, 8, 10, and 12, use the Lessons on pages 399-403.

128. How Well Can You Multiply and Divide with Fractions? (WRITTEN WORK.) Here are 17 examples in multiplication and division of fractions. Copy examples only when you need to, and be sure to watch the signs. **Time allowed:** 12 minutes. **Answers count as correct only when expressed in lowest terms.**

BASIC TEST IN MULTIPLICATION AND DIVISION OF FRACTIONS

1.	2.	3.	4.	5.
$\frac{1}{2} \times \frac{1}{3} = ?$	$\frac{1}{5} \div \frac{1}{2} = ?$	$\frac{2}{3}$ of $9 = ?$	$4 \div \frac{1}{3} = ?$	$4 \times \frac{2}{5} = ?$
6.	**7.**	**8.**	**9.**	**10.**
$\frac{2}{5} \times 3\frac{1}{3} = ?$	$\frac{5}{6} \div 7 = ?$	$\frac{3}{4}$ of $1\frac{1}{9} = ?$	$\frac{3}{8} \div 2\frac{1}{2} = ?$	$4\frac{1}{6} \times 2\frac{2}{5} = ?$
11.	**12.**	**13.**	**14.**	**15.**
$3\frac{1}{5} \div \frac{8}{9} = ?$	$3\frac{3}{8} \times 0 = ?$	$5\frac{1}{3} \div 4\frac{2}{9} = ?$	$2\frac{1}{2} \times 3\frac{9}{10} = ?$	$2\frac{1}{12} \div 15 = ?$

16. Two-thirds divided by three-fourths equals ?

17. $1\frac{1}{8} \times \frac{1}{2} \times 2\frac{1}{4} = ?$

STANDARDS

1-6 examples correct→poor sixth-grade work
7-12 examples correct→fair sixth-grade work
13-15 examples correct→good sixth-grade work
16-17 examples correct→excellent sixth-grade work

Have you forgotten the things you should know about multiplication and division of fractions? If you failed on the multiplication examples, use the Lessons on pages 403-411. For restudy of division, use pages 411-416.

INFORMAL TESTS

129. Informal Tests in Denominate Numbers. (WRITTEN WORK.) You will notice that there are no standards for the tests on the next few pages. Your teacher will tell you the correct answers, and you must judge for yourself whether you need restudy. Any pupil doing good work in sixth-grade arithmetic should be able to work the examples in Tests (*A*) and (*B*) correctly and easily.

TEST (A)

Changing the Form of Denominate Numbers

Write the numbers which should be on each blank in the 24 examples below. Number your papers carefully. **Suggested working time:** 10 minutes.

1. 90 min. = ... hr.
2. 24 qt. = ... pk.
3. 1 yd. 1 ft. = ... ft.
4. 32 oz. = ... lb.
5. 4000 lb. = ... T.
6. 18 in. = ... ft.
7. 1 cu. ft. = ... cu. in.
8. 9 gal. = ... qt.
9. 1 mi. = ... ft.
10. 10 pk. = ... bu.
11. $1\frac{1}{2}$ bu. = ... pk.
12. 1 ft. 7 in. = ... in.
13. 10 pt. = ... gal. ..... qt.
14. 38 in. = ... yd. ..... ft. ..... in.
15. 7 pt. = ... qt.
16. 288 sq. in. = ... sq. ft.
17. 98 da. = ... wk.
18. 14 ft. = ... yd. ..... in.
19. 1 lb. = ... oz.
20. 1 cu. yd. = ... cu. ft.
21. 1 min. = ... sec.
22. 9 sq. ft. = ... sq. yd.
23. $\frac{1}{2}$ sq. mi. = ... A.
24. 2 hr. = ... min.

If examples like those in Test (*A*) are hard for you, use the Self-Help Study and Practice Lessons on pages 417-418 and the Tables of Measure on page 456.

TEST (B)

The Form of Answers

Answers using denominate numbers should always be reduced when possible, as shown by the following samples. **Suggested working time:** 10 minutes.

First Found Answer	Good form of Answer
(*a*) 2 ft. 15 in.	3 ft. 3 in.
(*b*) 2 yd. 8 ft.	4 yd. 2 ft.
(*c*) 2 lb. 19 oz.	3 lb. 3 oz.
(*d*) 1 bu. 9 pk.	2 bu. 1 pk.

Change each answer below to the correct form for a final answer.

1. 2 lb. 22 oz.
2. 2 sq. ft. 432 sq. in.
3. 1 yd. 7 ft. 14 in.
4. 2 lb. 9 oz.
5. 2 gal. 3 qt. 6 pt.
6. 3 hr. 180 min.
7. 2 sq. yd. 10 sq. ft.
8. 2 bu. 13 pk. 5 qt.

If Test (*B*) was hard for you, use the Self-Help Study and Practice Lessons on pages 418-419 and the Tables of Measure on page 456.

130. Testing Your Ability to Add and Subtract Denominate Numbers. (WRITTEN WORK.) Your work on this test will show you whether or not you need to restudy the

addition and subtraction of denominate numbers. In this test, **answers must be in reduced form to count as correct.** **Suggested time:** 8 minutes.

TEST IN ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

1.

2 ft. 5 in.
3 ft. 11 in.
1 ft. 7 in.

2.

2 qt. 1 pt.
−1 qt.

3.

5 gal. 2 qt.
1 gal.
3 gal. 3 qt.

4.

Subtract:
4 yd. 1 ft.
2 yd. 2 ft.

5.

2 lb. 10 oz.
1 lb. 9 oz.
5 lb.
6 lb. 14 oz.

6.

Find the difference:
10 lb.
3 lb. 12 oz.

7.

Subtract:
6 gal.
2 gal. 2 qt.

8.

4 bu. 1 pk.
−2 bu. 3 pk.

9.

2 hr. 15 min.
5 hr. 25 min.
4 hr. 40 min.
1 hr. 10 min.

If Exs. 1, 3, 5, and 9 were hard for you, use the Self-Help Lessons on pages 419-421.

If Exs. 2, 4, 6, 7, and 8 were hard for you, use the Self-Help Lessons on subtraction of denominate numbers on pages 421-423.

131. Testing Your Ability to Multiply and Divide Denominate Numbers. (WRITTEN WORK.) Your work on the test below will show you how well you can multiply and divide denominate numbers.

If you copy these examples before working them, be sure to copy very accurately. **Only answers in reduced form count as correct.** Suggested time: 12 minutes.

TEST IN MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

1.	2.	3.
5 qt. 1 pt. $\times 8$	$4\overline{)5}$ hr. 20 min.	2 yd. 2 ft. $\times 6$
4.	**5.**	**6.**
8 min. 12 sec. $\times 7$	$3\overline{)8}$ pk. 5 qt.	4 lb. 6 oz. $\times 4$
7.	**8.**	**9.**
$7\overline{)11}$ yd. 2 ft.	$5\overline{)16}$ ft. 8 in.	3 bu. 2 pk. $\times 6$
10.	**11.**	**12.**
$9\overline{)19}$ T. 700 lb.	6 ft. 8 in. $\times 5$	$6\overline{)7}$ gal. 2 qt.

If Exs. 1, 3, 4, 6, 9, and 11 were hard for you, restudy Art. 32 and practice on Art. 33.

If Exs. 2, 5, 7, 8, 10, and 12 were hard for you, restudy Art. 36 and practice on Art. 37.

MENSURATION

132. Informal Tests in Mensuration. (WRITTEN WORK.) Here are three little tests in measuring. If you are doing good work in sixth-grade arithmetic, you will not have much trouble with them.

TEST (A). WORDS USED IN MENSURATION

Number your answers on your paper carefully. Write (as your answers) the word or words which should be on each blank line. **Suggested working time:** 5 minutes.

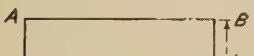


FIG. 1

1. Fig. 1 is called a.....

2. Fig. 1 has a.....o 24′.

3. Fig. 1 has an.....o 32 sq. ft.

4. In Fig. 1 the 8′ and the 4′ are the.....

5. In Fig. 1 lines AC and AB form a.....angle.

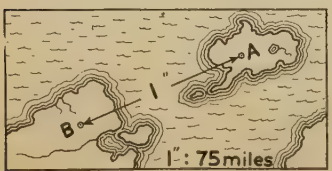


FIG. 2

6. In Fig. 2, the 1″: 75 miles is the.....o the map.

7. In Fig. 2, a distance of 2″ on the map represents .....miles.

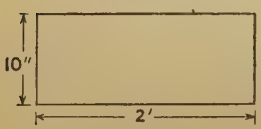


FIG. 3

8. In Fig. 2, the distance between points A and B is..... miles.

9. Before finding the area of Fig. 3, the dimensions would first be changed to the.....unit.

10. The dimensions of a rectangular solid are length, width, and..... ....

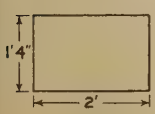
11. Volume or capacity is measured in units.

If you had trouble with Test (*A*), use the Self-Help Lesson on pages 424-425.

TEST (B). FINDING AREAS

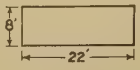
Time suggested: 10 minutes. Number each example carefully. You need write only the answers. Name answers carefully.

Find the areas of the figures below.



1.

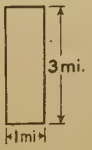
2.



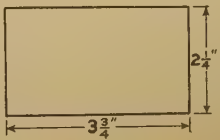
3.

4.

5.



6.



7.

If you need to restudy finding areas, use the Self-Help Study and Practice Lessons on pages 425-427.

TEST (C). FINDING VOLUMES OF RECTANGULAR SOLIDS

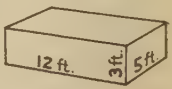
You need write only the answers. Number each answer on your paper. **Time suggested:** 10 minutes.

Find the volumes of the solids below.

1.

2.

3.



4.

5. Find the number of cubic yards in a rectangular bin which is $8' \times 10' \times 14'$.

6. Find the capacity in cubic feet of a rectangular swimming tank whose dimensions are $6' \times 24' \times 50'$.
The tank is the same depth throughout.

7. What is the capacity in cubic feet of a freight car whose inside dimensions are $9\frac{1}{3}' \times 12' \times 40'$?

Self-Help Study and Practice Lessons on finding volumes are on pages 427–428.

DECIMALS

133. Informal Test in Decimals. (WRITTEN WORK.) Your work on this test will show you how well you understand important facts about decimals. It is important that you correct any mistakes that you may make. **Suggested time:** 15 minutes.

TEST ON IMPORTANT FACTS IN DECIMALS

The figures in the answers of Exs. 1 to 4 are correct. Rewrite the answers so that they will be entirely correct.

1. $4.15+.62+3.18+14.07=2202$

2. $3.168+.243+5.827+4.+.711=13949$

3. $27.18-13.14=1404$

4. $70.00-24.17=4583$

Note that there are six parts to Example 5.

5. In the number 1789.04:

(*a*) The 7 is in the.....place. (*d*) The 8 is in the.....place.
(*b*) The 4 is in the.....place. (*e*) The 1 is in the.....place.
(*c*) The 0 is in the.....place. (*f*) The 9 is in the.....place.

6. In the number 43.8:

(*a*) The (.) is called a ....
(*b*) All figures on the right of the . are ....
(*c*) All figures on the left of the . are ....

7. Write each number below in words.

(*a*) 4.7 (*b*) .47 (*c*) .5 (*d*) .05

8. Write the numbers below in decimal form.

(*a*) $4\frac{1}{2}$ (*b*) $\frac{3}{4}$ (*c*) twenty-seven hundredths
(*d*) six and six-tenths

9. In adding and subtracting decimals, decimal points should be kept in a..... .....

If needed, use the Self-Help Lessons on pages 428–431.

134. Informal Test in Multiplication and Division of Decimals. (WRITTEN WORK.) Your work on this test will show you how well you can use the decimal point in multiplication and division of decimals. **Suggested time:** 9 minutes.

The figures in the products of Exs. 1-5 are correct. Copy and **point off** the products correctly.

1.	2.	3.	4.	5.
27.89	2.789	278.9	27.89	.2789
7.3	73	.73	.073	.73
8367	8367	8367	8367	8367
19523	19523	19523	19523	19523
203597	203597	203597	203597	203597

The figures in the quotients of Exs. 6-11 are correct. Copy and point off the quotients.

6. $73\overline{)2035.97}$ 2789

7. $7.3\overline{)2035.97}$ 2789

8. $.73\overline{)203.597}$ 2789

9. $.073\overline{)2035.97}$ 2789

10. $73\overline{)2.03597}$ 2789

11. $7.3\overline{)20359.7}$ 2789

In the examples below, find the quotients to two decimal places.

12. $8\overline{)267}$

13. $9\overline{)723}$

14. $15\overline{)2708}$

If Exs. 1-5 caused you any trouble, restudy Art. 40 and practice on Arts. 41, 42, and 43.

If Exs. 6-11 troubled you, restudy Arts. 64 and 65 and practice on Arts. 66 and 68.

For help with Exs. 12 to 14, see Arts. 72-74.

CHECKING

135. Informal Test in Checking. (WRITTEN WORK.) This test will show you how accurately you can check answers and correct them if your check shows that they are wrong. **Suggested time:** 12 minutes.

Notice that the answers to the examples below are printed. Do **not** work the examples. **Check them.** If your check shows that a printed answer is correct, put a ($\surd$) mark as your answer to that example. If an answer is wrong, write the correct one.

1.

Subtract:

43004

22346

20758

2.

7009

408

56072

280360

2859672

3.

493

87

408

6137

940

8075

4.

$900\frac{88}{89}$

$89\overline{)80189}$

5.

$48.7\overline{)18.506}$ *.38*

6.

Find the difference:

741.270

61.129

680.141

7.

489.7

18.7

32879

38176

4897

9053.39

8.

541.6

70.7

218.3

487.1

7.2

64.3

1389.2

If you do not know how to check or can not check accurately, use the Self-Help Lesson on page 388.

For checking Example 5, see Article 67.

PERCENTAGE

136. Informal Test in Percentage. (WRITTEN WORK.) Your work on Tests (*A*), (*B*), and (*C*) in the article below will show how well you remember important facts about percentage which you learned in the first half of your sixth-grade work.

TEST (A). RECOGNITION OF PER CENT EQUIVALENTS

Suggested time: 4 minutes.

Four answers are printed for each of the examples below. **Only one of these is correct.** Write as your answer the one you think is correct.

1. 25% of N means:
   (*a*) 25 times N (*b*) $N+25$ (*c*) $\frac{1}{25}$ of N (*d*) $\frac{1}{4}$ of N

2. 20% in fraction language would be:
   (*a*) $\frac{1}{20}$ (*b*) $\frac{1}{5}$ (*c*) .5 (*d*) .02

3. $\frac{1}{8}$ of N in per cent language would be:
   (*a*) 8% (*b*) 80% (*c*) $12\frac{1}{2}$% (*d*) $16\frac{2}{3}$%

4. $2\frac{1}{2}$% in decimal form would be:
   (*a*) .025 (*b*) .25 (*c*) 2.5 (*d*) .0225

5. The decimal 1.25 in per cent language would be:
   (*a*) $12\frac{1}{2}$% (*b*) $100\frac{25}{100}$% (*c*) $\frac{1}{8}$% (*d*) 125%

If Test (*A*) was hard for you, study Arts. 91, 99, and 100.

TEST (B). CHANGING THE FORM OF PER CENTS

Suggested time: 8 minutes.

Write as your answers to Exs. 1-10 the numbers which should be on each blank.

1. $\frac{1}{4}$ of N = ... % of N
2. $\frac{2}{3}$ of N = ... % of N
3. $\frac{2}{5}$ of N = ... % of N
4. 1.5 of N = ... % of N
5. $\frac{3}{100}$ of N = ... % of N
6. $1\frac{1}{2}$ or $\frac{3}{2}$ of N = ... % of N
7. .65 of N = ... % of N
8. .$33\frac{1}{3}$ of N = ... % of N
9. .06 of N = ... % of N
10. .045 of N = ... % of N

Write as your answers to Examples 11-20 the decimal or fraction which should be on each blank.

11. 50% as a fraction is .....
12. 4% as a decimal is .....
13. 125% as a fraction is .....
14. $3\frac{1}{2}$% as a decimal is .....
15. $12\frac{1}{2}$% as a fraction is .....
16. $7\frac{1}{2}$% as a decimal is .....
17. 11% as a fraction is .....
18. 81% as a decimal is .....
19. 150% as a fraction is .....
20. $37\frac{1}{2}$% as a fraction is .....

If Test (*B*) was hard for you, the following references are good ones for study: Arts. 91, 99, 100.

TEST (C). FINDING GIVEN PER CENTS OF NUMBERS

Suggested time: 10 minutes.

Be sure to name answers carefully.

1. Find 24% of \$650.
2. What is 6% of 2400?
3. $4\frac{1}{2}\%$ of \$5250 = ?
4. $12\frac{1}{2}\%$ of 3288 = ?
5. Find 150% of 600.
6. $3\frac{1}{2}\%$ of \$1500 is how much?
7. Find $19\frac{1}{2}\%$ of 1784.
8. What is $14\frac{1}{4}\%$ of 72?

If the examples in Test (C) were hard for you, restudy Arts. 100 and 110 and practice on Arts. 113 and 116.
For help with Examples 7 and 8, use Art. 112.

BANKING PRACTICE

137. Informal Test on Banking. (WRITTEN WORK.) You will not find this test very hard if you remember what you have studied about bank checks.

TEST ON BANK CHECKS

Suggested time: 10 minutes.

Make a copy of the blank check on page 195. Fill the blanks correctly with the facts given below and on the next page.

1. Use to-day's date.
2. The check's number is 72.
3. The check was made out to J. H. Wheaton.

Troy, New York,_____192___ No._____

THE RUSTON NATIONAL BANK

PAY TO THE ORDER OF_____ $_____

_____ DOLLARS

62-181

4. Its amount was $72.50. This was in payment of a doctor's bill.

5. This check was made by Helen Orton.

6. At the right is the stub belonging to the check you have made out. The stub for Check 71 showed that Miss Orton had a balance of $50 in her account. Before she wrote Check 72, she deposited $125.00. Make a copy of Stub 72, and fill in all the blanks correctly.

No _____ $_____

_____ 192__

To _____

For _____

	DOLLARS	CENTS
BAL BROT FORD.		
AMT. DEPOSITED		
TOTAL		
AMT. THIS CHECK		
BAL. CARD FORD.		

Write on your paper what belongs on the blanks in the next sentences.

7. Money paid for the use of money is called.....

8. Before you cash a check, you must.....it on the back.

9. Writing a check for more than the balance in your account is called.....your account.

10. A check that has been cashed by the bank is called a.....check.

If this test was hard for you, spend a little time now re-studying Arts. 85 and 86.

☞If you have taken the tests in this chapter honestly and have used the Self-Help Lessons as needed, you are ready to go on to Chapter VII.

CHAPTER VII

DISCOUNT AS AN APPLICATION OF PERCENTAGE. REVIEW OF ZERO DIFFICULTIES. FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

PERCENTAGE IN DISCOUNT

138. Discounts for Prompt Payments. (CLASS WORK.) A **bill** is a statement of the amount of money owed for goods or services. The **discount** is the amount by which a bill is reduced from its full amount. **Discounts** are sometimes given for prompt payment of bills. The **discount** in this case is the amount that you are allowed to take off your bill because of your prompt payment.

Discount is sometimes a stated amount, as in the bill on the next page, "1¢ per K.W." **Discount** is generally a certain per cent of the full amount of the bill.

A GOOD WAY TO SAVE MONEY

It is often possible to **save** many dollars a year by taking advantage of discounts for prompt payment of bills. Good business men almost never fail to get their bills discounted. Many housewives are good business women and save money in this way.

Study the bill on the next page to see one of the ways in which discounts are given for prompt payment. Answer the questions below the bill.

Failure to Receive This Bill Does Not Entitle Consumer to Discount

Please Bring This Bill to be Receipted

John A. Lee
436 E. Oak St.
City

To EDISON CITY LIGHT & POWER CO. Dr.

FOR ELECTRICITY SUPPLIED	METER READINGS			Sales	
To 4–22– 19 27	458 K.W.	15 K.W. @ 13c		1	95
From 3–23– 19 27	432 K.W.	11 K.W. @ 10c		1	10
Total Electricity Supplied	26 K.W.	K.W. @ 8c			
			Total	3	05
			1c per K.W. Dis.		26
			Net Amt.	2	79
			Arrears		
			Appliances		
			Total		

A DISCOUNT OF 1c PER K.W. will be allowed on electricity for payment, at the office of the Company on or before the *12th of the month*, of bills for consumption of *previous month only*.

Checks mailed in payment of Electricity Bills must bear postmark not later than 12th of month following that in which electricity is consumed to entitle consumer to discount.

Discount Limit on this Bill Expires May 12th 1927

ELECTRIC Apr. SALES

John A. Lee

EDISON CITY LIGHT AND POWER CO.

NO DISCOUNT AFTER
May 12th 1927

Gross	3	05
Discount		26
Net	2	79
Arrears		
Appliances		
Total		

Do Not Detach This Coupon

1. How large a discount was allowed on the bill?

2. Mr. Lee had to do what in order to get the discount

3. Why do you think that the discount was offered?

4. Do you notice this sentence on the bill: "Please brin this bill to be receipted"? Do you know or can you fin what "receipting" a bill means?

➨ Bring to class tomorrow bills showing discounts Be ready to explain how the discount is computed.

Discounts for prompt payment are usually a **per cen** of the total amount. Thus, you will find on some bills suc statements as these: "4 **per cent** off for cash," or "2% o for payment within ten days."

QUANTITY DISCOUNTS

139. Quantity Discounts. (STUDY PERIOD.) Discount are given for other reasons than prompt payment. Busines men who buy in large amounts usually get discounts.

Notice in the bill below that there are two discounts offered: The first is the 10% discount for buying in large quantities, and the second is a discount for cash, that is, immediate payment. (In business, "cash" often means payment within 30 days.)

Study the bill and answer the questions below it. (Golden Orange and Wild Cherry are syrups used at soda fountains.)

RALEIGH, NORTH CAROLINA. October 11, 1927

Cooper Supply Co.
Durham, North Carolina

BOUGHT OF **HART & HENDERSON**
WHOLESALE DRUGGISTS

TERMS: 1% for Cash (30 days). NO. 226-230 HARGETT ST.

80 Gal. Golden Orange,	@ $3.25—10%	(Quantity discount)	$234.00
80 Gal. Wild Cherry,	@ 3.00—10%	(Quantity discount)	216.00

1. One gallon of Golden Orange syrup costs $3.25. If no discount were given, how much would the 80 gal. cost?

2. What was the amount of the quantity discount on Golden Orange?

3. Was the amount of the discount on Wild Cherry figured correctly?

4. What would be the amount of the bill if the Cooper Supply Company paid cash? (See "Terms" on the bill.)

Two or more discounts are figured separately. Figure the larger discount first. Thus, if a bill came to $300 with a quantity discount of 10% and a cash discount of 1%, you would first deduct $30 (10% of $300) and then take away 1% of the remaining $270.

DISCOUNTS DURING SALES

140. Discounts During Special Sales. (STUDY PERIOD
Another use of discount is found in special sales in a store. Notice the advertisement of an "After Christmas Sale" shown at the right.

Overcoats, ¼ Off
A discount of 25 per cent will be allowed on all Overcoats.

1. Do "$\frac{1}{4}$ off" and "discount of 25%" mean the same?

2. If you were figuring a discount of 25%, would you multiply by .25 or by $\frac{1}{4}$?

3. What would be the price at the "After Christmas Sale" of a coat selling regularly at $100?

➽ 4. Find notices in newspapers of discounts at sales. Bring them to school and make and solve problems about them

TRADE DISCOUNTS

141. Trade Discounts. (STUDY PERIOD.) Another kind of discount is the **trade discount.** This is the discount the wholesaler gives to the store-keeper (the retailer). In wholesalers' catalogues the prices quoted are often the **retail prices,** that is, the amounts which the retailer must charge the customer for the article.

For an article marked $1.00 in the catalogue, the store-keeper cannot afford to pay $1.00 and then sell it for $1.00. In order to pay his business expenses and make a profit the store-keeper must buy the article **at a discount;** that is, he must buy of the wholesaler for **less** than $1.00.

For this reason catalogues usually give **retail prices, less a certain discount,** such as 20%, 25%, $33\frac{1}{3}$%, 40%, etc.

If the dealer buys this $1.00 article, less a 40% retailer

trade discount, he actually pays the wholesaler 60¢ for the article. He sells it for $1.00. The difference of 40¢ between what he pays for this article and what he receives for it, is **not all profit.** Out of the 40¢ the store-keeper must pay his clerks' wages, rent, advertising, and other expenses. What is left will be his **net profit.**

Study the wholesaler's bill shown below. Notice that it allows two kinds of discount on the same goods. These are figured separately. Answer the questions on page 202.

EXCELSIOR PORCH SHADE CO.

INVOICE NO. J 525

SOLD TO White Furniture Co., Spokane, Washington

JANESVILLE, WIS., May 21 19 27

CASES SHADES GROSS WT. LBS.

1 BOLES SHADES GROSS WT. 19 LBS.

GROSS WT. LBS.

SHIPPED VIA Express, Collect

TERMS: 60 DAYS NET FROM DATE OF INVOICE. 2 PER CENT DISCOUNT FOR CASH IN 20 DAYS FROM DATE OF INVOICE.

F. O. B. JANESVILLE, WIS.

YOUR ORDER NO. 2634

YOUR DEPT. NO. 6

SOLD BY Mail-May 19, 1927.

1 Spec. Size Shade-Color 2--4'5½"x7' @ .90 per drop ft.----	$ 6.30
1 " " " " 2--3'4" x7' @ .67 " " " ----	4.69
	$10.99
Less Dealer's Trade Discount 40%----------	4.40
	$ 6.59
Less discount for cash 2%--------	.13
	$ 6.46

"Drop ft." means length of the shade in feet. "Invoice" means bill.

In this bill, the Excelsior Porch Shade Company is the **wholesaler.** A wholesaler sells goods in large amounts to many **retailers.** The White Furniture Company is the retailer. A retailer sells directly to persons who actually use the article. Such persons are often called **consumers.**

QUESTIONS

1. Why are trade discounts given to store-keepers?

2. Figure the bill on page 201 to see if the work is correct

Where two discounts are given, figure the larger discount an then the smaller. In business you always figure discount **separately.**

3. If the White Furniture Co. took the discount fo prompt payment of the bill, what would be the amoun they would send to the Excelsior Porch Shade Company

Remember that discounts are given: (*a*) for promp payment of bills; (*b*) for purchases of large amounts (*c*) at "marked-down" sales in stores; (*d*) as a trade dis count to store-keepers.

Below is a common form of discount notice from a whole saler's catalogue, showing discounts given to retailers. "Lis price" is the price given in the catalogue.

Confidential Discount Information

Read Carefully

Prices of all items listed in this catalog are subject to a dealers' discount of **25%**

For example, see Item A3218, page 183.

List price	$12.00
Less 25%	3.00
Your net price	9.00

All repairing prices **NET, NO DISCOUNT.** This does not apply to Tennis Racket restringing; prices quoted on page 209 are subject to the usual discount. Mark the discount rate in code in your catalog.

Be sure to remove this sheet before taking orders!

142. Rapid Drill. (ORAL WORK.) The practice in this exercise will help you in your Self-Testing Drills. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$12\frac{1}{2}\%$ = what fraction?	146 −57	$85\overline{)733}$	126 −98	$87\overline{)621}$	How is division checked?
2.	162 −79	$4\overline{)5\text{ ft. }4\text{ in.}}$	*Good form of answer?* 2 qt. 7 pt.	158 −79	131 −38	134 −86
3.	$73\overline{)718}$	143 −46	$56\overline{)266}$	120 −36	8.6 ×.8	$56\overline{)368}$
4.	130 −48	$42\overline{)382}$	$\frac{2}{3}$ = what %?	$95\overline{)667}$	111 −45	What is a discount?
5.	*Read:* 3.48	6 is what % of 30?	*Reduce:* $\frac{10}{3}$	Does 8 = 200% of 4?	2 yd. 1 ft. ×4	$4\frac{1}{2}\%$ = what decimal?
6.	What is 300% of 8?	160 −81	What is retail price?	$74 \div 10$	$\frac{2}{3} \div 6$	$16\frac{2}{3}\%$ = what fraction?
7.	2 yd. = ...in.	$\frac{2}{3}$ of 15	$.85 \times 100$	$\frac{2}{3} + \frac{1}{4}$	$4\frac{1}{5} = \frac{}{5}$	13 $-4\frac{5}{12}$
8.	$\frac{2}{3} = \frac{}{24}$	.5 of 36	What is a reciprocal?	18.4 −8.9	$\frac{1}{3}$ of N = 6. N = ?	$59\overline{)534}$

SELF-TESTING DRILL NO. 14

143. Directions. Time allowed: 20 minutes. Follow th directions given for Drills 1 to 5. Rate yourself. **Try t beat all your past records.**

1.	2.	3.	4.	5.
$\frac{4}{5} \div \frac{2}{3} = ?$	12744	834	786	$7\frac{5}{6}$
	-7398	455	859	$-\frac{1}{8}$
		313	667	
6.	**7.**	868	432	**8.**
8912	$\frac{3}{5} + 4\frac{3}{8} + 3 = ?$	793	332	846
$\times 205$		434	278	$\times 264$
		329		

9.	10.	11.	12.
4.80	$96\overline{)8160}$	$177.2 - 9.43 = ?$	$1\frac{3}{5} \div \frac{3}{20} = ?$
$\times .673$			

13. $4 \times \frac{2}{5} = ?$

14. $\frac{7}{10}$
$+7\frac{11}{100}$

15. $7.3\overline{)335.8}$

16. 3 qt. − 1 qt. 1 pt. =

17. Find the volume of a tank $4' \times 4\frac{1}{2}' \times 5'$.

18. In a certain class 8 pupils, 25% or $\frac{1}{4}$ of the class, re ceived a grade of *B*. How many pupils were there in th class?

19. A farmer had 1492 bushels of wheat, of which h sold 45%. How many bushels did he sell?

20. 80 is what % of 240? (Think: 80 is what fraction of 240?

STANDARDS

Number correct.	0–4	5	6–7	8	9	10	11	12	13–14	15–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

REVIEW OF PERCENTAGE

144. Practice with Per Cents. (ORAL WORK.) Study the exercise below in order to give the correct answers quickly when you are called upon. To mark in the book is unfair.

EXERCISE

1. .25 equals ..... %.
2. 75% equals $\frac{3}{4}$.
3. .3 equals ..... %.
4. $\frac{1}{2}$ equals ..... %.
5. $\frac{1}{4}$ equals ..... %.
6. 50% of 84 is .....
7. 10% of 40 is .....
8. 20 is ..... % of 40.
9. 8 is ..... % of 4.
10. $12\frac{1}{2}$% of 24 is .....
11. 1% of 4356 is .....
12. $33\frac{1}{3}$% of 66 is .....
13. 15 is 50% of .....
14. 7 is 1% of .....
15. $\frac{3}{5}$ equals ..... %.
16. $\frac{1}{8}$ equals ..... %.
17. 40 is 200 ..... of 20.
18. 6% of \$300 is .....
19. 50% of $\frac{2}{3}$ is .....
20. $\frac{1}{6}$ equals ..... %.
21. 38% of means × .....
22. $66\frac{2}{3}$% equals the fraction .....
23. $62\frac{1}{2}$% equals the fraction .....
24. \$100 less 2% for cash is .....
25. $7\frac{1}{2}$% equals the decimal .....
26. \$200 less 40% trade discount is .....
27. A \$20 coat less $33\frac{1}{3}$% costs .....
28. $87\frac{1}{2}$% equals the fraction .....
29. $133\frac{1}{3}$% equals the improper fraction .....
30. 80 lb. is ..... % of 240 lb.
31. 200% means × .....
32. Discounts are often allowed for ....., ..., ..., ...

USEFUL FACTS TO REMEMBER

$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{1}{4} = 25\%$	$\frac{1}{5} = 20\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{1}{8} = 12\frac{1}{2}\%$
$\frac{2}{2} = 100\%$	$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{2}{4} = 50\%$	$\frac{2}{5} = 40\%$	$\frac{2}{6} = 33\frac{1}{3}\%$	$\frac{2}{8} = 25\%$
$\frac{3}{2} = 150\%$	$\frac{3}{3} = 100\%$	$\frac{3}{4} = 75\%$	$\frac{3}{5} = 60\%$	$\frac{3}{6} = 50\%$	$\frac{3}{8} = 37\frac{1}{2}\%$
$\frac{4}{2} = \ ?\ \%$	$\frac{4}{3} = 133\frac{1}{3}\%$	$\frac{4}{4} = 100\%$	$\frac{4}{5} = 80\%$	$\frac{4}{6} = 66\frac{2}{3}\%$	$\frac{4}{8} = 50\%$
	$\frac{5}{3} = \ ?\ \%$	$\frac{5}{4} = 125\%$	$\frac{5}{5} = 100\%$	$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$
		$\frac{6}{4} = \ ?\ \%$	$\frac{6}{5} = 120\%$	$\frac{6}{6} = 100\%$	$\frac{6}{8} = 75\%$
			$\frac{7}{5} = \ ?\ \%$	$\frac{7}{6} = 116\frac{2}{3}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$
				$\frac{8}{6} = \ ?\ \%$	$\frac{8}{8} = 100\%$
					$\frac{9}{8} = 112\frac{1}{2}\%$
					$\frac{10}{8} = \ ?\ \%$

To memorize these useful facts will save you work the rest of the year.

Some pupils like to think of arithmetic as using three languages: (*a*) fraction language, (*b*) decimal language, and (*c*) per cent language. Sometimes a problem is stated in one language, but the work is done in another. Thus:

Find 27% of 642. (Problem stated in per cent language.)

The work: $.27 \times 642$ (The work is done in decimal language.)

It is useful to be able to think in the three languages easily and correctly. The following exercise should be studied until it is very easy. What should be on each blank?

	Fraction Language	Decimal Language	Per Cent Language
1.	$\frac{3}{4}$		
2.			17%
3.		.06	
4.	$\frac{1}{8}$		
5.		$.16\frac{2}{3}$	
6.			250%

PROBLEM SCALE NO. IX

145. Problems Using Discount. (WRITTEN WORK.) You an earn 133 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. Find the discount for each of the bills below and then figure what amount must be paid if advantage is taken of the discount.

	BILL	DISCOUNT	
(*a*)	$546.00	6%	(6)
(*b*)	6.75	3%	(7)
(*c*)	350.00	7%	(7)
(*d*)	54.37	2%	(7)
(*e*)	194.35	$1\frac{1}{2}$%	(7)
(*f*)	87.86	8%	(7)

2. What are the sale prices on the following?

(*a*) Suit, regularly priced at $35; Spring Clearance Sale Discount, 20%. (7)

(*b*) Overcoat, regularly priced at $50; Spring Clearance Sale discount, 25%. (7)

(*c*) A pair of skates with shoes attached, regular price, $12; discount, $12\frac{1}{2}$%. (7)

(*d*) Shoes, regular price, $5.75; July Clearance Sale Discount, 15%. (7)

(*e*) Four shirts, priced, $1.50 each; discount, $33\frac{1}{3}$%. (7)

(*f*) A radio set, regular price, $35; discount, $37\frac{1}{2}$%. (8)

3. Study the following price list, taken from a catalogue of a wholesale house. Below it are given certain bills. Figure these out. Read the price list as often as you need to.

LUNCHEON MATS—WHITE FLANNEL

ROUND	PER DOZ.	OVAL	PER DOZ.
5 inch	$1.00	$4\frac{1}{4}$x 7	$1.20
6 inch	1.20	$5\frac{1}{2}$x $8\frac{1}{2}$	1.60
7 inch	1.50	$6\frac{1}{2}$x$10\frac{1}{2}$	2.20
8 inch	1.80	$7\frac{1}{2}$x11	2.50
9 inch	2.20	$8\frac{1}{4}$x$12\frac{1}{2}$	3.00
10 inch	2.80	$8\frac{1}{4}$x$14\frac{1}{4}$	3.80
12 inch	3.80	10x16	5.00

LAMP MATS—GREEN FLANNEL BOTH SIDES

MADE IN ROUND ONLY. PER DOZEN:

6 inch	$1.50	9 inch	$2.80
7 inch	1.80	10 inch	3.50
8 inch	2.20	12 inch	4.50

Dealer's discount, 30 per cent. Subject to change without notice.

Find the cost after discounting:

(*a*) 2 doz. round luncheon mats, 5-inch size. (7)

(*b*) 3 doz. 9-inch round, and 4 doz. oval $7\frac{1}{2}\times 11$ luncheon mats. (8)

(*c*) 5 doz. 12-inch round, 3 doz. $4\frac{1}{4}\times 7$ oval, and 1 doz. 10×16 oval luncheon mats. (8)

4. For how much more than he paid will the dealer sell each of the following? He sells at the prices shown in the list above and takes his discount as a dealer.

(*a*) 3 doz. 9-inch round luncheon mats. (8)

(*b*) 5 doz. 10-inch lamp mats. (8)

5. I bought:

8 gal.	Wild Cherry syrup,	@ $4.00 a gal.
3 gal.	Golden Orange,	@ $3.50 a gal.
3 gal.	Pineapple Crush,	@ $5.00 a gal.
5 gal.	Banana syrup,	@ $4.75 a gal.
Total 19 gal.		

In more than 10 gal. amounts on the total order I get 12% discount. If I pay my bill within ten days of receiving the goods, I get another discount of 3%. What will I pay after I deduct my discounts for buying in large quantities and for cash payment? (10)

STANDARDS

Superior work:	38–133 points	Slow work:	6–7 points
Good work:	14– 37 points	Inferior work:	0–5 points
Average work:	8– 13 points		

146. Estimating the Size of Fractions. (ORAL WORK.) Sixth-grade pupils should be so familiar with fractions that they can estimate the size of many fractions without using pencils. Can you do the work below correctly?

EXERCISE

1. Which of the following fractions are larger than $\frac{1}{2}$?

(*a*) $\frac{7}{13}$ (*b*) $\frac{406}{800}$ (*c*) $\frac{12}{32}$ (*d*) $\frac{40}{75}$ (*e*) $\frac{550}{1000}$

2. Which of the following fractions are smaller than $\frac{3}{4}$?

(*a*) $\frac{30}{48}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{4}{3}$ (*d*) $\frac{400}{900}$ (*e*) $\frac{45}{60}$

3. Which of the following fractions are about the same size as $\frac{1}{3}$?

(*a*) $\frac{10}{50}$ (*b*) $\frac{16}{49}$ (*c*) $\frac{21}{18}$ (*d*) $\frac{100}{299}$ (*e*) $\frac{350}{700}$

SELF-TESTING DRILL NO. 15

147. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Does your Progress Chart show that you are improving?**

1.
$$\begin{array}{r} 267 \\ \times 81 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 94583 \\ -38979 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 8\frac{4}{9} \\ -\frac{1}{3} \\ \hline \end{array}$$

4.
$$\begin{array}{r} 105.87 \\ -23.56 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 5054313 \\ 88000 \\ 9152 \\ \hline \end{array}$$

6. $1\frac{2}{3} \times \frac{2}{3} \times 1\frac{4}{5} = ?$

7.
$$\begin{array}{r} 974 \\ 945 \\ 675 \\ 482 \\ 763 \\ 892 \\ 962 \\ 573 \\ \hline \end{array}$$

8. $84\overline{)7308}$

9. $.73\overline{)5.913}$

10.
$$\begin{array}{r} 8 \\ -3\frac{1}{3} \\ \hline \end{array}$$

11.
$$\begin{array}{r} 487 \\ \times 76 \\ \hline \end{array}$$

12. $1\frac{4}{5} \div \frac{1}{6} = ?$

13.
$$\begin{array}{r} 6\frac{7}{10} \\ +10\frac{2}{3} \\ \hline \end{array}$$

14. Find 39% of \$708.

15. $20 \div 3\frac{3}{4} = ?$

16.
$$\begin{array}{r} 1 \text{ ft. } 3 \text{ in.} \\ 7 \text{ in.} \\ 4 \text{ ft. } 9 \text{ in.} \\ 2 \text{ ft.} \\ \hline \end{array}$$

17. $4\overline{)3 \text{ hr. } 20 \text{ min. } 12 \text{ sec.}}$

18. What is 27% of 349?

19. A dealer had 72 books. He sold 75% of them. How many books did he sell?

20. A rectangle is 4 ft. by 9 ft. 4 in. Find the area.

STANDARDS

Number correct..	0–4	5–6	7	8	9	10	11–12	13	14	15–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

A HINT ON STUDY HABITS

It is a good plan for you to watch your own work in arithmetic carefully. When you find that you are a little uncertain about a problem or a little afraid of a certain kind of example, review the articles which show you how to work that kind of example or problem. (See page 457.) **Do not wait to be told** to review work which will help you.

REVIEW FOR MASTERY

148. Denominate Numbers. (ORAL AND WRITTEN WORK.) Study the first three sets of review examples before the oral work begins. Use the tables on page 456 if you need them.

SET I (ORAL)

1. 6 mo. = ... of a yr.
2. 2 pk. = ... of a bu.
3. 8 qt. = ... gal.
4. 3 yd. = ... in.
5. 9 ft. = ... yd.
6. 1000 lb. = ... of a T.
7. 30 in. = ... ft.
8. 4 qt. = ... of a pk.
9. 2 lb. = ... oz.
10. 7 pt. = ... qt.
11. 10 oz. = ... of a lb.
12. 11 gal. = ... qt.

SET II (ORAL)

1. 4 ft. = ... in.
2. 8 mo. = ... of a yr.
3. 7 yd. = ... in.
4. $2\frac{1}{2}$ gal. = ... qt.
5. 9 in. = ... of a ft.
6. 16 pt. = ... gal.
7. 10 min. = ... sec.
8. 3 qt. = ... of a gal.
9. 2 T. = ... lb.
10. 3 hr. = ... min.
11. 6 bu. = ... pk.
12. $4 \times 2\frac{1}{2}$ in. = ... of a ft.

Set III (Oral)

1. 160 A. = ... of a sq. mi.
2. 8 in. = ... of a ft.
3. 30 sec. = ... of a min.
4. $3\frac{1}{2}$ yd. = ... ft.
5. 8 oz. = ... of a lb.
6. $2\frac{1}{2}$ ft. = ... in.
7. 7 gal. = ... pt.
8. $1\frac{1}{2}$ lb. = ... oz.
9. 4 bu. = ... pk.
10. 10 mo. = ... of a yr.
11. 18 in. = ... of a yd.
12. 5 min. = ... sec.

Set IV (Written)

Only answers in reduced form should be counted as correct.

1. 7×3 ft. 9 in.
2. 6×6 lb. 4 oz.
3. 8×9 qt. 1 pt.
4. 9×8 bu. 2 pk.
5. 5×4 T. 600 lb.
6. 2×12 yd. 2 ft.
7. 3×4 ft. 11 in.
8. 6×3 hr. 15 min.

149. Fundamentals of Division of Fractions. (Written Work.) Agree to count as correct only answers in simplest form. Decide what score you must make in order to be voted "competent in fractions." Your teacher will help you set up a class standard. **Time allowed:** 15 minutes.

If you fail to come up to the standard voted by the class, use the Self-Help Index on page 457.

1. $10 \div \frac{1}{3}$
2. Three sevenths divided by two.
3. $\frac{3}{8} \div 2\frac{1}{4}$
4. $10\frac{1}{2} \div 1\frac{1}{6}$
5. Three quarters divided by nine sixteenths.
6. $15 \div 1\frac{1}{2}$
7. $\frac{1}{5} \div \frac{1}{2}$
8. $4 \div \frac{1}{2}$
9. Three and one-half divided by two and one-tenth.

10. $\frac{6}{7} \div \frac{3}{4}$
11. $3\frac{1}{5} \div 1\frac{1}{2}$
12. $1\frac{9}{24} \div 3\frac{2}{3}$
13. $\frac{3}{16} \div \frac{9}{40}$
14. $4\frac{1}{6} \div 7\frac{1}{2}$
15. $\frac{1}{2} \div \frac{2}{3}$
16. $\frac{3}{10} \div \frac{2}{9}$
17. $1\frac{3}{5} \div 1\frac{1}{15}$
18. $1\frac{1}{6} \div \frac{3}{5}$
19. $8 \div 3\frac{3}{7}$
20. $5 \div \frac{1}{10}$
21. $\frac{1}{18} \div \frac{7}{9}$
22. Fifteen divided by one and one-half.
23. $\frac{7}{16} \div \frac{3}{8}$
24. $\frac{2}{3} \div \frac{3}{11}$
25. $1\frac{1}{3} \div \frac{1}{2}$
26. Five sevenths divided by one-half.
27. $\frac{1}{20} \div \frac{3}{4}$
28. Seven divided by two fifths.
29. $\frac{3}{8} \div 9$
30. $2\frac{1}{3} \div 2\frac{1}{2}$

After you have finished and have corrected the 30 examples above, see if you can answer the following questions.

1. What rule would use these words?

reciprocal *to divide* *multiply by* *fraction*

2. What improper fractions equal these mixed numbers?

$4\frac{1}{2}$ $9\frac{2}{3}$ $6\frac{4}{5}$ $7\frac{1}{4}$ $5\frac{5}{6}$ $2\frac{7}{12}$ $1\frac{1}{11}$

3. How would the fractions and mixed numbers below be changed in working the division examples?

$3 \div \frac{1}{4}$ $6 \div \frac{3}{5}$ $5 \div 2\frac{1}{2}$ $7 \div 6\frac{3}{5}$ $2 \div 8\frac{1}{3}$

REVIEW OF ZERO DIFFICULTIES

150. Practice with Zero. (STUDY PERIOD.) There are certain facts about zero which make it worth especial study.

I. In addition, the zero really makes the work easier, because it can be skipped. Copy and add the following numbers, and you will understand this. $246+350+600+40$

II. ***In subtraction,** zero must be watched carefully. Study Sample (*a*), where zeros occur in the minuend.

Sample (*a*).	*Subtract:* 9000 2087

By changing the form of the 9000, Sample (*a*) can be thought as: ⟶

1
899 0
208 7
691 3

Sometimes pupils forget that when they borrow from a number two or three places away, all the 0's are changed; so when the pupil gets to 9000, he sees $0-0=0$, which is so easy

$$\begin{array}{r} 2087 \\ \hline 13 \end{array}$$

that he forgets he has borrowed. The best protection against this blunder is always to check subtraction.

In Sample (*b*), the zero occurs in the subtrahend.

Sample (*b*).	*Subtract:* 2478 1206

Sometimes this blunder is made:

2478
1206
1202

Of course the correct answer is 1272.

Sample (*c*).	*Subtract:* 7214 4214

The correct difference is 3000. A blunder would be to write only the 3.

Work these examples:

1.	2.	3.	4.	5.
6000 −2137	2182 −1091	8000 −2006	5673 −4092	20000 −10029

*Teachers using Austrian Subtraction should omit this work on subtraction. See comment Art. 150, in *Teacher's Guide Book*.

III. Zero (0) in multiplication. Pupils sometimes have difficulty in writing the work in an example like Sample (*a*).

Sample (a).	409
	×25

Think $5\times9=45$. Write 5. $5\times0=0$ $0+4$ (carried from the 45) equals 4.

409
25
2045
818

Sample (b).	234
	×506

The work at the left below shows how a sixth-grade boy worked Sample (*b*). Notice that a 0 in 11,700 was ignored.

234
×506
1404
1170
3104

The work should be done as shown at the right.

He forgot to take care of the zero (0) in the multiplier because there happened to be another zero in the multiplication 5×4.

234		234
506		506
1404	or	1404
000		11700
1170		118404
118404		

Never hurry in multiplying when there are 0's in the multiplier.

The boy did this example correctly:	But this one he did incorrectly:
234	234
516	506
1404	1404
234	1170
1170	2574
120744	

Work these examples:

$14\times0=$	7. $37\times0=$		
$26\times0=$	8. $37+0=$		
$14+0=$	9. $0\times15=$		
$26+0=$	10. $0+15=$		
$14-0=$	11. $\frac{1}{2}\times0=$	13. $\frac{1}{2}+0=$	15. $2419\times609=$
$26-0=$	12. $37-0=$	14. $3008\times24=$	16. $234\times506=$

IV. Zero (0) in division. Study these samples:

Sample (*a*).	Here is a common error in division →	
206	**Watch out for this error!**	2 6
24)4944		24)4944
48		48
144		144
144		144

Wrong Thinking. Forgetting to write the **0** in the quotient in Sample (*a*) above when 24 will not divide into the **14** is a mistake. Always write the 0 in the quotient when you ought.

Sample (*b*). $26\overline{)4163}$ $16\frac{3}{26}$ Another error to watch out for! Work this example and you will see that the correct quotient is $160\frac{3}{26}$. To omit a zero at the end of a quotient when it belongs there is a bad mistake.

Sample (*c*). $.033\overline{)297}$ Think: first take care of decimal points by shifting, as: $.033\overline{)297.000}$. Since .033 is now a whole number, the 0 in .033 may be neglected. The 000 in the dividend must be written.

Sample (*d*). $24\overline{).192}$ No decimal point in divisor; so decimal point in quotient is above the point in the dividend, as: $24\overline{).192.}$ A blunder is to forget to write the 00 in a decimal quotient, as: $24\overline{).192.}$ $.008$ Do not think that, because 00 is not written in whole number division, as: $24\overline{)192}$ 8, the 00 can be

left out here: $24\overline{).192}$. If the zeros are left out, you get $24\overline{).192}$. It reads eight tenths, while eight thousandths (.008) is the correct answer.

Work the examples below. Notice that zeros hold other figures in their **proper places.**

1.	2.	3.	4.	5.
$24\overline{).0024}$	$56\overline{).112}$	$62\overline{)43875}$	$.025\overline{)1275}$	$.08\overline{)736.8}$
6.	**7.**	**8.**	**9.**	**10.**
$.06\overline{)35.22}$	$19\overline{)76095}$	$.09\overline{)36450}$	$.307\overline{)276.914}$	$92\overline{)3.036}$

V. Study these statements:

$4\% = .04$, but $40\% = .40$ or $.4$.

40 is 10 times larger than 4; .4 is ten times larger than .04.

$7.4 \times 10 = 74$. Think: $\times 10$ means **move point 1 place to right.**

$74 \times 10 = 740$. Think: $\times 10$ means 740. or 740

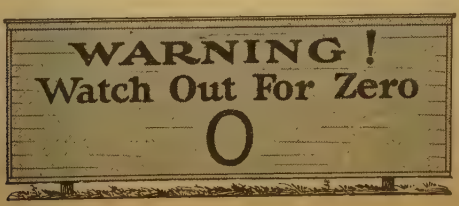
$.04 \times 100 = 4$. Why? $.7234 \times 100 = 72.34$ Why?

$872 \times 1000 = 872000$. Why? $.0007 \times 1000 = .7$ Why?

$93 \div 10 = 9.3$. $\div 10$ means **move point 1 place to left.**

$.4 \div 10 = .04$. Why? $920 \div 10 = 92$. Why?

$75 \div 100 = .75$. Why? $.04 \div 100 = .0004$. Why?



SELF-TESTING DRILL NO. 16

151. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

Watch out for the zeros in Example 3.

1.
970
×28

2.
853
63
474
352
541
295
356

3.
705
×506

4.
812289988
84670
746456362

5.
.814
×4.9

6.
.4522
−.1376

7.
$8\frac{5}{7}$
$-\frac{3}{5}$

8.
$4\frac{1}{4} \div 3\frac{1}{4} = ?$

9.
$\frac{3}{8} \times 7\frac{1}{3} = ?$

10.
2 pk. 7 qt.
×4

11.
Receipts $294.32
Expenses 137.75
Balance

12.
$46\overline{)2626}$

13.
$5 \div 1\frac{1}{2} = ?$

14.
$\frac{7}{8} + \frac{5}{24} = ?$

15. On a bill of $890, a certain dealer was to receive 45% discount. Find the amount of his discount.

16.
$4\frac{5}{7} \times 2\frac{2}{3} = ?$

17.
115750
−64929

18.
$116\frac{2}{3}\%$ equals what fraction?

19.
$9156 \div .037 = ?$

20. What is $3\frac{1}{2}\%$ of $147.50?

STANDARDS

Number correct..	0–2	3–4	5	6–7	8	9	10	11	12–13	14–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Are you correcting mistakes and using the Self-Help Index?

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER: THE SECOND USE OF PERCENTAGE

152. How to Find What Per Cent One Number Is of Another. (Study Period.) Study the whole article by yourself. Think carefully of any questions you want to ask your teacher when the study period is over. This article teaches you the second general use of percentage. In school and later on in life you must know how to find **what per cent one number is of another.**

Here is a good way to think of any question like this one: "56 is what per cent of 112?"

Think the question: "56 is what per cent of 112?", as **"56 is what fraction of 112?"** "56 is what per cent of 112" is simply another way of saying: **"56 is what part** or **what fraction of 112?"**

56 is what $\left\{\begin{matrix}\text{per cent}\\ \text{or}\\ \text{fraction}\end{matrix}\right\}$ of 112?

56 is $\frac{56}{112}$ of 112.

$\frac{56}{112}$ is changed to "decimal language" like this $112\overset{\bullet}{)}56.$

Any question like: "56 is what $\left\{\begin{matrix}\text{per cent}\\ \text{fraction}\end{matrix}\right\}$ of 112?" can be thought of as $\frac{56}{112}$ or $112\overline{)56}$.

We can see that 56 is $\frac{1}{2}$, or 50%, of 112. It is also .50 of 112. Let us see if 56 as a per cent of 112 would work out right as $\frac{56}{112}$ or $112\overline{)56}$.

```
      .50, or 50% Ans.
112)56.00
    56 0
    ----
       0
```

Do you see that this is an example in the **division of decimals?**

The .50 is changed to 50% because the question asks "what per cent?"

See if everything is done correctly in this example: 8 is what per cent of 24?

Think: 8 is what fraction of 24? Think: $\frac{8}{24}$, or $\frac{1}{3}$.
8 is $\frac{1}{3}$ of 24. 8 is $.33\frac{1}{3}$ of 24. 8 is $33\frac{1}{3}\%$ of 24.

We know that the question: "8 is what per cent of 24?" can be thought: "8 is what fraction of 24, or $\frac{8}{24}=?$" We see that $\frac{8}{24}=\frac{1}{3}$ and that $\frac{1}{3}=33\frac{1}{3}\%$.

Many examples in finding what per cent one number is of another, can be solved by writing the fraction, reducing the fraction, and stating the per cent which is equal to the fraction. Thus: 9 is what per cent of 36?

Think: $\frac{9}{36}=\frac{1}{4}=25\%$. 9 is 25% of 36.

ORAL EXERCISE

1. 2 is what per cent of 8?
2. 6 is what per cent of 36?
3. 9 is what per cent of 18?
4. 17 is what per cent of 34?
5. 3 is what % of 24?
6. 21 is what % of 28?
7. 75 is what % of 100?
8. 22 is what % of 55?

Sometimes you will have a problem in finding what per cent one number is of another, when the fraction is too large to be handled mentally. In such cases the problem is worked like the following two samples.

Sample 1. 93 is what % of 124? Think: $\frac{93}{124}$ or $124\overline{)93.}$

```
        .75, or 75%  Ans.
124)93.00
    86 8        .75 = 75/100
    -----       or 75%
     6 20
     6 20
     ----
```

93 = 93. = 93.00. Divide as shown at the left. Do not forget the decimal point in the quotient. **Change the quotient from a decimal to a per cent,** because you are asked what %, not what decimal.

Check: $.75\times124$

Check Sample 1 to see if 93 really is $\frac{3}{4}$, or 75%, of 124.

Sample 2. 45 is what % of 218? Think: $\frac{45}{218}$ or $218\overline{)45}$.

.206+, or about 20.6%

$218\overline{)45.00}$

43 6

1 400

1 308

$.206 = \frac{20\frac{6}{10}}{100}$ or 20.6%

20.6% may be written as follows: $20\frac{6}{10}\%$.

Be sure you see that .206 equals 20.6%.

The % stands for 2 decimal places, or for hundredths.

Is 45 about $\frac{1}{5}$ of 218?

The rule given below always works. Remember it.

Rule: **To find what per cent one number is of another—**

Think what fraction is made by the numbers. Reduce the fraction if you can. Try to see the per cent which equals the fraction.

When you cannot reduce to a small fraction, then divide the numerator by the denominator. Point off quotient carefully. Change quotient to per cent form by moving decimal point 2 places to the right and attaching per cent sign.

Study the work that follows on this page and the next.

1. 35 is what per cent of 56? (35 is what fraction of 56?)

$\frac{35}{56} = \frac{5}{8} = 62\frac{1}{2}\%$. Or $\frac{35}{56} \rightarrow 56\overline{)35.00}$

.62$\frac{1}{2}$, or 62.5% *Ans.*

33 6

1 40

1 12

28

$\frac{28}{56}$, the remainder, is reduced to $\frac{1}{2}$

2. What % of 70 is 42? (42 is what fraction of 70?)

.60, or 60% *Ans.*

$\frac{42}{70}=\frac{6}{10}=60\%$. Or $\frac{42}{70}\rightarrow 70\overline{)42.00}$ Find 60% of 70. It should equal 42. Does it?

420

0

3. 63 is what % of 72? (63 is what fraction of 72?)

.875, or 87.5%, or $87\frac{1}{2}\%$ *Ans.*

$\frac{63}{72}=\frac{7}{8}=87\frac{1}{2}\%$. Or $\frac{63}{72}\rightarrow 72\overline{)63.000}$

576

540 *Check:*

504 Find $87\frac{1}{2}\%$ of 72. What should it equal?

360

360

Any fraction can be changed to a decimal by dividing its numerator by its denominator. The decimal may then be written in per cent form.

Thus: $\frac{5}{12}$ **(fraction)** $=$ $12\overline{)5.00}$ $.41\frac{2}{3}$ **(decimal)** $=$ $41\frac{2}{3}\%$

Work the following:

4. 19.92 is what % of 24? (19.92 is what fraction of 24?)

$\frac{19.92}{24}$ or $24\overline{)19.92}$ Change quotient to a %. Why?

5. 24.45 is what % of 67? Or $67\overline{)24.45.}$ (Change quotient to %.)

6. Change $\frac{19}{95}$ to a decimal; then to a per cent. $(95\overline{)19.00})$

7. Change $\frac{21}{50}$ to a decimal; then to a per cent.

8. Change $\frac{11}{12}$ to a decimal; then to a per cent.

153. Practice in Finding What Per Cent One Number Is of Another. (STUDY PERIOD.) Review the table of Useful Facts to Remember on page 206 before you begin working these examples.

This work is to be done without pencil. Think what ought to be on each blank. In Example 1, think: 4 is what fraction of 8? Then think: $\frac{4}{8}$ or $\frac{1}{2}$ is the same as what per cent?

Practice on the examples in this exercise until every one is easy for you. Some of your answers will be more than 100%.

1. 4 is.....% of 8. 4 is.....% of 16. 4 is.....% of 12.
2. 7 is.....% of 21. 7 is.....% of 28. 7 is.....% of 35.
3. 9 is.....% of 18. 9 is.....% of 36. 9 is.....% of 72.
4. 11 is.....% of 33. 11 is.....% of 44. 11 is.....% of 55.
5. 8 is.....% of 24.
6. 80 is.....% of 160.
7. $\frac{1}{3}$ is.....% of 1.
8. 7 is.....% of 8.
9. 25 is.....% of 50.
10. $\frac{2}{5}$ is.....% of $\frac{4}{5}$.
11. 3 is.....% of 2.
12. 6 is.....% of 16.
13. $\frac{5}{8}$ is.....% of 1.
14. 50 is.....% of 80.
15. 16 is.....% of 16.
16. 20 is.....% of 80.
17. 6 is.....% of 10.
18. 8 is.....% of 12.
19. 100 is.....% of 500.
20. 5 is.....% of 30.
21. 200 is.....% of 300.
22. 12 is.....% of 20.
23. 8 is.....% of 6.
24. 18 is.....% of 16.
25. 9 is.....% of 45.
26. 15 is.....% of 20.

154. Review of the First Two Uses of Percentage. (CLASS WORK.) You have learned these two uses of percentage:

I. To find a given per cent of a number.

II. To find what per cent one number is of another.

Your work with these two uses will be made clearer if you keep the following suggestions in mind.

Suggestion 1. Problems and examples calling for either one of these two uses of percentage may be stated in different ways.

Thus, we would find 16% of 759 or $\begin{array}{r} 759 \\ \times .16 \\ \hline \end{array}$ for each of the following questions:

(*a*) What is 16% of 759?

(*b*) If 759 equals 100% of a number, what is 16% of it?

(*c*) What number equals 16% of 759?

No matter what way a problem is stated, if the real question is to find a certain per cent of a number, think: "Per cent of a number equals decimal times the number."

The question, "50.12 is what per cent of 83?" can be thought of as: "50.12 is what fraction of 83?" To solve this example, we would write it as $\frac{50.12}{83}$, or $83\overline{)50.12}$.

We could ask the question in several ways: "What per cent of 83 is 50.12?" or, "If 83 is the whole, what per cent of the whole is 50.12?" or, "If 83 equals 100% of a number, what per cent of the number is 50.12?"

No matter what way the question is asked, if you are to find what per cent one number is of another, you may think: What fraction is the one number of the other? Then write the fraction as a division example.

Suggestion 2. It pays to study work which is done correctly. Do you understand the next work?

Sample of Use I

Find 136% of \$362.

Think: "per cent of" means "decimal times."

\$362
×1.36
2172
1086
362
\$492.32 *Ans.*

\$492.32 is 136% of \$362.

Sample of Use II

\$492.32 is what per cent of \$362?

Think: "what per cent of" means "what fraction of."

$\frac{492.32}{362}$ or $362\overline{)492.32}$ $\quad 1.36 = 136\%$

362
1303
1086
2172
2172

\$492.32 is 136% of \$362.

These two uses of percentage work just as well when the per cent is larger than 100% as when smaller per cents are called for. In the Sample of Use I above, why is the decimal 1.36 used? In the Sample of Use II above, why is the decimal 1.36 in the quotient changed to 136%?

PRACTICE IN USING JUDGMENT

Suggestion 3. It helps to be able to see what question in percentage the solving of an example will answer. In the work below tell what question in percentage would be answered by solving each example.

SUGGESTION: The example at the right would answer the question, "What number is 58% of 567?"

567
×.58

1. \$783	2. \$783	3. \$783	4. \$783
×.04	×.045	×.40	×.0425

5. $\$783 \times 4.25$

6. $\frac{837}{1674}$ or $1674\overline{)837.}$

7. $\frac{318}{173}$ or $173\overline{)318.}$

8. $\frac{1525}{305}$ or $305\overline{)1525.}$

9. $3702 \times .07$

10. $\frac{299.41}{3702}$ or $3702\overline{)299.41}$

Which of the examples above have answers in decimal form that should be changed to per cents?

Which examples above have answers that are more than 100%?

PRACTICE IN SELECTION

Suggestion 4. Good workers can tell whether an example is written correctly. For each example below, several solutions are shown. Only one is correct. Which one is it?

1. Find $4\frac{1}{2}\%$ of 936.

(*a*) $936 \times 4\frac{1}{2}$ (*b*) 936×45 (*c*) $\frac{45}{936}$ or $936\overline{)45.}$ (*d*) $936 \times .045$

2. 362 is what per cent of 1351?

(*a*) 1351×3.62 (*b*) $\frac{362}{1351}$ or $1351\overline{)362.}$ (*c*) $\frac{1351}{362}$ or $362\overline{)1351.}$

3. What per cent of 450 is 932?

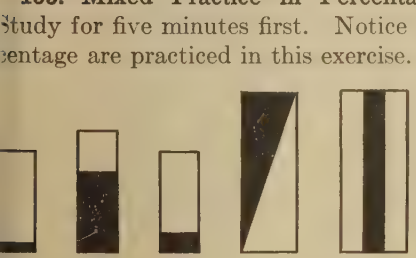
(*a*) $\frac{450}{932}$ or $932\overline{)450.}$ (*b*) 932×4.50 (*c*) $\frac{932}{450}$ or $450\overline{)932.}$

4. What number equals 125% of 249?

(*a*) 249×125 (*b*) 249×1.25 (*c*) $\frac{125}{249}$ or $249\overline{)125.}$ (*d*) $249 \times .125$

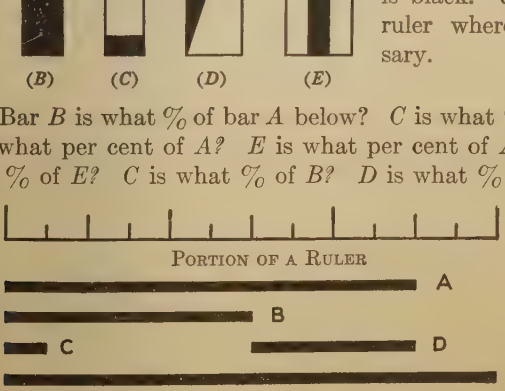
DRILLS AND PROBLEMS USING PERCENTAGE

155. Mixed Practice in Percentage. (ORAL WORK.) Study for five minutes first. Notice that two uses of percentage are practiced in this exercise.



1. In each of the figures at the left tell what per cent is black. Use your ruler where necessary.

2. Bar *B* is what % of bar *A* below? *C* is what % of *A*? *D* is what per cent of *A*? *E* is what per cent of *A*? *B* is what % of *E*? *C* is what % of *B*? *D* is what % of *E*?



3. 4% of 200 is.....
4. $12\frac{1}{2}$% of 8 bu. is.....
5. 28 is.....% of 14.
6. $33\frac{1}{3}$% of 666 is.....
7. 6 in. is.....% of 2 ft.
8. $87\frac{1}{2}$% of 24 is.....
9. 80 is.....% of 40.
10. $16\frac{2}{3}$ is.....% of $33\frac{1}{3}$.
11. $83\frac{1}{3}$% of 30 is.....
12. 5 is 50% of.....
13. 40 is.....% of 60.
14. \$2.50 is 25% of.....
15. 12 is.....% of 36.
16. 6A. is.....% of 18A.
17. 1 pt. is.....% of 2 qt.
18. 1 bu. is.....% of 1 bu.

156. Practice on the First Two Uses of Percentage. (WRITTEN WORK.) Remember: (*a*) Find 14% of 1926, means $.14 \times 1926$; (*b*) What per cent of 136 is 34? means 34 is what fraction of 136?

1. Find 14% of 1926.
2. What per cent of 136 is 34?
3. What % of 216 is 72?
4. Find 88% of 2167.
5. What per cent of 264 is 33?
6. 24 is what % of 600?
7. 26.5 is what % of 300?
8. 24 is what % of 150?
9. Find 125% of 84.
10. 84 is what % of 42?
11. Does $4\frac{1}{2}\%$ of 451 = 202.95 or 2.0295 or 20.295?

157. Problems Using Percentage. (WRITTEN WORK.) Study all the problems and plan your work before you begin.

1. The standing of the clubs in the National League during one season was determined from this table:

CLUB	GAMES WON	GAMES LOST	PER CENT WON
Chicago	104	50	67.5%
New York	91	63	
Pittsburgh	86	67	
Philadelphia	78	75	
Cincinnati	75	79	
Brooklyn	64	90	
St. Louis	63	90	
Boston	53	100	

Find for each club what per cent of the total games **played** were won. Carry per cents to one decimal place.

How will you find the total number of games played? How many decimal places will you have in the quotient **if** the **per cent** is to have **one** decimal place?

2. Find the per cent of accuracy of the pupils named in ne report below. Figure to nearest whole per cent. Ar- ange your answers so that the highest per cent (with pupil's ame) will come at the top of the list, etc.

PUPIL'S NAME	PROBLEMS TRIED	PROBLEMS CORRECT	PER CENT OF ACCURACY
Henry	94	80	
Mary	64	47	
Stephen	80	72	
William	106	81	
Lucy	75	53	
Florence	100	91	
Margaret	79	65	
John	80	70	
Harry*	54	27	

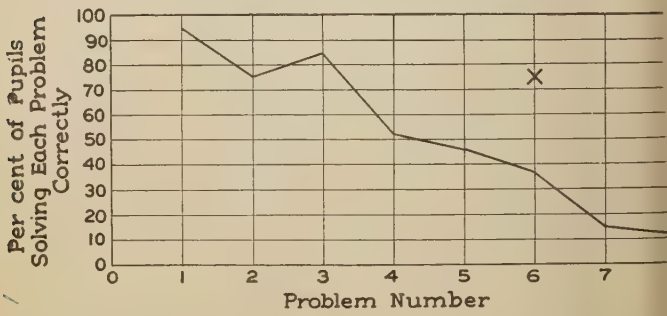
*Harry was absent from school 4 weeks.

3. Draw a graph showing the per cent of correct prob- ems of the pupils in Problem 2. Arrange your graph so that ne pupil with the largest per cent will come first, etc. For graph drawing, see Art. 83.)

4. In the table below, column (*A*) shows the number of ays in the school year for certain schools in various parts f the country. Column (*B*) shows the number of absences n days per pupil. Find the per cent of absence from chool in each case. Carry per cents to one decimal place.

(*A*) DAYS IN SCHOOL YEAR	(*B*) ABSENCES IN DAYS
182	6
180	16
176	3
172	11
160	9

158. Reading Per Cents from Graphs. (ORAL WORK. Study this graph. Answer all the questions about it.



1. What do the 0, 10, 20, etc., to 100 on the graph mean?

2. What do the 1, 2, 3, 4, 5, 6, 7, 8 on the graph mean?

3. What per cent of pupils did Problem 1 correctly?

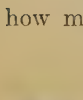
4. About what % of the pupils did each problem correctly?

5. What per cent of pupils would have done Problem 6 correctly if the line had gone through the mark ×?

6. Which was the hardest problem? The easiest problem?

7. Suppose 100 pupils took the examination. About how many of them did each problem correctly?

SOME CAUSES OF ERRORS



159. Some Stubborn Errors in Multiplication and Division. (CLASS DISCUSSION.) You may remember that in Art. 96 you were put on your guard against some frequent errors in addition and subtraction. See if the errors listed on the next 3 pages

e the ones you have to fight against in your work with ultiplication and division.

MULTIPLICATION ERRORS

Error 1. You may think that it is strange, but it seems at the most frequent error in multiplication is getting ie wrong product for simple multiplication facts, such as $\times 8$, 9×8, or 6×9. It does little good to know some the multiplication facts and keep getting others of them rong.

Watch yourself as you work multiplication examples. If iere are certain facts which bother you, practice on them ntil you know them as well as you know that $2 \times 2 = 4$.

➡ The Self-Help Index on page 457 will show you where find practice work on multiplication.

Error 2. Mistakes are often made in adding the number arried. Samples of this type of error are shown below.

SAMPLE (*A*)

478	*Error:*
$\times 36$	$6 \times 7 + 4 = 46$,
58	not 45
↑	

SAMPLE (*B*)

377	*Error:*
$\times 27$	$7 \times 7 + 4 = 53$,
29	not 52
↑	

It is poor policy to let mistakes in addition continue to ouble you. To carry this handicap in your arithmetic ork would be like trying to win a foot race while carrying heavy bag of sand on your shoulders.

➡ Use the Self-Help Index on page 457 to find practice ork on multiplication if you need such work.

Error 3. Zero in the multiplier seems to cause pupils altogether too much trouble. Read again the note on zero in the multiplier (page 215). Study the next samples.

SAMPLE (*A*)

```
 372   Error:
×406   0×2=0,
2232   not 2
   2
   ↑
```

SAMPLE (*B*)

```
 372   Error:
×406   Even if 0 is not actually used
2232   as a multiplier, the product of
   8   4×2 must start under the 4;
   ↑   not under the 0.
```

DIVISION ERRORS

Error 1. Many pupils have trouble with quotient figures. To decide on the correct quotient figures seems to be the hardest thing in division. Do you see the mistakes made in the samples below?

SAMPLE (*A*)

```
   9   Error:
17)108 17 goes into 108
       6 times, not 9
       times.
```

SAMPLE (*B*)

```
   9   Error:
28)2784 The 9 is the correct
        figure, but it is mis-
        placed. It should
        be over the 8.
```

If you dislike division, you may suspect that you have not mastered it well enough. Most people do not like to do what they can not do well. However, they rather like to do those things which they can do easily and correctly.

Watch your work in division and see if finding the correct quotient figures bothers you. If so, your Self-Help Index will refer you to articles for study and practice.

Error 2. Mistakes are often made in the multiplication and subtraction necessary in dividing. Of course you will not divide very well if you make mistakes in the multiplication and subtraction. Study the two samples below.

SAMPLE (*A*)

```
   6      Error:
46)31745  6×46=276,
  266     not 266.
   ↑
```

SAMPLE (*B*)

```
   6      Error:
46)31745  The 3 is wrong.
  276        ↑
   31     It should be 4.
   ↑
```

Do you think there is any excuse for a sixth-grade pupil making mistakes like those in the two samples above?

Error 3. Our "old friend Zero" seems to trouble us in quotients. Do you see the mistakes in Samples (*A*) and (*B*)?

SAMPLE (*A*)

```
  45      Error:
26)10530  Quotient is 405,
  104     not 45.
  130
  130
```

SAMPLE (*B*)

```
  45      Error:
26)11700  Last 0 in dividend not used.
  104
  130     Quotient is 450,
  130     not 45.
```

☛ From now on insist on accurate work in multiplication and division. You will catch mistakes by checking, and you may use the Self-Help Index on page 457 at any time for a little independent study and practice when you think you need it.

SELF-TESTING DRILL NO. 17

160. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

Remember to reduce answers to proper form.

1. 94×201

2. $98\overline{)4508}$

3. $378 \times .40$

4. $1709 - 975$

5.
$$\begin{array}{r} 984 \\ 394 \\ 795 \\ 295 \\ 871 \\ 866 \\ \hline \end{array}$$

6. $9 \times 1\frac{5}{6} = ?$

7. $16.593 \div 2.2 = ?$

8. $\frac{3}{7} \div 2 = ?$

9.
$$\begin{array}{r} \frac{1}{10} \\ \frac{2}{15} \\ \frac{1}{2} \\ \frac{1}{30} \\ \hline \end{array}$$

10. $6.03 \times .95$

11. $.7468 - .3567$

12. Find the volume of a wheat bin $6' \times 9' \times 15'$.

13. $7\frac{2}{5} - 3\frac{5}{6} = ?$

14.
$$\begin{array}{r} 8\frac{3}{5} \\ 6\frac{1}{2} \\ 12\frac{3}{4} \\ \frac{9}{10} \\ \hline \end{array}$$

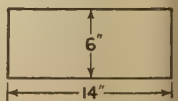
15.
$$\begin{array}{r} .749 \\ .688 \\ .377 \\ .658 \\ .969 \\ .379 \\ \hline \end{array}$$

16. $\frac{14}{15} \div 2\frac{1}{10} = ?$

17. *Add:*

3 yd. 1 ft.
1 yd. 2 ft.
4 yd. 2 ft.
5 yd.

18. Find the area of this rectangle. →



19. An agent sold $29,652.00 worth of books at a commission of 15%. What was his commission? (A commission is money paid an agent or salesman for his work.)

20. A team played 18 games and won 12 of them. What per cent of the games were won?

STANDARDS

Number correct..	0–3	4–5	6–7	8	9	10	11	12–13	14	15–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

USING PERCENTAGE

161. Finding and Reporting Per Cents. (Problem Work.) You may need to study Art. 152 to help you with the work below.

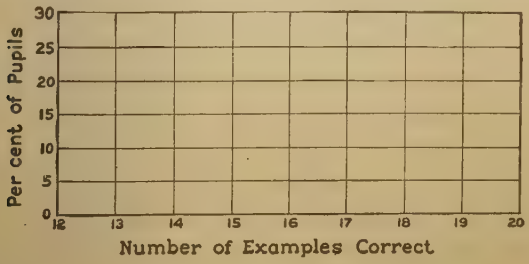
In the sixth grade of the Williams School, the scores made on Drill No. 17 are recorded below.

3 pupils did 20 correctly.	4 pupils did 16 correctly.
6 pupils did 19 correctly.	4 pupils did 15 correctly.
9 pupils did 18 correctly.	2 pupils did 14 correctly.
6 pupils did 17 correctly.	2 pupils did 13 correctly.

1. Find what per cent of pupils did correctly 20 examples, 19 examples, etc. Figure to the nearest whole per cent.

(3 pupils is what per cent of all the pupils?)

2. Report the per cents asked for in Problem 1 on a graph like the sample below, but draw yours much larger. Notice carefully what the numbers on the graph mean.



3. Report on a graph like the one in Problem 2, above, the following class record from a St. Paul, Minn., school.

4 pupils did 19 correctly.	8 pupils did 15 correctly.
4 pupils did 18 correctly.	5 pupils did 14 correctly.
10 pupils did 17 correctly.	4 pupils did 13 correctly.
5 pupils did 16 correctly.	

PROBLEM SCALE NO. X

162. Problems about School Subjects. (WRITTEN WORK.) You can earn 50 points by working every problem correctly. Find your score and rate yourself as directed for Scales I to III.

A group of sixth-grade pupils named the one subject that they liked best. When the results were made into a table (tabulated), the following facts were shown.

SCHOOL SUBJECT	NUMBER LIKING SUBJECT BEST	
	Boys	*Girls*
Drawing or Art	6	8
Geography	15	23
English	17	20
Nature Study	7	9
History and Civics	10	5
Household Arts	0	15
Manual Training	21	0
Arithmetic	20	11
Music	3	7
Penmanship	1	2

1. How many girls voted for the subject that was liked best by the girls? (4)

2. Name two school subjects which were liked best by about 20% of the girls. (4)

3. Name two school subjects which were liked best by about 20% of the boys. (4)

4. What per cent of girls liked arithmetic best? (5)

5. What per cent of boys liked geography best? (5)

6. Suppose that 14% of the boys had voted for penmanship. How many boys would have voted for it? (6)

7. How many girls would have had to vote for household arts in order to give it 80% of the girls' votes? (6)

8. How many boys voted for the subject that was best liked by boys and girls counted together? (7)

9. Draw a graph showing the interest of the boys in each subject. On the same graph show the ratings for girls. (Use a solid line for boys and a dotted line for girls.) (9)

STANDARDS

Superior work:	25–50 points	Slow work:	6–13 points
Good work:	22–24 points	Inferior work:	0– 5 points
Average work:	14–21 points		

163. Percentage Ratings of School Subjects. (A CLASS PROJECT.) Make a list of the subjects that your class studies. Have every pupil check the one subject that he thinks he likes the best. Then make a table of these votes, like the table in Art. 162.

PROBLEMS

1. How many boys voted?
2. How many girls voted?
3. What subject had the most votes from the boys?
4. What per cent of the total boys' vote was this vote?
5. Find per cent of votes given each subject by boys.
6. Find per cent of votes given each subject by girls.
7. Draw a graph showing the per cent of votes given to each subject by the boys. (Use a solid line.)
8. On the same graph draw a dotted line showing the per cent of votes given to each subject by the girls.

164. A "Hard Spot" in Writing Per Cents. (CLASS WORK.) Sometimes we find in print per cents like $6\frac{1}{4}\%$, $12\frac{1}{2}\%$, $33\frac{1}{3}\%$, $87\frac{1}{2}\%$, etc. Such mixed per cents and common fractions must at times be changed to decimal form, as in computing the discount on an article sold at $12\frac{1}{2}\%$ off the regular price.

Expressing numbers like $4\frac{1}{2}\%$ in decimal form seems to cause some pupils a great deal of trouble.

➡ It is said that for every ten pupils who can change 4% to a decimal correctly there are but three who can express $4\frac{1}{2}\%$ as a decimal.

At first it may be best for you simply to study the decimal equivalents of some of the most widely used mixed per cent and fraction expressions commonly occurring in business. In studying the following, think of N as meaning any number that you may be working with.

$4\frac{1}{2}\%$ of N means $\frac{4\frac{1}{2}}{100}$ of N, or $\frac{4.5}{100}$ of N, or $.045 \times N$

$12\frac{1}{2}\%$ of N means $\frac{12\frac{1}{2}}{100}$ of N, or $\frac{12.5}{100}$ of N, or $.125 \times N$

$6\frac{3}{4}\%$ of N means $\frac{6\frac{3}{4}}{100}$ of N, or $\frac{6.75}{100}$ of N, or $.0675 \times N$

$37\frac{1}{2}\%$ of N means $\frac{37\frac{1}{2}}{100}$ of N, or $\frac{37.5}{100}$ of N, or $.375 \times N$

$3\frac{3}{4}\%$ of N means $\frac{3\frac{3}{4}}{100}$ of N, or $\frac{3.75}{100}$ of N, or $.0375 \times N$

$33\frac{1}{3}\%$ of N means $\frac{33\frac{1}{3}}{100}$ of N, or $\frac{33.3\frac{1}{3}}{130}$ of N, or $.333 \times N$

In the case of $33\frac{1}{3}\%$, it is more accurate to write $.33\frac{1}{3}$. Similarly $66\frac{2}{3}\%$ becomes $.66\frac{2}{3}$.

Thus far you have learned that 7% of a number means **7 hundredths** (.07) of that number. If this is true, $7\frac{1}{2}\%$ of a number equals $7\frac{1}{2}$ hundredths ($.07\frac{1}{2}$) of that number. However, it is easier to express $.07\frac{1}{2}$ as .075 when actually working with per cents.

Sixth-grade pupils should not only know that $7\frac{1}{2}\%$ is ritten as .075, but also the reason why this is true. If you) know the reason, it will help you to overcome this "hard ›ot" in percentage. Study the following work very carelly so that you can see each step and why it is correct.

$7\frac{1}{2}\%$ of $N = \frac{7\frac{1}{2}}{100}$ of $N = \frac{7.5}{100}$ of $N = .075 \times N$

$6\frac{1}{4}\%$ of $N = \frac{6\frac{1}{4}}{100}$ of $N = \frac{6.25}{100}$ of $N = .0625 \times N$

$7\frac{1}{2}\%$ of $N = \frac{137\frac{1}{2}}{100}$ of $N = \frac{137.5}{100}$ of $N = 1.375 \times N$

Even more rarely you will have to use such expressions 4.6% or 13.2%. If you remember that "per cent of a mber" means "hundredths of that number," expressions e these will not trouble you.

You can express 4.6% as .046 (its decimal equivalent), us: 4.6% of a number $= \frac{4.6}{100}$ of the number = .046 times e number.

➡ The secret of overcoming the difficulties in changing h expressions as $8\frac{3}{4}\%$, $87\frac{1}{2}\%$, 6.2%, 55.8%, etc., to ir decimal equivalents is remembering that given per its of a number mean hundredths of the number.

EXERCISE

Change these per cents to their decimal equivalents:

1. $2\frac{1}{2}\%$	6. $16\frac{2}{3}\%$	11. $87\frac{1}{2}\%$
2. 6.5%	7. $33\frac{1}{3}\%$	12. 72.1%
3. $28\frac{3}{4}\%$	8. $62\frac{1}{2}\%$	13. $137\frac{1}{2}\%$
4. $\frac{1}{4}$ of 1%	9. 12.5%	14. $\frac{1}{2}$ of 1%
5. $37\frac{1}{2}\%$	10. 45.6%	15. 6.75%

SELF-TESTING DRILL NO. 18

165. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

Mistakes should be corrected. Checking will reduce errors.

1. $7.197 - 5.222 = ?$

2.
34371
65136
53249
29168

3.
7005
-1007

4. $\frac{4}{15} - \frac{1}{10} = ?$

5.
$4\frac{5}{24}$
$+\frac{3}{4}$

6.
$6\frac{3}{7}$
$+7\frac{3}{14}$

7.
80235
68021
25431
91064

8. *Add:*
1 gal. 3 qt.
4 gal.
2 gal. 1 qt.

9.
689
$\times 579$

10. $\frac{2}{7} \div \frac{1}{2} = ?$

11.
9735
$\times 68$

12. $68\overline{)625634}$

13. $277.9 \div 49 = ?$

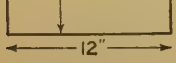
14.
$1\frac{1}{15}$
$\times 3$

15.
.698
$\times 574$

16. Find $7\frac{1}{2}\%$ of \$324.

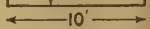
17. $\frac{1}{3}$ divided by $9 = ?$

18. Find area of this rectangle:



19. 84 is what % of 420?

20. Find area of this rectangle:



STANDARDS

Number correct.	0–4	5-6	7	8	9	10	11	12–13	14–15	16–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

SUMMARY

166. General Review. (Class Work.) To be sure that you have learned the new work in the chapter, answer the questions below.

1. Does $84 \times \frac{1}{4}$ equal $84 \times .25$?

2. Is 200% of a number 200 times that number, or is it twice that number?

3. Which of the three following processes is the correct work for this example? Find 16% of 96.

(*a*) $96\overline{)16}$ (*b*) $\begin{array}{r} 96 \\ \times .16 \\ \hline \end{array}$ (*c*) $16\overline{)96}$

4. What decimal equals 4%? .0025 = what%?

5. $62\frac{1}{2}\%$ equals what fraction?

6. Name three kinds of discounts.

7. What does the following phrase mean: "2% off for cash"?

8. 84 can be thought of as what fraction of 252?

9. What does 1% of 728.4 equal?

10. Make a problem in percentage which uses the multiplication shown at the right.

$$\begin{array}{r} 896 \\ \times .36 \\ \hline \end{array}$$

11. Make an example in percentage which uses the fraction shown at the right.

$$\frac{708}{1180}$$

12. What per cent equals $.62\frac{1}{2}$?

13. What does "net profit" mean?

14. What is meant by a "consumer"?

15. On a Problem Scale could the number of points earned equal 125% of the total possible points?

16. Does a discount ever increase the amount of a bill?

17. What is meant by the word "commission"?

18. What is the difference between a discount for cash and a discount for quantity purchase?

19. Could a discount be more than 100%?

20. 87% equals a fraction with what numerator and denominator?

21. Which division below shows how to change $\frac{4}{7}$ to a decimal?

Division I	Division II
$4\overline{)7.00}$	$7\overline{)4.00}$

22. What question in percentage does the work below suggest?

$$\frac{24}{136} \text{ or } 136\overline{)24.00}$$

23. What is the reciprocal of $\frac{8}{9}$?

CHAPTER VIII

MENSURATION OF AREAS AND VOLUMES. REVIEW OF PERCENTAGE

MENSURATION OF AREAS AND VOLUMES

167. Review of Mensuration. (Oral Work.) Give the following questions a few minutes of silent study, and be ready to answer them quickly when your teacher asks for answers. If you need to do so, do not fail to study the article or articles, the numbers of which are given after the questions.

QUESTIONS

1. Does the word "area" mean the length, width, or surface of an object? (Art. 314.)

2. How much would 4 inches on a map represent if the scale were $1'' : 1000'$? (In reading scales the sign : stands for the preposition "to." Thus, $1'' : 1000'$ would be read "one inch to a thousand feet.") (Art. 59.)

3. Could a carpet $9' \times 12'$ be laid in a room whose dimensions are $7'11'' \times 12'8''$?

4. If L = length, and W = width, $L \times W$ = what? (Art. 315.)

5. What would you do first in working this example? $L = 7'5''$, $W = 9'7''$. Find the area. (Art. 317.)

6. In the example $6' \times 7' = 42$....., what should be on the blank? (Art. 315.)

7. What are four units which are used to measure length? (Art. 317.)

8. What are four units which are used to measure area? (Page 456.)

9. What are four units which are used to measure width? (Art. 317.)

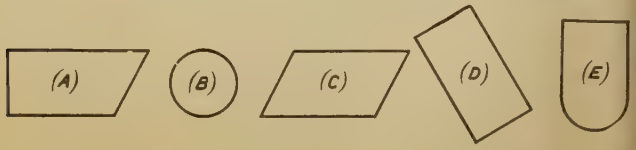
10. If L = length, W = width, H = height, $L \times W \times H$ = what? (Art. 319.)

11. Is the cubic foot a unit of measure of length, breadth, area, or volume? (Art. 319.)

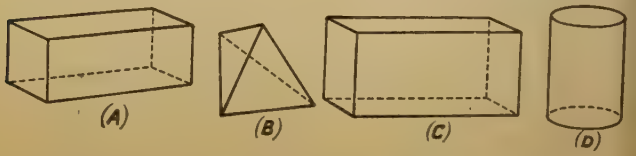
12. $6'' \times 7'' \times 10'' = 420$ ..... What should be on the blank? (Art. 319.)

13. Why do careful pupils memorize the facts given in the Tables of Measure on page 456?

14. Which of the surfaces shown below is a rectangle?

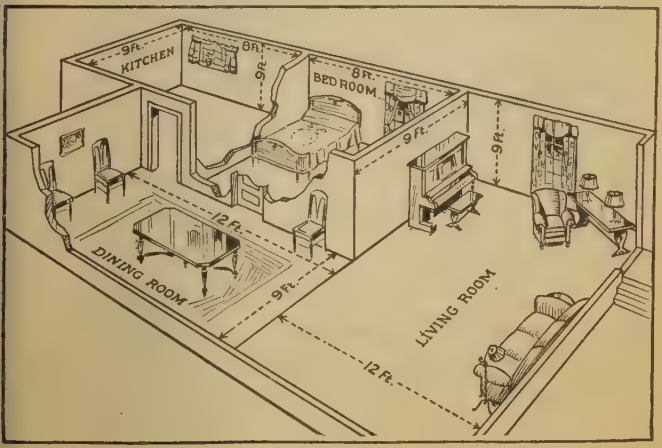


15. Which of the volumes below are rectangular solids?



16. Do "sq. mi." and "A." measure area or volume?

168. House Measurements. (PROBLEM WORK.) Look carefully for the right dimensions to use in the problems.



PROBLEMS

1. How much will it cost to cover the entire kitchen floor with linoleum at $16\frac{1}{2}¢$ a sq. ft.?

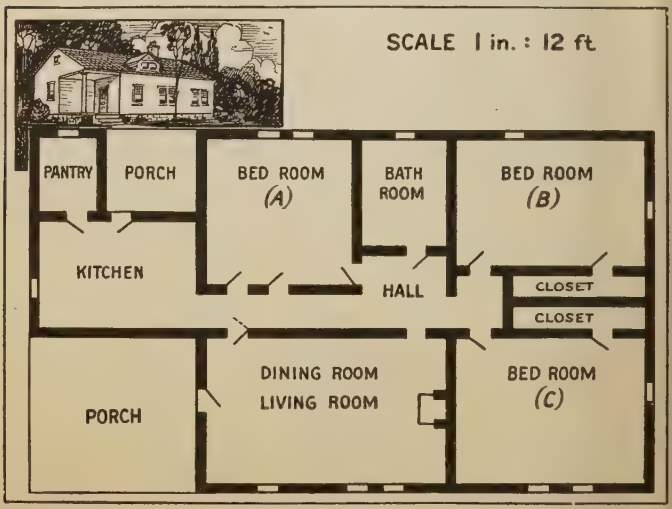
2. Find the total floor space of the 4 rooms.

3. How much would a hard-wood floor cost in the dining room, if the materials and labor cost \$1.12 a sq. ft.?

4. How many square feet are there in the walls of the living room, subtracting nothing for doors and windows? (Think of the walls as not having been cut away.)

5. How many rolls of wall paper would be needed to paper the dining room walls? (Subtract 2 rolls from total for doors and windows.) A roll of wall paper is 15′ long and 18″ wide.

169. Floor Plans. (CLASS WORK.) Study the plan and the questions below it before the oral work begins. You will need to use your ruler in answering the questions.



QUESTIONS

1. What is the total ground area covered by the house, including porches?

2. What is the total floor space given to bed rooms?

3. How does the ground area of this house compare with the floor area of your school room?

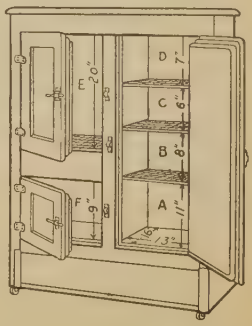
4. (SPECIAL WORK.) Prepare a floor plan of your own home. Be careful to measure correctly. Choose a convenient scale for your drawing.

170. Review of Measurement of Volumes. (STUDY PERIOD.) This article will give you practice in finding volumes. You may need to review Art. 319.

EXERCISE

Dimensions of the refrigerator:
Outside height, 43″
Outside width, 35″
Outside depth, 19″

Compartments E and F have the same depth and width as the other compartments.



1. Could a block of ice 12″ × 12″ × 16″ be put into Compartment E?

2. In which compartment besides E could a jar 10″ tall be placed?

3. What is the amount of floor space that is taken up by this refrigerator? (See *Dimensions of the refrigerator* above.)

4. What is the area of the bottom of Compartment F?

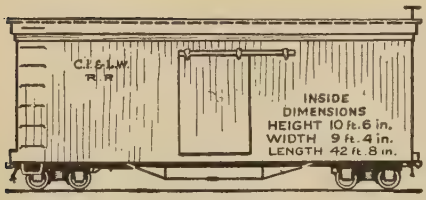
5. What is the outside area of the back of this refrigerator?

6. What is the outside area of one side of this refrigerator?

7. What is the volume of $A+B+C+D$?

8. The volume of Compartment E is how much greater than the volume of Compartment F?

9. The refrigerator takes up how many cubic inches of space?



10. What is the inside volume of the freight car?

11. The volume of your school room is equal to the inside volume of how many freight cars like this one?

12. (SPECIAL WORK.) Bring to class tomorrow a drawing of your refrigerator or of a trunk or wagon box, showing dimensions. Make your drawing so that it can be understood easily. Take the measurements carefully. Make a rough drawing first.

PROBLEM SCALE NO. XI

171. Practical Measurements. (WRITTEN WORK.) You can earn 85 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. How many cubic feet of air are there in a room $8' \times 10' \times 15'$? (5)

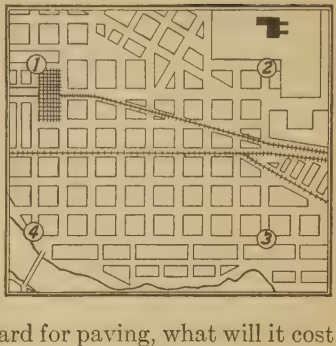
2. Find the volume of a rectangular solid with dimensions 8 ft. $\times 9\frac{1}{2}$ ft. $\times 4$ yd. (6)

3. Find the area of a rectangle 6′4″ wide and 7′ long. (7)

4. The dimensions of a trunk are $2.5' \times 2.8' \times 4'$. It is full of clothes, books, and equipment for a summer trip. It weighs 210 lb. What is the weight per cu. ft.? (7)

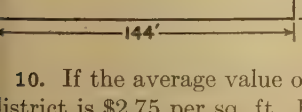
5. If the weight of the trunk and contents in Problem 4 had averaged 9.25 lb. per cu. ft., what would have been the total weight? (8)

6. Find the area of the business district in square feet. The business district is the rectangle inclosed by the numbers 1, 2, 3, and 4. From 1 to 2 is 1000 yd. From 2 to 3 is 750 yd. (8)



7. What is the volume of a building 280′ wide, 310′ long, and 824′ high? (8)

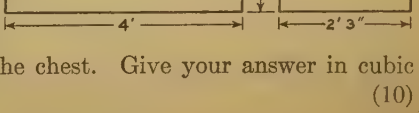
8. Allowing \$3 a square yard for paving, what will it cost to pave a section of a street 30′ wide and 80′ long? (8)



9. At \$2 a square foot, find the cost of the plot of ground shown in the drawing. (9)

10. If the average value of the land in a certain business district is \$2.75 per sq. ft., how much is a lot 36′ × 144′ in the business district worth at average value? (9)

11. The dimensions of the tool chest, as shown in the diagram, are all inside dimensions.



Find the volume of the chest. Give your answer in cubic feet and inches. (10)

STANDARDS

Superior work:	18–85 points	Slow work:	6–7 points
Good work:	12–17 points	Inferior work:	0–5 points
Average work:	8–11 points		

172. Rapid Drill. (ORAL WORK.) Exercises such as this one will give you practice in working without pencil. Give only the first quotient figure in the division examples.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	196 −99	8+19+6	120 −24	181 −94	8 is what % of 16?	6.7 ×8
2.	$\frac{2}{3} \div 2\frac{5}{6} =$ $\frac{2}{3} \times$ ----	$69\overline{)684}$	$74\overline{)636}$	69 ×4	5.6 ×.9	$47\overline{)235}$
3.	19+8+9	66 ×6	163 −94	$48\overline{)367}$	16+8+7	13.7 −7.8
4.	131 −59	163 −66	$79\overline{)209}$	175 −78	164 −79	154 −87
5.	172 −89	25+9+6	*What is wrong?* Area = 18 in.	2.7+.1 +.7	122 −54	142 −88
6.	9+17+5	121 −67	143 −95	87 ×4	$98\overline{)588}$	112 −63
7.	43 ×.9	$.59\overline{)1.77}$	101 −13	$37\overline{)149}$	99 −40	*Best form of Ans.* 4 yd. 5 ft.
8.	$7\frac{1}{2}\%$ = what decimal?	.0425 = what %?	$\frac{36}{712}$ or $712\overline{)36.}$ Answers what question in percentage?		\$972 ×.73 Answers what question in percentage?	

SELF-TESTING DRILL NO. 19

173. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Are you correcting mistakes carefully?**

1. 524×6

2. $22\overline{)308}$

3. $816 + 30 + 1870 + 50 + 583$

4. $1200 - 582$

5. $167 + 358 + 580 + 163 + 421 + 278$

6. 374×75

7. $4\frac{1}{3} - 1\frac{1}{3}$

8. *Write the correct answer:*

$$\begin{array}{r} 9.4 \\ \times 3.2 \\ \hline 188 \\ 272 \\ \hline 29.08 \end{array}$$

9. *Reduce to lowest terms:* $\frac{18}{4}$

10. $\frac{2}{3} \times \frac{9}{10} = ?$

11. 3 qt. 1 pt. $-$ 2 qt. $= ?$

12. $\frac{1}{6} \div 4 = ?$

13. $72.7 - 1.489 = ?$

14. 528×3.8

15. $4\frac{1}{3} + \frac{9}{10} + 2 = ?$

16. $.059\overline{)27.75}$

17. *Write in figures:* forty-four thousand six hundred and nineteen hundredths.

18. $.78\overline{)7286}$

19. 8 rd. 3 yd. 1 ft. $\div 3 = ?$

20. The list price of a bill of goods was $984, less a dealer's discount of 40%. What was the net cost?

STANDARDS

Number correct.	0–4	5	6–7	8	9	10	11	12–13	14–15	16–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

PRACTICE ON FRACTIONS AND DECIMALS

174. Fundamentals in Fractions. (WRITTEN WORK.) You should work this exercise in less than 10 minutes without a mistake. If you cannot do so, you should spend extra time in reviewing fractions. Use the Self-Help Index on page 457.

1. $\frac{1}{3}+\frac{1}{3}$
2. $\frac{7}{8}\div\frac{3}{4}$
3. $\frac{1}{2}-\frac{1}{4}$
4. $\frac{1}{5}\times\frac{2}{3}$
5. $\frac{1}{2}$ of 3
6. $\frac{3}{4}$ of $1\frac{2}{5}$
7. $1\frac{1}{2}\div 5$
8. $2\frac{3}{8}-1\frac{1}{4}$
9. $1\frac{5}{12}\div 1\frac{7}{8}$
10. $\frac{2}{3}\times\frac{1}{5}$
11. $3\frac{1}{2}\times 1\frac{3}{5}$
12. $\frac{3}{4}+\frac{1}{2}+\frac{5}{6}$
13. $12\frac{1}{3}-8\frac{1}{4}$
14. $1\frac{1}{6}\times 2\frac{2}{3}$
15. $5\frac{2}{4}$
    $\frac{1}{2}$
    $\frac{3}{6}$
16. $4\frac{2}{5}-1\frac{1}{2}$
17. $3\frac{1}{3}\times 2$
18. $4-2\frac{1}{2}$

175. Practice on Applications of Decimals. (WRITTEN WORK.) In order to remember how to use decimals it is a wise plan to review them frequently. There are decimals in the Self-Testing Drills, but here is a little extra practice to make sure that you have remembered about decimals.

If it takes you more than 15 min. to work this exercise correctly, find out what kind of examples bother you and then use the Self-Help Index on page 457.

1.	2.	3.	4.
20.9×2.85	$1.25\overline{)68.75}$	Find 73.6% of \$581.	$.89\overline{)51.62}$

5. Subtract each of the numbers below from 100:

(*a*) .93 (*b*) .27 (*c*) .54 (*d*) .38

6.

Checks for April:

\$50.00 rent
31.80 food
6.90 insurance
4.91 milk
27.25 clothes
25.00 cash
4.58 gas

Total

7.

A man spends:

17.5% of salary for rent
1.9% of salary for vacation
10. % of salary for clothing
36.6% of salary for food
5.8% of salary for miscellaneous expenses

Remainder is saved.
What % does he save?

8. Brown earned in May \$369.50. He spent \$260.75 nd saved the rest. How much did he save?

9.

.63 + 4.25 + 7.48 + 6.47 + 8.46 = ?

10.

36 is what % of 90?

11.

Find the area of a tract of land which is 3.76 mi. long and 10.4 mi. wide.

12.

$.16\overline{)\$426}$

Check the answers of the examples below and **correct any mistakes** you find.

13. $5.9 + 7.3 + 5.9 + 4.3 + 6.4 = 298$

14. $1.11 - 9.6 = .15$

15. $10.2 - 6.4 = 4.8$

16. $129 - 35 = 94$

17. $1.111 - .124 = .0987$

18. $.67\overline{)5.0518}$ (754)

19.

$$\begin{array}{r} 83.6 \\ \times 2.35 \\ \hline 196.460 \end{array}$$

20. $.479\overline{)2.8837}$ (6.3)

REVIEW OF TWO USES OF PERCENTAGE

176. Mixed Fundamentals of Percentage. (WRITTEN WORK.) In working the exercise below use the samples whenever you are not sure of the correct method.

Many percentage examples can be worked like one of these two samples.

SAMPLE (*A*)	SAMPLE (*B*)
FIRST USE Find 27% of 356. "27% of" means $\times .27$ 356 $\times .27$ 2492 712 96.12 *Ans.*	SECOND USE 96.12 is what per cent of 356? "What per cent of" means "what fraction of." $\frac{96.12}{356}$ or $356\overline{)96.12}$ $.27 = 27\%$ $356\overline{)96.12}$ 71 2 24 92 24 92

EXERCISE

1. Find 6% of \$1900.
2. 17 is what % of 68?
3. 125 is what per cent of 200?
4. 392 is what % of 3920?
5. $.04 \times N =$ what % of N?
6. Find 24% of 856.
7. If $y = 76$, 200% of $y = ?$
8. 83.6 is what % of 418?
9. \$54.45 is what % of \$1089?
10. If $x = 362$, 54% of $x = ?$

11. Find $17\frac{1}{2}\%$ (.175) of 428.
12. 36 is what % of 288?
13. Find 135% of 673.
14. 96.12 is what % of 356?
15. \$450 = what per cent of \$5000?
16. 216 is what % of 576?
17. Find $12\frac{1}{2}\%$ of 192.
18. Find 27% of 356.
19. 900 is what % of 300?
20. $\frac{4}{5}$ of N = what % of N?
21. 100% = 839; 2% = ?
22. Find $87\frac{1}{2}\%$ of 5938.
23. What % of 105 is 84?
24. Find $\frac{1}{2}\%$ of 943.

PROBLEM SCALE NO. XII

177. Percentage in Life. (WRITTEN WORK.) You can earn 90 points by working every problem correctly. Find your score and rate yourself.

1. In a recent election 1900 votes were cast. Mr. Jessup received 75% of the votes. Mr. Smith received 20% of the votes. The rest were scattered.

(*a*) How many votes did Mr. Jessup get? (5)
(*b*) How many votes went to Mr. Smith? (5)
(*c*) How many were scattered? (6)

2. Mrs. Wagner baked 148 cakes last year. She frosted 75% of them. How many cakes did Mrs. Wagner frost? (6)

3. Mrs. Kent plans on saving 15% by buying in large quantities. Her food bill when she buys in small quantities is \$48.50 a month. How much does she plan to save by buying in large quantities? (6)

4. Mr. McHenry bought some land for $1000. He sold it for 130% of what he paid for it. For how much did Mr. McHenry sell it? (6)

5. Mr. White gets 40% of the price of certain goods he sells as his gross profit for selling them. Last week he sold $450.50 worth of goods. What was the gross profit? (7)

("Gross profit" means the profit before the cost of doing business is taken out.)

6. Mary weighs 57 lb. The normal weight for her age is 60 lb. Her weight is what per cent of normal? (7)

7. Because Helen's uncle owns the store, she can buy books at 85% of the regular cost. The regular price for a book entitled *A Girl's Trip Around the World* is $2.00. What must Helen pay for it? (7)

8. The total amount spent one year by a village for all purposes was $11,650. 42% of this was spent for schools. How much did the village spend for schools? (8)

9. James has built up a "magazine subscription" trade to 68 subscriptions a year. 75% of these were repeat orders. What per cent were new orders? (8)

10. While Alice was in the sixth grade, she attempted 216 problems and did 152 of them correctly on the first trial. What % did she do correctly on the first trial? (9)

11. Ralph's pet cow, Lazy Lou, yields 4.3% butter fat from her milk. Last August Lazy Lou gave 1252 pounds of milk. How much fat did she give in August? (10)

STANDARDS

Superior work:	28–90 points	Slow work:	5 points
Good work:	15–27 points	Inferior work:	0 points
Average work:	6–14 points		

SELF-TESTING DRILL NO. 20

178. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Why do careful workers often check examples?**

1.	2.	3.	4.	5.
$5 \times \frac{1}{6} = ?$	59235	$57\overline{)3935}$	10530	898
	-57871		-3565	986
				717
6.				767
$\frac{8}{21}$	7.	8.		768
$-\frac{5}{21}$	$\frac{1}{7}$ of $8 = ?$	$\frac{7}{50} + \frac{11}{50} + \frac{31}{50} + \frac{3}{50} = ?$		987

9.	10.	11.	12.	13.
$10 \div \frac{1}{3} = ?$	738	$3\frac{1}{2}$	789	$125 \div 45 = ?$
	$\times 165$	$-\frac{1}{7}$	$\times .961$	

14.	15.	16.
$83\overline{)41.09}$	$26\overline{)2178}$	$515 + 54.85 + 86.7 = ?$

17. What is the amount of a 2% discount for cash on a bill of $8.50?

18. Divide 93 by 18 and carry the quotient to two decimal places.

19. If there are 40 pupils enrolled in a class and 35 are present, what per cent are absent?

20. In June, 12 days were rainy. What per cent of the month was rainy?

STANDARDS

Number correct..	0–3	4–5	6	7–8	9	10	11	12	13–14	15–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Are you remembering to use the Self-Help Index and to practice on drills on which you made a low rating? Do you correct errors at once?

SUMMARY

179. General Review. (Class Work.) Pupils who can answer the questions below correctly may feel that they are getting along well in their arithmetic work.

1. In adding decimals, what is one thing to remember about the decimal points?

2. In the example at the right, what should be done before the divisor is inverted?

$$2\tfrac{1}{2} \div 5\tfrac{3}{4}$$

3. To what multiplication example would the example in Question 2 be changed?

4. What common denominator would you use in adding $\frac{}{4}$, $\frac{}{6}$, and $\frac{}{8}$?

5. What fractions would the following become if you changed them to fractions having the denominator you would use in Question 4 above? $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$.

6. Before multiplying dimensions to get area or volume, what one thing must be remembered about the units of the dimensions?

7. $9+0=?$ $9-0=?$ $9\times0=?$

8. What do the following words and phrases mean? "Indorse," "overdraw an account," "stub," "monthly statement," "balance at the bank."

9. How can you tell how many decimal places to point off in a product?

10. Where do the decimal points belong in the following quotients?

(*a*) $2.71\overline{)97.831}$ = 361

(*b*) $50.3\overline{)2.1126}$ = 42

(*c*) $.53\overline{)1113}$ = 21

11. What per cents equal these decimals?

(*a*) .25 (*b*) .04 (*c*) 1.5 (*d*) .065 (*e*) .0325

12. What decimals equal these per cents?

(*a*) 15% (*b*) $3\frac{1}{2}\%$ (*c*) $\frac{1}{4}$ of 1% or $\frac{1}{4}\%$ (*d*) 215% (*e*) $2\frac{1}{4}\%$

13. What questions in percentage would be answered by solving each of the examples below?

(*a*) $\$7025 \times .07$ (*b*) $\frac{361}{1721}$ or $1721\overline{)361.}$ (*c*) $\$2150 \times 1.10$

(*d*) $\frac{528}{1831}$ or $1831\overline{)528.}$ (*e*) $\frac{726}{338}$ or $338\overline{)726.}$

14. Which example below is the correct one to use in solving this problem? Find $3\frac{1}{2}\%$ of 450.

(*a*) 450×35 (*b*) $450 \times .035$ (*c*) 450×350 (*d*) $\frac{35}{450}$ or $450\overline{)35.}$

15. Which example below would you use to solve this example? What per cent of 913 is 67?

(*a*) $\frac{67}{913}$ or $913\overline{)67.}$ (*b*) $\frac{913}{67}$ or $67\overline{)913.}$ (*c*) $913 \times .67$

16. In the example at the right, what number is the minuend? The subtrahend? The difference?

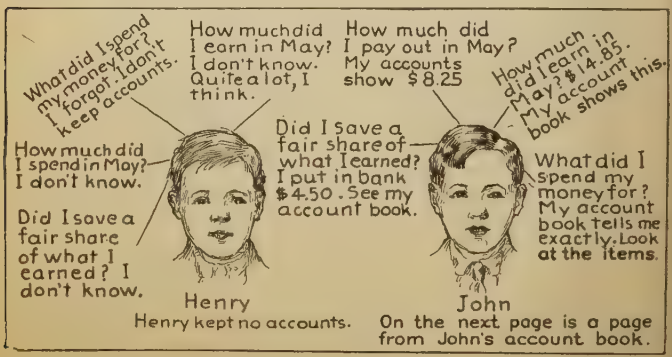
$$\begin{array}{r} 794 \\ -616 \\ \hline 178 \end{array}$$

CHAPTER IX

ACCOUNTS. ESSENTIALS OF INTEREST

THE PERSONAL ACCOUNT

180. Keeping Account of Your Money. (STUDY PERIOD.) Study the picture below and the one on the next page; then answer the questions that are asked.


What did I spend my money for? I forgot. I don't keep accounts.
How much did I earn in May? I don't know. Quite a lot, I think.
How much did I pay out in May? My accounts show $8.25
How much did I earn in May? $14.85. My account book shows this.
How much did I spend in May? I don't know.
Did I save a fair share of what I earned? I don't know.
Did I save a fair share of what I earned? I put in bank $4.50. See my account book.
What did I spend my money for? My account book tells me exactly. Look at the items.
Henry
Henry kept no accounts.
John
On the next page is a page from John's account book.

1. Can John tell how much money he received in May?

2. What balance did John carry over to start the June account?

3. Can John tell when he spent money?

4. Can he tell for what he spent money?

1927 - Money Received				1927 - Money Paid Out			
May 2	Saved from April	1	85	May 5	Baseball glove	1	75
" 16	Cutting lawn	3	15	" 17	Savings Bank	4	50
" 17	Paper route	8	85	" 26	Baseball	1	00
" 31	Allowance	1	00	" 30	Mother's Birthday	1	00
	Total				Total		
	Balance (to start June account)						

A PAGE FROM JOHN'S ACCOUNT BOOK

5. Why would money put in a savings bank show as "Paid Out" in this account?

6. Can John tell the total amount he paid out?

7. Where will the monthly balance (amount left over) be written in John's account for the next month (June)?

8. Does the account show for what the money was received?

9. Does the account show when each amount of money was received?

10. What is the difference of the two totals called?

11. Check over John's account to see if everything is correct.

12. What things about your money would an account book of your own show you?

181. Practice in Keeping Accounts. (WRITTEN WORK.) Make your papers look like the pages of an account book, and copy the two accounts shown here.

1. William White's account for October, 1927:

Carried over from September→$4.75.
Earned from errands→25¢, Oct. 4.
Received monthly allowance→$1.00, Oct. 10.
Sale of old baseball glove→60¢, Oct. 14.
Saturday work at Sloan's Grocery→$2.00, Oct. 15.
Painting screens→75¢, Oct. 20.
Spent for hat→$1.25, Oct. 6.
Bought instrument for radio→75¢, Oct. 10.
Repair of punctured bicycle tire→50¢, Oct. 15.
Spent for Father's birthday→50¢, Oct. 25.
Put in savings bank→$4.00, Oct. 28.
Bought stamps, 10¢, Oct. 29.

Show the balance of the account on October 31.

2. Mildred Smith's account for December, 1927:

Balance brought forward from November→$8.90.
Received for caring for baby→$2.00, Dec. 1.
Errands→75¢, Dec. 2.
Allowance received→$2.00, Dec. 2.
Paid for Father's Christmas present→$1.00, Dec. 20.
Paid for Mother's Christmas present→$1.00, Dec. 20.
Present for Mary→25¢, Dec. 21.
Paid for material for John's present→55¢, Dec. 21.
Bought Tuberculosis Seals→$1.00, Dec. 22.
Bought stationery→75¢, Dec. 23.
Received present from Uncle Joe→$5.00, Dec. 26.
Stamps (bought)→35¢, Dec. 27.
Paid insurance on package→8¢, Dec. 27.
Deposited in Bank→$5.00, Dec. 31.

Show the balance to be used for the January account.

3. Compute and write the balance for this account:

1928.	Receipts (Money taken in)		1928.	Expenditures (Money paid out)	
Jan. 2	Bal. from Dec.	$ 74 82	Jan. 2	Carfare and meals on trip	$ 11 50
" 2	Sale of auto	425 00	" 2	Room rent	20 00
" 4	Interest	18 50	" 5	Clothes	17 65
" 7	Loan paid back	10 00	" 6	Movies	80
" 16	Salary	125 00	" 10	Savings account	425 00
" 30	Salary	125 00	" 17	Church	5 00
			" 18	Two books	2 50
			" 20	Hospital fund	5 00
			" 31	Food	32 00
			" 31	Dentist	17 50

4. Check the account below. If there are any mistakes, give the correct figures.

A man has on hand at the beginning of February, 1928, $250. He receives during the month $200 for salary and $60 from the rent of two houses. During the month he pays $72.50 on an insurance policy, $110 for household expenses, $25.50 for incidentals, and $5 for books and stationery. His account book shows the following entries:

1928.	Receipts (Money taken in)		1928.	Expenditures (Money paid out)	
Feb. 1	Bal. from Jan.	$250 00	Feb. 7	Insurance	$ 72 50
" 15	Salary	200 00	" 17	Books and Sta.	5 00
" 27	Rent	60 00	" 29	Household Ex.	110 00
			" 29	Incidental "	25 50
		$510 00			$213 00
Feb. 29	Bal. (Mar. 1)	$307 00			

PROBLEM SOLVING

182. A Hint on Problem Solving. (Class Discussion.) There are several reasons why problems are sometimes difficult. One of these is that we really do not understand the meaning of the problem. This failure to understand the problem may be caused by careless reading or by the fact that we do not understand the words used in the problem.

Too rapid reading or failure to read the problem several times in order to know exactly what is asked for is one cause of failure to understand a problem. You have been cautioned about too hasty reading of problems many times.

Another kind of trouble is not knowing what the problem is talking about. Below are two problems which call for exactly the same computation, but most pupils find Problem II the easier, for they know what Problem II is about.

Problem I. Mr. Standish raises kruckaws for the tricut. He has four separate guindishers in which he keeps his kruckaws. In one there are 45, in another 63, in a third 89, and in the fourth guindisher there are 37 kruckaws. How many has he in all?

Problem II. Mr. Standish raises chickens for the market. He has four separate pens in which he keeps his chickens. In one there are 45, in another 63, in a third 89, and in the fourth pen there are 37 chickens. How many chickens has he in all?

We not only find problems more interesting when we know what they are about, but we seem to understand them better. It seems easier for most of us to think about chickens than about kruckaws. It is easier to work with a problem about chicken pens than with one about guindishers.

When you read a problem which refers to things you do not know much about, it is a good plan to spend some time in investigating those things. Such investigations often

make good practice in the use of the dictionary and other reference books.

Here is a list of things used in problems from pages 197 to 259. How many of them do you know something about, and how many of them may be "kruckaws" and "guindishers," for all you know about them?

Light and Power Company
Wholesale druggist
Quantity discount
A retailer
Porch shades
Luncheon mats
National League
Graphs
Days in a school year
An agent who sells on commission
Manual training
To lay a carpet
Hardwood floor
Linoleum
Wall paper
A floor plan
Business district
Paving
Bill of goods
An election
Normal weight
Magazine subscription
Butter fat
A balance of an account
A bicycle-tire puncture
A payroll
Wages
A contractor
Plowing
Seed corn
Husking corn
Shocks of corn
Fodder
Quality of land
Harrowing

When you find a word or phrase in a problem which you do not understand, it will make the problem more interesting and the work will seem clearer and easier if you take the trouble to investigate the meaning of the word or phrase. ☞Every sixth-grade pupil should know how to use a dictionary and other reference books so well that he can do this investigating on his own responsibility.

PROBLEM SCALE NO. XIII

183. Problems about Labor and Wages. (WRITTEN WORK.) You can earn 170 points by working every problem correctly. Find your score and rate yourself as directed for Scales I to III.

Notice that some of the problems have more than one part. Number and letter your answers plainly.

Accounts are kept for many different purposes. The following account was kept by a contractor in order that he might make up the payroll for the men whom he hired. Study it and do the work asked for.

ACCOUNT OF TIME CARPENTERS WORKED (WEEK OF JAN. 14, 1928)

S. B. SULZER, CONTRACTOR

	HOURS WORKED PER DAY						WAGES
	Mon.	*Tues.*	*Wed.*	*Thurs.*	*Fri.*	*Sat.*	
Adams	$7\frac{1}{2}$	8	8	$8\frac{1}{2}$	9	4	
Benson	8	$8\frac{1}{2}$	$8\frac{1}{4}$	$8\frac{3}{4}$	$8\frac{1}{2}$	$3\frac{1}{2}$	
Boyd	0	4	6	6	$6\frac{1}{2}$	4	
Claussen	8	$8\frac{1}{2}$	$8\frac{1}{2}$	$8\frac{1}{2}$	9	3	
Denning	$8\frac{1}{2}$	9	$8\frac{1}{2}$	$8\frac{1}{4}$	9	4	
Doan	8	8	8	8	8	$3\frac{1}{2}$	

1. Find the number of hours each man worked during the entire week.

(*a*) Adams (3) (*c*) Benson (4) (*e*) Boyd (4)
(*b*) Claussen (3) (*d*) Denning (4) (*f*) Doan (4)

2. (*a*) Which man worked the smallest number of hours during the week? (4)

(*b*) Which man worked the greatest number? (4)

3. At 90¢ an hour, find the Thursday's wage for each man.

(*a*) Adams (5) (*c*) Benson (5) (*e*) Boyd (5)
(*b*) Claussen (5) (*d*) Denning (5) (*f*) Doan (5)

4. Find the total number of hours the men worked during the week. (5)

5. Find the amount due each man at the close of the week's work.

(*a*) Adams (6) (*c*) Benson (6) (*e*) Boyd (6)
(*b*) Claussen (6) (*d*) Denning (6) (*f*) Doan (6)

6. Counting 8 hr. to the day, with 4 hr. on Saturday, what is the full amount any one man would earn in a week at 90¢ an hour? (6)

7. How many hours did Boyd lose for the week? (8 hr. day with 4 hr. on Saturday is standard work.) (6)

8. How much less did Boyd earn for the week than if he had worked 8 hr. a day with 4 hr. on Saturday? (7)

9. Counting 8 hr. to the day, with 4 hr. on Saturday, how much over-time did each man work?

(*a*) Adams (7) (*c*) Benson (7) (*e*) Boyd (7)
(*b*) Claussen (7) (*d*) Denning (7) (*f*) Doan (7)

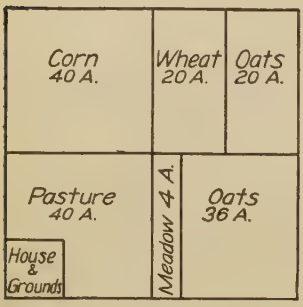
10. Find the total amount due the six men at the close of the week. (This would be the contractor's payroll.) (8)

STANDARDS

Superior work:	115–170 points	Slow work:	23–43 points
Good work:	78–114 points	Inferior work:	0–22 points
Average work:	44– 77 points		

DAY-BOOKS

184. An Account of a Modern Farmer. (Written Work.) The following forms show how Mr. Tom Jones, a modern farmer, keeps a systematic account of receipts and expenditures for one of the fields shown in the figure. The items for which money was paid out or received were put down with the dates of payment or of receipt in a **day-book** as shown below. A day-book is a daily record of income and expenditures.



Day-Book for Forty-Acre Cornfield

1. Paid for 2 da. work burning stalks @ $2.00, Mar. 18.
2. Paid for 10 da. plowing @ $5.00, Apr. 30.
3. Paid for 2 da. harrowing @ $5.00, May 1.
4. Paid for $2\frac{1}{2}$ da. planting corn @ $6.00, May 18.
5. Paid for 6 bu. seed corn @ $3.00, May 18.
6. Paid for $1\frac{1}{2}$ da. harrowing corn @ $5.00, May 29.
7. Paid for 6 da. cultivating @ $5.00, June 15.
8. Paid for 5 da. cultivating @ $5.00, June 30.
9. Paid for 4 da. cultivating @ $5.00, July 15.
10. Paid for cutting 240 shocks corn @ $12\frac{1}{2}$¢, Sept. 15.
11. Paid for husking 1392 bu. corn @ 9¢, Nov. 20.
12. Paid for shelling 1336 bu. corn @ $2\frac{1}{4}$¢, Jan. 10.
13. Sold 1336 bu. corn @ 80¢, Jan. 10.
14. Rec'd for 3 mo. pasture rent @ $12.00, Feb. 28.
15. Sold 240 shocks of fodder @ 20¢, Feb. 28.

These items in the day-book are here arranged in the form of a receipt and expenditure account.

Fill out on a separate sheet an account-form like the one below, and find the totals and the net profit for this forty-acre cornfield.

IN ACCOUNT WITH FORTY-ACRE CORNFIELD

RECEIPTS (Money received)			EXPENDITURES (Money spent)	
Jan. 10	1336 bu. corn @ 80¢	Mar. 18	Burning stalks, 2 da., @ $2.00	
Feb. 28	3 mo. pasture rent @ $12.00	Apr. 30	Plowing, 10 da., @ $5.00	
" 28	240 shocks fodder @ 20¢	May 1	Harrowing, 2 da., @ $5.00	
	Total	" 18	Planting corn, 2½ da., @ $6.00	
	Expenditures to be deducted	" 18	Seed corn, 6 bu., @ $3.00	
	Net profit from 40 acres	" 29	Harrowing corn, 1½ da., @ $5.00	
	Net profit per A.	June 15	Cultivating, 6 da., @ $5.00	
	Account closed March 1, 1927.	" 30	Cultivating, 5 da., @ $5.00	
		July 15	Cultivating, 4 da., @ $5.00	
		Sept. 15	Cutting 240 shocks corn @ 12½¢	
		Nov. 20	Husking 1392 bu. corn @ 9¢	
		Jan. 10	Shelling 1336 bu. corn @ 2½¢	
			Total	

185. Rapid Drill. (ORAL WORK.) Can you work each example and answer each question? You should be able to do so. Give only the first figure in quotients.

	(A)	(B)	(C)	(D)	(E)
1.	$\frac{1}{5}+\frac{3}{10}$	8 is what % of 40?	$27\overline{)216}$	44 $\times .9$	36 -29
2.	Rule for area of rectangles.	100 -27	Find 400% of 60.	28 $\times 6$	$3\frac{1}{2}\%$ = what decimal?
3.	1 bu. = ...pk.	438 -208	$\frac{5}{8}+\frac{3}{8}$	*Reduce:* $\frac{21}{6}$	$29+4+6$
4.	$\frac{2}{3}\times\frac{9}{10}$	55 -26	42 -18	$18+8+7$	$\frac{7}{8}-\frac{1}{4}$
5.	36 $\times 8$	$3\frac{7}{8}=\frac{}{8}$	200 -8	5.7 $\times 7$	$36\overline{)338}$
6.	43 -17	$\frac{4}{5}\div 3$	44 -26	*What is wrong?* $4''\times 2'=$ 8 sq. in.	38 -19
7.	$2'6''$ $\times 4$	$18\overline{)131}$	$28+9+4$	If 10% of $N=6$, $N=$.....	$17+8+6$
8.	$9+9+6$	If $\frac{1}{5}$ of $N=6$, $N=$.....	$49\overline{)160}$	2 yd. -1 yd. 1 ft.	107 -98

SELF-TESTING DRILL NO. 21

186. Directions. Time allowed: 20 minutes. Follow the ections given for Drills 1 to 5. Rate yourself.

1.
7153
6864

2. $3\frac{1}{17}+\frac{7}{17}+6\frac{4}{17}=?$

3.
919
861
259

4.
478
×708

5. $\frac{7}{16}\div\frac{3}{8}=?$

7.
419
×258

8.
60.90
−4.673

9.
58
14

10.
.405
×.751

11. $7\frac{4}{9}-3\frac{1}{6}=?$

12. $38\overline{)8.9528}$

13. *Reduce:* 26 lb. 6 oz. to oz.

14. $\frac{1}{6}$ of $1\frac{1}{6}$ of $\frac{1}{2}=?$

15. $28\overline{)8925}$

16. 3 pk. 6 qt. $\div 2=?$

17. 94 is what per cent of 470?

18. *Express as fractions:* (*a*) $33\frac{1}{3}\%$ (*b*) $87\frac{1}{2}\%$ (*c*) $83\frac{1}{3}\%$ 10% (*e*) 1%

19. What is the commission at 15% on a sale of goods ounting to $15,950?

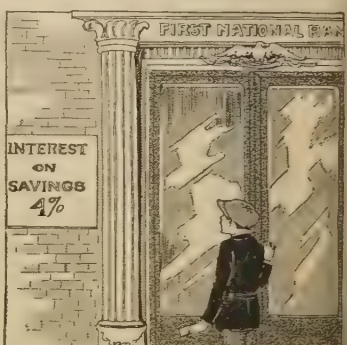
20. Twice a number is what per cent of it?

STANDARDS

umber correct...	0–3	4–5	6	7	8	9	10	11	12–13	14–19	20
ting.....	0	1	2	3	4	5	6	7	8	9	10

INTEREST AS PERCENTAGE

187. Interest: A Business Application of Percentage. (STUDY PERIOD.) I. What do you suppose the "Interest on Savings 4%" in the picture means?



When you deposit money in an account in a savings bank, two things are done:

(1) Your money is kept safely.* It will not be lost.

(2) The bank will pay you money for the use of your money. This is called **interest** on the money. Thus money put into the savings bank will **earn** interest.

Interest is reckoned by the year (**per annum**), and usually is paid by banks by the half-year. Thus, if a bank pays 4% interest **per annum** on your money, it may pay you 2% every 6 months.

$100 is put in a bank which pays 4% interest each year. If the $100 is deposited on Jan. 2, 1928, how much interest will it earn by Jan. 2, 1929? (What is 4% of $100?)

Suppose you loan a friend $500 for a year. He agrees to pay you 6% interest. You give him the money on Jan. 16, 1928. On Jan. 16, 1929 he should pay you the $500 and how much more as interest? (What is 6% of $500?)

*Most savings banks are safe. They are managed by skillful and honest men. On rare occasions, however, a savings bank "fails," i.e., it is unable to return all the money deposited. One should investigate a bank before depositing any money in it.

Below is a picture of a page from a savings bank-book. oes the account show that any interest was paid to Mr. arrison by the bank? How much money was earned by e deposits?

o. 1073 GEORGE HARRISON

IN ACCOUNT WITH

The Farmers Loan and Trust Company

MANHATTAN, KANSAS

DATE 192 7	WITHDRAWN	DEPOSITED	BALANCE
uly 1		10	185
" 2		14 75	199 75
" 5		30 25	230
uly 1	Int.	2 20	232 20

II. Study Problems *A*, *B*, and *C* on this page and the xt. They are about interest. You should make sure at you understand everything that is stated.

PROBLEM (*A*). What would be the interest on $400.00 for e year at the rate of 3%?

The Work	
$400	Interest is reckoned by the year.
×.03	3% interest for one year is 3% of $400.
$12.00, interest	Does $12.00, interest, answer the question asked in the problem?

What would be the interest on $400 at 3% for 2 yrs.?
What would be the interest on $400 at 3% for 6 months?

PROBLEM (*B*). How much interest would $650 earn i two years at 4% interest?

The Work	*Questions*
$650	1. Does .04 = 4%?
×.04	2. Why is $26.00 multiplied by 2
$26.00, interest for one year.	3. Does $52.00, interest, answe
×2	the question the proble
$52.00, interest for two years.	asks?

PROBLEM (*C*). I borrowed $250 at the rate of 7% interest At the end of six months I was to pay back the loan wit interest. How much money should I have paid back?

The Work	
$250	$250 + $8.75 = $258.75, th
×.07	amount to be paid back
2)$17.50, interest for one year.	Is $258.75 a reasonabl
$8.75, interest for 6 mo. ($\frac{1}{2}$ year).	answer for Problem *C*?

What amount should be on each blank?

4% interest for 1 year on $100 is .....

4% interest for 2 years on $100 is .....

4% interest for 6 mo. on $100 is .....

4% interest for 3 mo. on $100 is .....

4% interest for 1 year 6 mo. on $100 is .....

Find the interest on each of the following:

1. $900 @ 7% for 1 yr.
2. $250 @ 5% for 6 mo.
3. $1600 @ 6% for 2 yr.
4. $500 @ 4% for 3 yr. 6 mo
5. $100 @ $6\frac{1}{2}$% for 1 yr.
6. $750 @ $4\frac{1}{2}$% for 1 yr. 6 mo

188. Further Study of Interest. (STUDY PERIOD.) ften money is loaned or borrowed for other lengths of me than years or half-years. is wise to know how to find terest for such times as 1 mo., mo., 7 mo., etc.

Here is a good way to figure the terest for 8 months. →

Find the interest on \$600 for 1 ar, 8 months at 7%.

\$600
×.07
\$42.00, interest for 1 yr.
21.00, interest for 6 mo. ($\frac{1}{2}$ yr.)
7.00, interest for 2 mo. ($\frac{1}{6}$ yr.)
\$70.00, interest for 1 yr., 8 mo.

To think of 8 mo. as $\frac{8}{12}$ or $\frac{2}{3}$ of year is correct. But it is easier think of 8 mo. as: →

6 mo. = $\frac{1}{2}$ of a year,
2 mo. = $\frac{1}{6}$ of a year.
8 mo.

To think of 9 mo. as $\frac{9}{12}$ or $\frac{3}{4}$ a year is correct. But it is sier to think of 9 mo. as: →

6 mo. = $\frac{1}{2}$ of a year,
3 mo. = $\frac{1}{4}$ of a year.
9 mo.

How would you think of 1 mo.? Of 2 mo.? Of 5 mo.? 3 mo.? Of 4 mo.? Of 10 mo.? Of 7 mo.?

11 months can be thought of as:

6 mo. = $\frac{1}{2}$ of a year.
4 mo. = $\frac{1}{3}$ of a year.
1 mo. = $\frac{1}{12}$ of a year.

How would you think of 3 yr. 6 mo.? Of 2 yr. 3 mo.? 4 yr. 8 months?

Think the following as years and fractions of years:

) 2 yr. 9 mo. (*c*) 5 yr. 6 mo. (*e*) 4 yr. 4 mo.
) 1 yr. 8 mo. (*d*) 3 yr. 1 mo. (*f*) 2 yr. 3 mo.

FACTS TO BE REMEMBERED ABOUT INTEREST

I. Interest is simply a business application of percentage

II. Interest is reckoned by the year, or fractions of a year

(Compare 4% for 1 year with 4% for 2 years, for 3 years, fo 4 years, and for one-half a year.)

III. To find interest for a number of months, divide th months in a way that will give fractions easy to work with

IV. Interest is money earned by money that is loaned

V. Interest is also money that is paid for the use o borrowed money.

VI. The usual interest rates are 3%, $3\frac{1}{2}$%, 4%, $4\frac{1}{2}$% 5%, $5\frac{1}{2}$%, 6%, $6\frac{1}{2}$%, 7%, and $7\frac{1}{2}$%.

VII. The usual times for loans are 30 da. (1 mo.), 2 mo. 3 mo., 6 mo., 1 year, and over. (The interest rate is alway stated in % **per annum.** "Per annum" means "by th year" or "yearly.")

189. Practice in Finding Interest. (WRITTEN WORK. The examples on this page and the next will be your firs work in interest. Try to get them correct the first time When you are grown men and women, it will be importan for you to be skillful in computing interest.

EXERCISE

How much interest must be paid for the use of each o the following? CAUTION: Interest is reckoned per annum.

1. **$600 for 1 yr. at 7 per cent.**
2. **$350 for 2 yr. at 8 per cent.**
3. **$575 for 2 yr. 6 mo. at 6 per cent.** (6 mo. $= \frac{1}{2}$ yr.)

4. \$860 for 3 yr. 9 mo. at 5%. (9 mo. $= \frac{1}{2}$ yr. $+ \frac{1}{4}$ yr.)

5. \$300 for 2 yr. 4 mo. at $4\frac{1}{2}$ per cent. ($4\frac{1}{2}\% = .045$)

6. \$160 for 3 yr. 7 mo. at $6\frac{1}{2}$ per cent.

7. \$850 for 3 yr. 2 mo. at $5\frac{1}{2}$ per cent.

Find the ***interest*** *each of the following would earn:*

8. \$700 for 1 yr. 7 mo. at 3 per cent.

9. \$400 for 2 yr. at 3 per cent.

10. \$210 for 2 yr. 5 mo. at 6 per cent.

11. \$150 for 1 yr. 11 mo. at 7 per cent.

12. \$280 for 1 yr. 6 mo. at $4\frac{1}{2}$ per cent.

13. \$360 for 1 yr. 4 mo. at 4 per cent.

14. \$260 for 2 yr. 3 mo. at $3\frac{1}{2}$ per cent.

15. \$300 for 2 yr. 7 mo. at 5 per cent.

16. \$500 for 3 yr. 6 mo. at 6 per cent.

17. Esther Brown deposited \$150 in a Savings Bank in Portland. This bank pays 4% on deposits. How much will the \$150 earn for Esther in six months?

18. How much will Esther's money earn for her in 2 years and 6 months?

19. Joseph Mahan borrowed \$300 to start a shoe store. He paid 7% interest. How much was the interest per year on the loan?

20. How much interest must Mr. Mahan pay in all if he keeps the \$300 for 2 years and 8 months?

21. What fractions of a year are each of the following: 9 mo., 8 mo., 3 mo., 2 mo., 6 mo., 4 mo.?

REVIEW OF PERCENTAGE

190. The First Two Uses of Percentage. (A READING LESSON.) Study the two uses of percentage listed below until everything is clear.

I. **To find any per cent of a number.** Think: **% o** means **times a decimal.** Thus:

4% of 76 means: $\begin{array}{r} 76 \\ \times .04 \end{array}$ $6\frac{1}{2}\%$ of 38 means: $\begin{array}{r} 38 \\ \times .065 \end{array}$

$7\frac{1}{2}\%$ of 62.50 means: $\begin{array}{r} 62.50 \\ \times .075 \end{array}$

Do you see that **finding interest for a year at a given rate (%) of interest** is really the first use of percentage?

Thus, 7% interest on an amount means, the amount $\times .07$ **per year.**

6% interest on \$900 for a year means: $\begin{array}{r} \$900 \\ \times .06 \end{array}$

$4\frac{1}{2}\%$ interest on \$875 for a year means: $\begin{array}{r} \$875 \\ \times .045 \end{array}$

3% interest per annum on \$460 for 6 mo. means: $\begin{array}{r} \$460 \\ \times .03 \\ 2\overline{)\quad} \end{array}$ interest 1 yr. / interest 6 mo.

II. **A certain number is what per cent of another?** Think: A certain number is **what fraction** of another number? Thus:

"8 is what % of 24?" means "8 is **what fraction of** 24?"

Think: $\frac{8}{24}$ or $24\overline{)8.00}$

19 is what % of 190? Think: 19 is what fraction of 190?

Think: $\frac{19}{190}$ or $190\overline{)19.00}$

Could examples and problems in interest be made which ould apply the second use of percentage? Does the llowing example apply the second use of percentage?

EXAMPLE: A savings account of $80 earned $3.20 interest for ie depositor in one year. What per cent of interest did the oney earn?

$3.20 is what % of $80? Think: $\frac{3.20}{80.00} = \frac{.04}{80\overline{)3.20}}$ or 4%

Can you make up another interest problem which is really ie second use of percentage?

Now work the exercise below. It gives practice on some ses of percentage.

EXERCISE

1. Find 4% of 1924.
2. 95 is what % of 950?
3. What is 55% of 737?
4. 200% of 64 = ?
5. 486 is what % of 729?
6. What % of 87 is 58?
7. Find $66\frac{2}{3}$% of 87.
8. Find $87\frac{1}{2}$% of 2496.
9. 44 is what % of 352?
10. $\frac{1}{3}$ of N = what % of N?
11. 26 is what % of 78?
12. 625 is what % of 750?
13. What is the amount of the interest on $1565 for 1 ear at 9%?
14. Loan, $500. Time, 1 year. Interest rate, $4\frac{1}{2}$%. ind the interest.
15. 40% of $2184 would be what amount?
16. Deposited $200 in bank. Interest at 4%. Money rned in 1 year.....
17. Loaned $750 @ 6% for 6 months. When paid will ceive a total of .....

18. My savings of \$28.00 earned me \$1.40 interest last year. What per cent interest did my savings earn?

19. I decided to save 15% of my earnings each year. I earn \$3500 a year. How much money should I save each month?

20. (*a*) 25% of N = what fraction of N?
(*b*) 75% of N = what fraction of N?
(*c*) $12\frac{1}{2}$% of N = what fraction of N?
(*d*) $62\frac{1}{2}$% of N = what fraction of N?
(*e*) 7% of N = what decimal of N?
(*f*) $4\frac{1}{2}$% of N = what decimal of N?

If any of these percentage problems bother you, review percentage. Use the Self-Help Index on page 457 to find the articles that you should study.

PROBLEM SCALE NO. XIV

191. Problems in Percentage. (Written Work.) You can earn 60 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. A school report said: "Of the 460 pupils in school, 65% keep written accounts of their own money." How many keep written accounts of their money? (6)

2. How much interest will Mr. Bailey have to pay on a loan of \$750 at 6%, for 2 years and 6 months? (6)

3. The records show that 357 workers in a certain factory save money every month. There are 420 workers. What per cent of the workers save money? (8)

4. What is the amount of the discount on the bill at the top of the next page? (6)

Jan. 11, 1927.

L. B. Hall bought of Smith Bros.:

20 pr. Boys' Boots @	$3.75	*List Price*	$ 75.00
15 pr. Slippers @	2.50		37.50
25 pr. Ladies' Shoes @	8.00		200.00
25 pr. Rubbers @	1.50		37.50
		Total	$350.00
		Dealer's Discount	25%

5. Eight pupils were perfect in a history test. 32 pupils ook the test. What per cent made perfect scores? (7)

6. The Amazon River is 3612 miles long. The Mississippi iver is 4200 miles long. The length of the Amazon is what r cent of the length of the Mississippi? (8)

7. On July 15, 1925, Henry Walker deposited $750 in e Farmers Savings Bank. On Jan. 15, 1926, Mr. Walker ok his money out. The money earned $4\frac{1}{2}\%$ while on eposit. How much interest did the bank pay to Mr. alker when he took his money out? (9)

8. Browne, Steele & Co. offer 2% discount if bills are id within 10 days from date of purchase. Mr. French id his bill when he bought his goods. How much did e save? His bill showed these items: (10)

5 yd. Silk @	$1.75	4 papers Pins @	.07
3 yd. Ribbon @	.75	2 papers Needles @	.08
7 yd. Velvet @	4.50		

STANDARDS

Superior work:	19–60 points	Slow work:	6 points
Good work:	13–18 points	Inferior work:	0 points
Average work:	7–12 points		

192. Rapid Drill. (ORAL WORK.) Careless pupils often make more mistakes in easy work than in hard work. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	180 −89	95)728	60%= what decimal?	147 −79	73)5.84	138 −39
2.	95)814	97 ×.9	124 −88	97)485	86)655	.9+2.6 +.7
3.	39)372	19+9+6	*What is wrong?* Volume = 24 ft.	8+19+4	29+9+2	7+29+8
4.	113 −59	64)437	130 −51	24+1+9	*What mistake?* $\frac{5}{6}+\frac{3}{4}=1\frac{9}{12}$	96)768
5.	96)897	176 −79	74)519	164 −95	143 −56	165 −68
6.	13.6 −.68	142 −89	How is subtraction checked?	132 −78	.68 ×7	97 ×7
7.	53 ×8	98)392	.16)1.28	"Rate of interest" means what?	32)246	52)345
8.	124 −27	$\frac{1}{4}\div 3\frac{2}{5}=\frac{1}{4}\times\cdots$	2′ 7″ ×3	Explain uses of accounts.	68)304	.0425= what %?

SELF-TESTING DRILL NO. 22

193. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

Watch out in Ex. 4 or you will add wrong numbers.

1. $4.684 - 3.469$

2. 129×37

3. $382 + 461 + 831 + 760$

4. $5855177799444 + 6756858952638$

5. $\frac{1}{2} \div \frac{8}{15} = ?$

6. $4.2505 - .7667$

7. $28\overline{)817}$

8. $\frac{1}{7} \times 12 = ?$

9. $\frac{3}{8} + \frac{1}{12} = ?$

10. $8\frac{1}{3} + 2\frac{1}{4} + 3\frac{5}{6} + 8\frac{1}{2} + 9\frac{7}{12}$

11. $4.410 \div 45 = ?$

12. $.650 \times 4.70$

13. $.852 \times 920$

14. *Subtract:* $\frac{1}{18}$ from $\frac{7}{9}$

15. *Carry to 2 decimal places:* $7\overline{)22}$

16. 3 bu. 1 pk. $\times 8$

17. If a man borrowed \$150 for a year and paid \$7.50 for the use of it for that time, what was the rate of interest?

18. In a certain problem the dividend was 8 and the divisor was $5\frac{1}{3}$. What was the quotient?

19. 92 is $12\frac{1}{2}\%$ or $\frac{1}{8}$ of what number?

20. Find 4.5% of \$1650.

STANDARDS

Number correct...	0–4	5	6–7	8	9	10	11	12	13	14–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

SUMMARY

194. General Review. (Class Work.) To be sure that you understand the new work you have studied in this chapter, answer the questions and do the work in this article.

1. Why is it a good plan to keep accounts of your own money?

2. What is meant by the word "balance" in personal accounts?

3. If a boy takes in \$14.76 during October and spends \$11.42, what is his "balance" for the month?

4. Why is interest paid on loans and savings?

5. What per cents of interest are commonly paid for money?

6. Does 4% interest mean 4% every day, every week, every month, or every year?

7. There are two reasons why it is a wise plan to put money in a good savings bank. One reason is that the money will be safely kept. What is the other reason?

8. What should be on each blank below?

(*a*) 4 mo. = ... year.

(*b*) 1 mo. = ... year.

(*c*) 6 mo. = ... year.

(*d*) 2 mo. = ... year.

(*e*) 3 mo. = ... year.

(*f*) 30 da. = ... year.

(*g*) 5 mo. $= \frac{1}{4}$ year plus ..... year, or $\frac{5}{12}$ of a year.
(*h*) 9 mo. $= \frac{1}{2}$ year plus ..... year, or $\frac{3}{4}$ of a year.
(*i*) 7 mo. = ... year plus ..... year, or $\frac{7}{12}$ of a year.
(*j*) 8 mo. = ... year plus ..... year, or ..... of a year.
(*k*) 10 mo. = ... year plus ..... year, or ..... of a year.
(*l*) 11 mo. = ... year plus ..... year, plus ..... year, or ..... of a year.

9. What is the interest for each example below?
(*a*) \$400 at 4% for 1 year.
(*b*) \$900 at 7% for 2 years.
(*c*) \$600 at 5% for 1 year 6 months.
(*d*) \$500 at 6% for 2 years 9 months.

10. $5+0=?$ $5-0=?$ $5\times 0=?$

11. Which of the two solutions below, when it is completed correctly, would answer the question: What % of 35 is 67?

Solution I	Solution II
$\frac{6\,7}{1\,3\,5}$ or $135\overline{)67.00}$	$\frac{1\,3\,5}{6\,7}$ or $67\overline{)135.}$

12. (*a*) $\frac{1}{4}+\frac{1}{2}=?$ (*b*) $\frac{1}{4}\times\frac{1}{2}=?$ (*c*) $\frac{1}{4}\div\frac{1}{2}=?$

13. Study each word or phrase below and be ready to give a two-minute talk about it.
(*a*) Dimensions.
(*b*) Savings banks.
(*c*) Graphs.
(*d*) Interest.
(*e*) Checks for accuracy.
(*f*) Discounts.
(*g*) Hints on problem solving.

14. What should be in each blank space in the chart below?

	Fraction Language	Decimal Language	Per Cent Language
(*a*)			125%
(*b*)	$\frac{1}{3}$		
(*c*)		.5	
(*d*)			7%
(*e*)		.02	
(*f*)	$\frac{7}{8}$		
(*g*)		$.66\frac{2}{3}$	

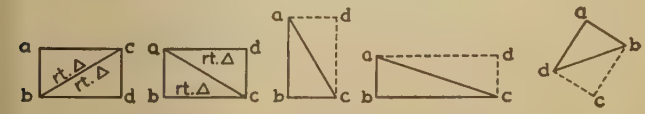
CHAPTER X

MENSURATION OF TRIANGLES AND PARALLELOGRAMS

TRIANGLES AND PARALLELOGRAMS

195. The Right Triangle. (Silent Reading.) Notice that you are to study a new topic. Be sure that you really understand this reading lesson.

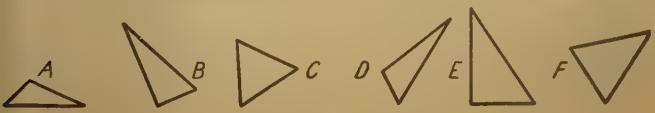
A right triangle is $\frac{1}{2}$ of a rectangle, the rectangle being divided as in the following figures:



("Rt. △" means "right triangle.")

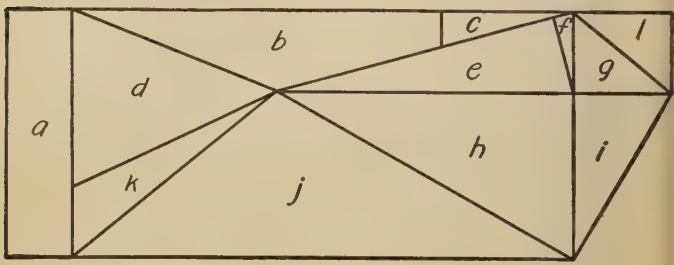
Sometimes the complete rectangle is not shown, but you can always think what the rectangle would be if it were drawn. Study the rectangles above. The dotted lines show the complete rectangle; the heavy lines show the right triangle.

A triangle is any closed figure which has just three straight sides. A right triangle is a **special** kind of triangle. A **right** triangle has 3 straight sides, and **one of its angles is a right angle.** Which of the triangles below are right triangles? (Use a square corner of a sheet of paper to test with.)



Find in the figure below at least 7 right triangles and 2 triangles which are not right triangles.

If you are not sure that an angle is right, or square, test it with the corner of a rectangular sheet of paper or card.



A right triangle can be thought of as one-half of a rectangle, thus: . Triangles which are not right triangles are one-half of figures like those below:

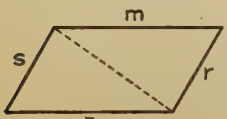


Fig. 1

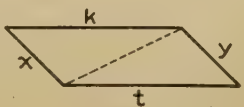


Fig. 2

Fig. 3

Figures 1, 2, 3, above are called **parallelograms.** Parallel lines are lines like x and y or k and t above.

Note: Parallel lines mean lines which keep always the same distance apart. Parallel lines never meet.

The lines x and y in Fig. 2 are **parallel lines.**

The lines m and n in Fig. 1 are **parallel lines.**

Line k is parallel to line t in Fig. 2.

What kind of lines are s and r in Fig. 1?
What kind of lines are 3 and 4 in Fig. 3?
What special kind of parallelogram is a rectangle?
What special kind of triangle is a right triangle?

FACTS TO BE REMEMBERED

I. A parallelogram is a figure bounded by two pairs of parallel lines. □ ▱

II. A rectangle is a **special kind** of parallelogram, all four angles being right angles. A □ not a ▱.

III. A right triangle is a special kind of triangle, **one of the angles being a right angle.** or , not

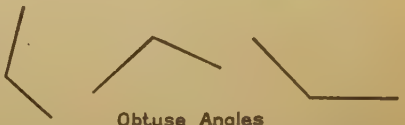
IV. Angles smaller than a right angle are called **acute angles, thus:**

Right Angle

Acute Angles.

V. Angles larger than a rt. ∠ are called **obtuse** angles.

Right Angle


Obtuse Angles

VI. Parallelograms and triangles exist not only in the arithmetic textbooks but in everyday life.

On your way home from school today try to see some figures such as fields, sides and roofs of houses, streets, etc., which are parallelograms, rectangles, right triangles. Look also for right, acute, and obtuse angles.

196. Figures from Everyday Life. (Class Work.) Many classes find it interesting to pin on the board the material asked for in Problem 2 below.

1. In the picture pick out the triangles, right triangles, parallelograms, rectangles, parallel lines and lines which are not parallel, acute angles, and obtuse angles.

Use the letters in the picture in describing a figure.

2. Bring to class tomorrow pictures, street maps, snapshots, etc., showing rectangles, parallelograms, right triangles, and triangles which are not right triangles. Be ready to answer questions about the material you bring.

AREA OF A RIGHT TRIANGLE

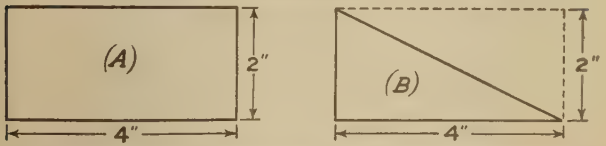
197. How to Find the Area of a Right Triangle. (Study Period.) Study the whole article by yourself. It is a long one. Notice what is oral work and what is to be written. Do the written work and be ready to answer the questions.

"Rt. △" is "short" for "right triangle."

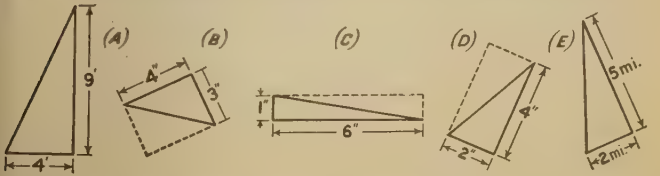
"Rt. ◬" is "short" for "right triangles."

ORAL EXERCISE

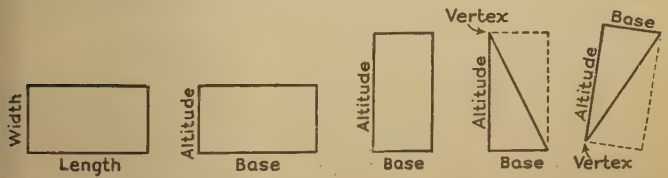
1. What is the area of rectangle A? Of rt. $\triangle$ B?



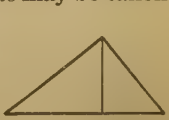
2. Find the areas represented by these right triangles:



3. Two new words: ***base*** and ***altitude.***



A base of a triangle is the side upon which the triangle is thought of as resting or standing. Any side **may** be taken as the base. If the **longest** side is chosen as base, then the altitude is **not** a side; it is the line drawn **perpendicular** (at right angles) to the base from the **vertex** of the right angle, as shown at the right.

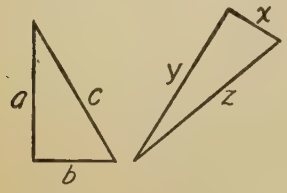


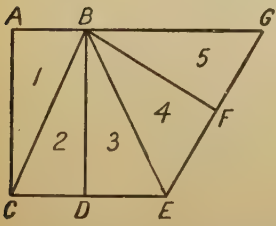
We usually think of the base and the altitude of a right triangle as the **two shorter sides of the right angle.** This fact will be clear if you look at the rt. $\triangle$ above.

Rule: **Area of a rectangle = base × altitude. Dimensions of base and altitude must be expressed in same units.**
Area of a rt. △ = $\frac{1}{2}$ of the base × the altitude.
Area is expressed in square units.

The two shorter sides are the base and altitude that are to be used in finding the area of a right triangle.

4. Name the **base** and **altitude** which you would use in finding the areas of the four rt. ◬ below.





5. Read the **base** and **altitude** which you would use in finding the area of each of the rt. ◬ shown in the figure at the left.

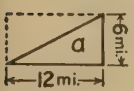
A line is read by naming its end letters; thus, a base of triangle 1 is AB, an altitude is AC.

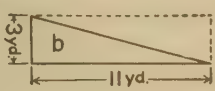
6. Describe some piece of land or a part of a building you have seen which is a right triangle. In your description make a good estimate of its dimensions; that is, state its approximate dimensions.

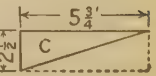
In getting the area of a rt. △, remember that it is $\frac{1}{2}$ of a rectangle, thus:

WRITTEN EXERCISE

1. Find the areas represented by these rt. △.







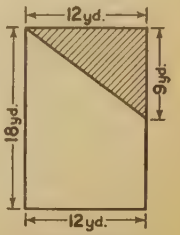
2. Right triangle: base = 5 in., altitude = 7 in., area = ?

3. Rt. △: $b = 19$ mi., $a = 24$ mi., area = ? (b = base; a = altitude.)

4. Find the area of Rectangle Q.

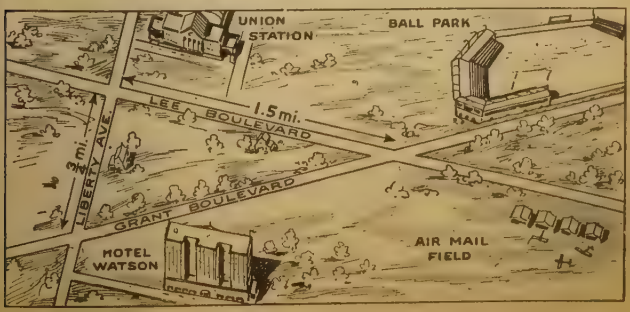
5. Find the area of the right triangle in Rectangle Q.

6. Find the area of the unshaded part of Rectangle Q.



RECTANGLE Q

7. The "heart of the city" is shown below. It is in the shape of a rt. △ formed by Liberty Ave. and Lee and Grant Boulevards. What is the area in **square miles** of the heart of the city?



ESTIMATING ANSWERS

198. Practice in Estimating. (Oral Work.) We know that in solving problems it is a good plan to estimate the answer before actually doing the computation. After the work is done, we can compare the final answer with our estimate. When there is a big difference between our estimate and our answer, it is wise to look over our work to make sure that we have not made some foolish mistake.

It is one thing to want to estimate and quite another thing to be able to estimate well enough really to help us in problem solving. Useful estimating uses two things. These are: (1) a thorough understanding of the problem in order to know upon what facts the answer depends, (2) ability to get an answer which is somewhere near the correct one without using paper and pencil.

Four answers are given to each of the next examples. One of them in each case is "somewhere near right." After studying them carefully, try to select the best estimate.

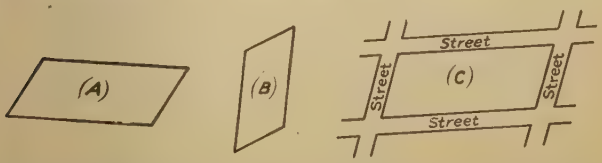
1. The product of 38.9×4009 is about:
   (*a*) 1600 (*b*) 16,000 (*c*) 160,000 (*d*) 1,600,000
2. The area of a rectangle $9' \times 3\frac{3}{4}'$ is about:
   (*a*) 3 sq. ft. (*c*) 35 sq. ft.
   (*b*) 27 sq. ft. (*d*) 37 sq. ft.
3. 261% of 8002 is about:
   (*a*) 20 (*b*) 200 (*c*) 2000 (*d*) 20,000
4. $34.5\overline{)35793}$ is about:
   (*a*) 100 (*b*) 1000 (*c*) 10,000 (*d*) 100,000
5. $\frac{3}{4} \times 4.04$ is about:
   (*a*) 3 (*b*) 30 (*c*) 120 (*d*) 400

6. 3.09×8.08 is about:
   (*a*) .0024 (*b*) .24 (*c*) 24 (*d*) 240
7. $.871\overline{)4.385}$ is about:
   (*a*) 500 (*b*) 50 (*c*) 5 (*d*) .5
8. 101% of 876 is about:
   (*a*) 87 (*b*) 90 (*c*) 900 (*d*) 9000

AREA OF A PARALLELOGRAM

199. The Parallelogram. (STUDY PERIOD.) We sometimes need to know how to find the area of a surface that is not a rectangle. This article teaches you how to find the area of surfaces which have the form of a **parallelogram.**

Parallelograms are figures like these:



A parallelogram is a figure bounded by two pairs of parallel lines. You may think of a parallelogram as a figure like a rectangle, except that the angles do not have to be right angles.

Figures like *A*, *B*, and *C* above, are called parallelograms because their opposite sides are parallel. Is the next sentence true? Lines or sides are parallel when they are everywhere the same distance apart; thus this pair of lines ═══ is parallel, but these lines ═══ are not parallel.

You will remember that the altitude of a figure means the height of that figure measured along a line drawn at **right angles** to the base.

NOTE: The word "altitude" when used in geography means the distance of a place above sea level measured by a line drawn at right angles to the earth's surface from that place to the level of the sea.

What do the next two sentences mean? A mountain has an altitude of 7864 feet. The airplane was flying at a very low altitude.

Study the figures below.

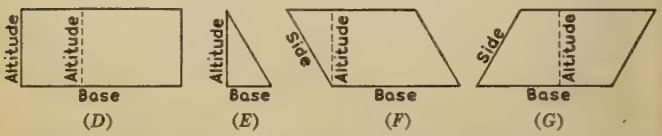


FIG. *D*. One side of a rectangle is also its altitude, because the side makes a right angle with the base.

Area of a rectangle = base × altitude.

The answer is expressed in square measure.

FIG. *E*. One of the sides which makes the right angle of a rt. △ is also its altitude.

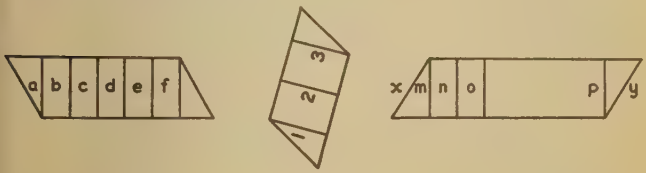
Area of a right triangle = $\frac{1}{2}$ of base × altitude.

The answer is expressed in square measure.

FIGURES *F* AND *G*. The side of a parallelogram is **not** its altitude, because the side does not tell its height, except when the parallelogram is a rectangle. The altitude of a parallelogram must make a right angle with its base.

An altitude may be drawn anywhere along the base, but it must be drawn at **right angles to the base.** Another way of saying "drawn at right angles to the base," is to say "drawn **perpendicular** to the base."

Study the figures below.



(*H*) (*I*) (*J*)

FIG. *H*. Lines *a*, *b*, *c*, *d*, *e*, *f* are all altitudes.

FIG. *I*. Turn your book around to see if lines 1, 2, and 3 are all altitudes.

FIG. *J*. What are lines *m*, *n*, *o*, *p*? Would the line *x* or *y* be an altitude? Why?

200. Finding the Area of the Parallelogram. (CLASS WORK.) Study Figures 1 to 4 on this page and the next to see if this statement is true: **The area of a parallelogram equals its base × its altitude.**

If Triangle *A* were cut off the left end of the parallelogram in Figure 1 and placed at the other end of the parallelogram, would Figure 1 look like Figure 2?

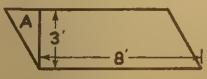


FIG. 1

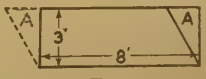


FIG. 2

Do you see that Figure 2 has a different shape from that of Figure 1, but that the figures have the same area?

Study Figs. 3, 4, and 5 in order to see why the area of a parallelogram can be thought of as equal to the area of a rectangle having the same base and altitude.

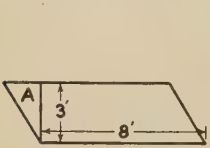


FIG. 3

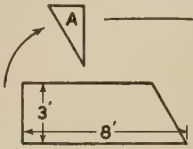


FIG. 4

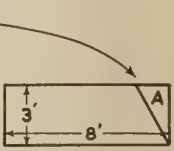


FIG. 5

Does the **area** of Fig. 3 (parallelogram) equal the **area** of Fig. 5 (rectangle)? Give your reasons for your answer.

What is the area of Fig. 5?

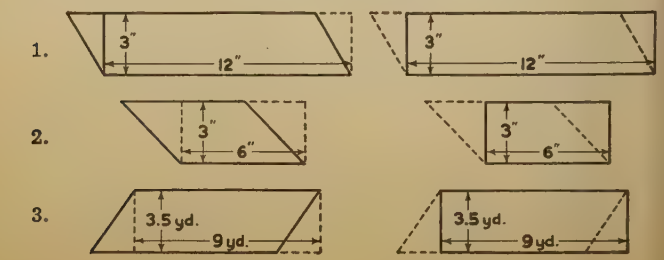
What must be the area of Fig. 3?

Is the area of Fig. 3 equal to the product of its base and its altitude? Why is a side of a parallelogram **not always** its altitude?

> ***Rule:*** **To find the area of a parallelogram, multiply base by altitude. Dimensions must be in the same unit.**

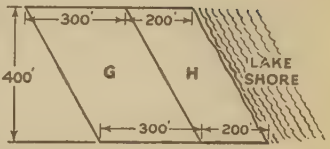
EXERCISE

What are the areas of the parallelograms below?



4. Find the area in square feet of Lot *G*. Of Lot *H*.

5. Lot *H* sells for \$2 a sq. ft. Find the selling price of the lot.

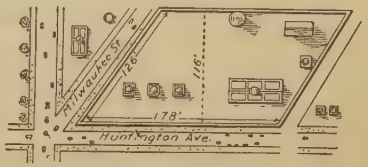


6. At \$1.50 a square foot what would Lot *G* sell for?

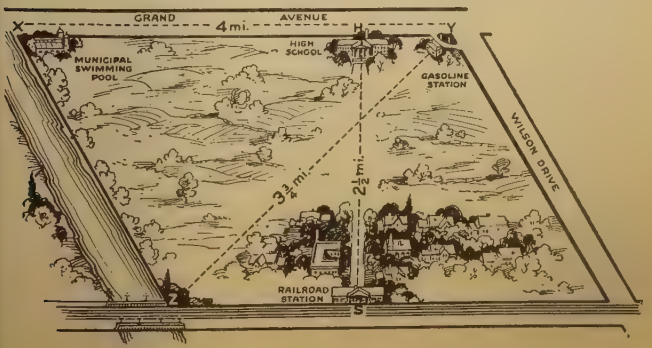
7. Which lot is worth more, and how much more?

8. The length of a plot of ground is 725 ft. Its shape is a parallelogram whose altitude is 326 ft. Find the area of the plot. (Will the answer be ft. or sq. ft.?)

9. Find the area of the roof shown in the drawing at the right. (You are looking at the roof from an airplane.)



10. About how many square miles are there in this city?



201. Rapid Drill. (ORAL WORK.) Ability to do examples like these quickly and accurately is a good test of mastery of arithmetic. Give only the first figure in quotients.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	Why correct mistakes?	45×8	$7+16+6$	86×9	$167 - 98$	79×9
2.	97×7	$167 - 79$	$.6+2.8+.9$	56×7	How is division checked?	$8+49+7$
3.	$9+49+5$	$18+8+3$	$98\overline{)564}$	$166 - 68$	$59+6+8$	$1.54 - .79$
4.	$96\overline{)637}$	Meaning of altitude.	98×8	$9+17+9$	$27+8+6$	$96\overline{)769}$
5.	54×9	$175 - 78$	$69\overline{)500}$	$87\overline{)835}$	$146 - 59$	$89\overline{)356}$
6.	$87\overline{)600}$	$78\overline{)548}$	$46\overline{)417}$	$9+8+29$	*Best form of answer:* $\frac{3\,8}{6}$	$74 \times .6$
7.	$133 - 89$	$.78\overline{)6.24}$	$54\overline{)432}$	$1.53 - .67$	$69\overline{)485}$	$122 - 95$
8.	$6\frac{1}{2}\%$ = what decimal?	$7 - 2\frac{1}{4}$	$87\overline{)748}$	What % = 4× a number?	$95\overline{)918}$	$\frac{1}{2}$ sq. mi. = ...A.

SELF-TESTING DRILL NO. 23

202. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1.	2.	3.	4.	5.
528 $\times 64$	$10\frac{1}{3}\frac{7}{5} + 5\frac{1}{3}\frac{3}{5} = ?$	$\frac{2}{9} \times 21 = ?$	98964 -55943	121 273 465 423 597 579 753
6.	**7.**	**8.**	**9.**	
91790 -52292	$35\overline{)8295}$	$61\overline{)3658}$	$\frac{2}{15} \div \frac{1}{9} = ?$	
10.	**11.**	**12.**	**13.**	**14.**
70.5 $\times .294$	*Subtract:* $\frac{3}{8}$ from $\frac{2}{3}$	.096 .772 .717	$\frac{5}{6} - \frac{4}{18} = ?$	$2\frac{3}{5} \times 6\frac{1}{4} = ?$
15.	**16.**		**17.**	
$.023\overline{)621}$	$.369 \div 50 = ?$		1 pt. is what % of 2 qt.?	

18. A farmer borrowed one thousand dollars for one year at $7\frac{1}{2}\%$ interest. How much did he have to pay for the use of the money?

19. A rectangle 2 ft. by 5 ft. is cut from corner to corner by a diagonal. What is the area of each piece?

20. If a dealer's discount is 40%, what per cent of list, or printed, price does the dealer pay?

STANDARDS

Number correct..	0–4	5	6–7	8	9	10	11	12–13	14–15	16–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

AREA OF ANY TRIANGLE

203. Finding Area of Any Triangle. (STUDY PERIOD.) You know that the area of a right triangle $= \frac{1}{2}$ base $\times$ altitude.

Study the right triangle, Fig. 1.

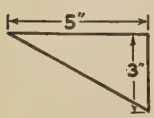


FIG. 1

Area $= \frac{1}{2}$ of $5'' \times 3''$, because the rt. $\triangle$ is $\frac{1}{2}$ of the rectangle shown in Fig. 2. One side is an altitude. One side is a base.

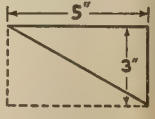


FIG. 2

Study the triangles, Fig. 3 and Fig. 4.

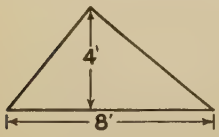


FIG. 3

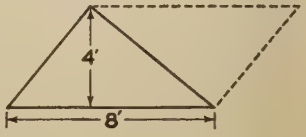


FIG. 4

Fig. 3 is not a rt. $\triangle$. Its area $= \frac{1}{2}$ of $8' \times 4'$, however, because the triangle is $\frac{1}{2}$ of the parallelogram shown in Fig. 4, and the area of the parallelogram $= 8' \times 4'$ (base $\times$ altitude).

Any triangle is always half of some parallelogram; so the area of a triangle is always $\frac{1}{2}$ the area of some parallelogram. The altitudes of triangles which are not rt. $\triangle$ can be drawn as in Fig. 3, above.

The area of any parallelogram equals what? The area of any triangle equals what?

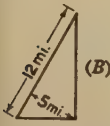
What kind of angle is formed by a base and an altitude? Why must we draw the altitude of a triangle that has no right angle?

Find the areas of the $\triangle$ shown at the top of the next page.

(*A*)

Think this △ as $\frac{1}{2}$ of ⟶

Area of (*A*) = ...sq. ft.

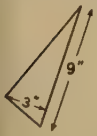


(*B*)

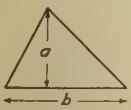
Think this △ as $\frac{1}{2}$ of ⟶

Area of (*B*) = ...sq. mi.

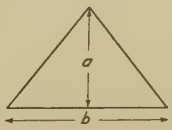
Find the areas of △ *C*, *D*, *E*, *F*. (Use your ruler with *D*, *E*, *F*.)



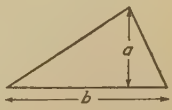
(*C*)



(*D*)



(*E*)



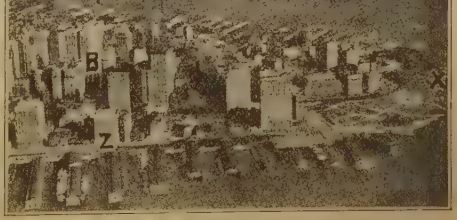
(*F*)

204. Problems in Finding Areas. (WRITTEN WORK.) In the problems below you will see that a triangle can be of **any size.**

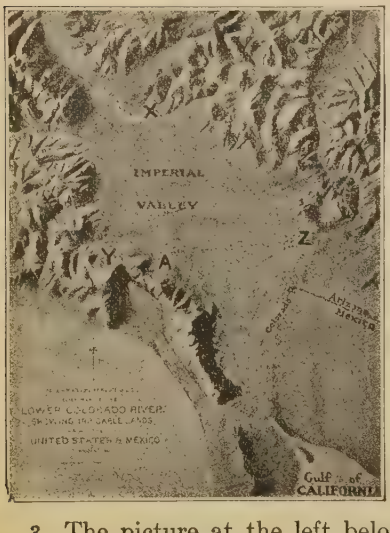
1. Lower New York City is called "the world's busiest triangle." What is the area of triangle *XYZ*?

Y to *Z* = $1\frac{1}{2}$ mi.
X to *B* = $1\frac{3}{4}$ mi.

Can you see any rectangles and rectangular solids in the picture?



LOWER NEW YORK CITY



2. A Triangle from Geography.

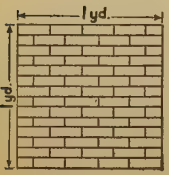
$Y \rightarrow Z = 64$ mi.
$A \rightarrow X = 46$ mi.

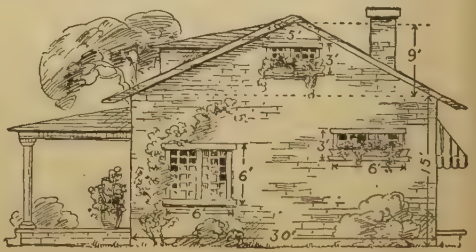
(*a*) Compute the approximate area of the Colorado Imperial Valley.

(*b*) Compare its area with the area of your state.

(*c*) How many lower N. Y. triangles could be put into the Imperial Valley triangle? (See Problem 1 on page 303.)

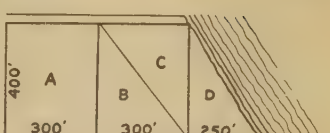
3. The picture at the left below shows the number of bricks per square yard. Count them.





First: **Estimate** number of bricks in the side of this house (subtract for windows). Some pupils guess within 250 bricks. Second: **Find** the number of bricks as accurately as you can. (End of house is a rectangle and a triangle.)

4. What are the shapes of areas A, B, and C? Find the areas represented by A, B, C, and D.



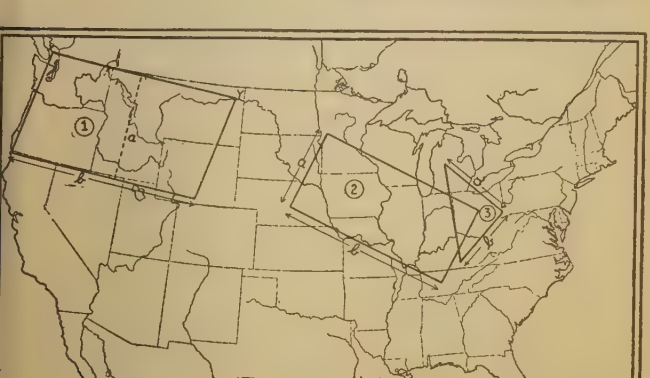


Figure 1: The Great Northwest Parallelogram. Figure 2: The Great Corn-Producing Region. Figure 3: A Great Sheep-Raising Section.

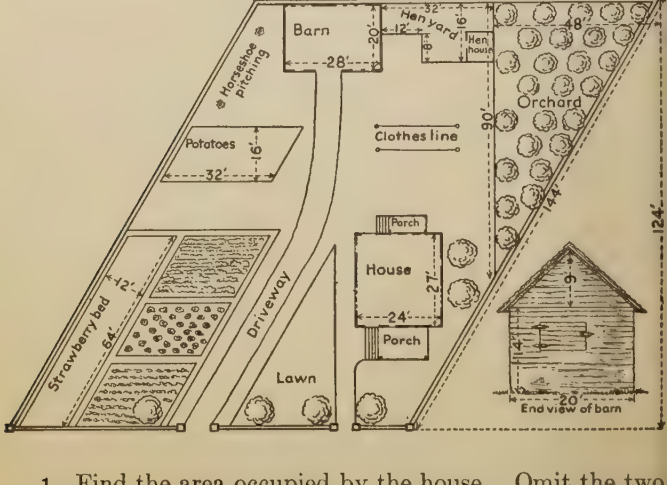
5. Estimate the area of the great corn-producing region of the U. S. (For Problems 5, 6, and 7 find a good map and estimate dimensions by using the scale on the map.)

6. Estimate the area of the sheep-raising section.

7. Estimate the area of the Great Northwest.

➡ Examples requiring finding the area of a square, of a rectangle, of a parallelogram, of a right triangle, or of any triangle, will appear in the Self-Testing Drills from now on. Restudy these topics now if you are a little "uneasy" about them.

205. Problems in Mensuration. (WRITTEN WORK.) In working these problems use the facts shown below in the picture of Mr. Williams's property. Name answers carefully.



1. Find the area occupied by the house. Omit the two porches.

2. Find the area of the barn floor.

3. Find the area of the hen yard, including ground used by the hen house.

4. Mr. Williams allows 24 sq. ft. per hen. Less space would result in overcrowding. How many hens could be kept in the hen house and yard? (Be sure your answer is a reasonable one.)

5. Find the area of the strawberry bed.

6. How many strawberry plants can Mr. Williams plant this bed if he allows $\frac{3}{4}$ sq. ft. per plant?

7. How many strawberries will the bed yield at 1 qt. per plant?

8. Mr. Williams plans to put young peach trees in the orchard. He allows only 64 sq. ft. per tree, because he expects to take some of them out and replant them when they are more fully grown. How many peach trees should he buy?

9. Mr. Williams hopes to average a peck of potatoes from each four hills. How many bushels does he expect from his potato patch? He allows $1\frac{1}{2}$ sq. ft. per hill.

10. In buying rock dust for surfacing his driveway Mr. Williams figured:

Length of driveway—160 ft.
Width of driveway—8 ft.
1 cu. yd. of rock dust to cover 80 sq. ft. of surface.
Cost of dust delivered—$1.25 per cubic yard.

What was the cost of the rock dust needed to cover the driveway?

11. How much paint should Mr. Williams order for painting the barn? (Allow 1 qt. per 125 sq. ft. of surface. Get length of barn from floor plan.)

12. The dimensions of the lawn (a right triangle) are: base, 32′, altitude, 52′. About how much grass seed will be needed to plant the lawn? Allow 1 lb. of seed for every 75 sq. ft.

13. What is the total area of Mr. Williams's property? Does he have about one acre, seven tenths acre, five tenths acre, three tenths acre, or one tenth acre?

PROBLEM SCALE NO. XV

206. Problems about an Old Store Bill. (WRITTE
WORK.) You can earn 60 points by working every proble
correctly. Find your score and rate yourself as directed f
Problem Scales I to III.

Below is a copy of a bill of goods, purchased in 184
at Dubuque, Iowa, from a firm named Thomson & Finle
Work the problems in connection with the bill.

2 pitchforks @ 75¢	$1.50
2 lb. starch 19¢, 1 pr. shoes $1.00	1.19
$3\frac{1}{3}$ yd. blue drilling	.50
1 whip lash	.25
1 set cups and saucers	.32
3 pipes 6¢, palm hat 25¢	.31
1 lb. Imperial tea	.80
7 lb. sugar 50¢, 1 plug tobacco 13¢	.63
$\frac{1}{2}$ lb. salts 12¢, 7 yd. brown shirting 63¢	.75
Thread 6¢, $2\frac{1}{2}$ lb. Rio coffee 25¢	.31
9 lb. coffee $1.00, tobacco 13¢	1.13
$\frac{1}{2}$ lb. Gunpowder tea	.50
Coffee 19¢, needles 7¢, rhubarb 20¢	.46

1. How much would a pound of Gunpowder tea hav
cost? (

2. How many cents did one pipe cost? (

3. How much a yard was the brown shirting? (

4. What was the total of this bill? (

5. The 9 lb. of coffee cost how much per pound? (

6. What was the cost of the sugar per pound? (

7. What was the cost of Rio coffee per pound? (

8. What did the drilling cost per yard? (5)

9. If $\frac{1}{2}$ lb. of salts cost 12¢, what would $2\frac{1}{2}$ lb. cost? (5)

10. Six cups and saucers equal a set. What did one cup and saucer cost? (6)

11. What would be the amount of a 5% discount for cash on this bill? (See Problem 4.) (7)

12. After the discount had been deducted, what would the store-keeper have received for this bill of goods? (7)

STANDARDS

Superior work:	43–60 points	Slow work:	12–21 points
Good work:	32–42 points	Inferior work:	0–11 points
Average work:	22–31 points		

Would it be a good plan to spend a few minutes in review of this chapter to make sure that you understand everything in it? Would the following suggestion be interesting work?

Form committees to make out questions like those in Arts. 179 and 194. Then see if the class can answer all the questions.

CHAPTER XI

GETTING READY FOR THE SEVENTH GRADE

The work of this chapter is very important. It gives you an opportunity to practice upon the arithmetic you will need next year in the seventh grade. Some of you may go out to earn a living without further study of arithmetic. You, too, will need this arithmetic.

207. Careless Mistakes. (CLASS WORK.) If you think carefully, you will see just how careless are the mistakes listed below. Correct each one.

1. *Find $2\frac{1}{2}\%$ of 800.* "That's simple," John thought; "$2\frac{1}{2}$ is the same as 25. 25 is $\frac{1}{4}$ of 100%. $\frac{1}{4}$ of 800 is 200."

2. *Find the product of $2.4 \times .5$.* "I can do that in my head," said Mary, as she went ahead without looking carefully. She said the product was 120.

3. *Add:* $\frac{3}{4}+\frac{3}{4}$. Esther had just been multiplying fractions, where you multiply **both** numerators and denominators; so she carelessly said, "The sum is $\frac{6}{8}$, or $\frac{3}{4}$ if you insist on a good form for the answer."

4. *Find the area of a rectangle 8″ by 11″.* "Baby work," thought James as he wrote "88 in." for this answer.

5. *What is the quotient of $8\overline{)1616}$?* "I wish all arithmetic was as easy," thought Harold, as he wrote the quotient, 22.

6. *Find the difference:* $\begin{array}{r} 346. \\ 228 \\ \hline \end{array}$ "I need not pay any attention to such a third-grade example as this one," thought Thomas, as he put down 108 for the answer.

7. *Find 4% of 350.* "Anyone can multiply by 4," boasted Mary, as she hurried to write 1400 as the answer.

8. *78 is what % of 130?* "Any fraction is correct," thought Bill as he wrote down this work: $\frac{130}{78}$ or $78\overline{)130}$.

9. *Find 300% of 60.* "Really, this work is too simple to bother about," thought Frank as he promptly divided by 3.

10. *What is the area of a right triangle whose two shorter sides are 4 ft. and 6 ft.?* George said this problem could not be worked because the altitude is not given.

SELF-TESTING DRILL NO. 24

208. Directions. Time allowed: 20 minutes. **Careful pupils insist on understanding and correcting mistakes.**

1.
784
$\times 7$

2.
287
72
379

3.
128151
-78998

4.
3 gal. 3 qt.
-1 gal. 2 qt.

5.
$\frac{1}{16}$
$\frac{3}{16}$
$\frac{5}{16}$

6.
193
345
434
$\div \frac{9}{16} = ?$

7.
6.53
$\times 8$

8.
$3\frac{1}{2}$
4
$2\frac{1}{2}$
3
$11\frac{1}{2}$

9.
731
$\times .408$

10.
$.59 - 35.86 = ?$

11.
$15 \div 1\frac{1}{2} = ?$

12.
$91\overline{)1804}$

13.
.681
.145
.571
.608
.688

14.
$9.32 \times 1000 = ?$

15.
$12\frac{12}{15} - \frac{7}{15} = ?$

16.
Find five-sixths of three-fourths.

17.
$23\overline{)628.5}$

Go on to the next page.

18. Which of the following statements is incorrect?

(*a*) $50\% = \frac{1}{2}$ (*c*) $\frac{1}{6} = 12\frac{1}{2}\%$

(*b*) $25\% = \frac{1}{4}$ (*d*) $\frac{1}{3} = 33\frac{1}{3}\%$

19. The base of a parallelogram is 12 ft. and the altitude is 7.5 ft. What is the area?

20. What per cent of the area of the circle as shown in the diagram, is shaded?

STANDARDS

Number correct	0–5	6–7	8–10	11	12	13	14	15	16	17–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

GOOD HABITS FOR SOLVING PROBLEMS

209. Problem Solving. (Study Period.) In order to solve the problems which everyday life will bring to you you must be able to do several things accurately. You can learn to do these things by thinking carefully.

This article gives practice in thinking about problems. You will find three answers to each question about the problems (pages 313-315). Be ready to tell which answer is correct.

You must think carefully just what the problem asks for, and how to do each step. Remember in solving problems that **all** the work must be some kind of addition, subtraction, multiplication, or division. Some of the things you must think clearly about are:

1. In this step will I need to **add?** If so, what numbers must I add?

2. In this step will I need to **subtract?** If so, which number is subtracted from the other?

3. Does this step call for **multiplication?** If so, what umbers are to be multiplied?

4. Does this step use **division?** If so, what numbers re to be divided? Which number is the divisor?

5. Is my answer a reasonable one, and is it expressed good form?

6. Did I check for mistakes?

PROBLEMS

Tell which of the answers to the questions are correct.

I. A man worked 312 da. a year at $3.75 a day. He ved $117 during the year and spent the rest. What r cent of his earnings did he spend?

(*a*) What will the product of 312×$3.75 be?

1. It will not be anything until you add $117 to it. Then it will be the amount earned during the year.
2. The product of 312×$3.75 equals amount spent.
3. 312×$3.75 = earnings for the year. It is $1170.

(*b*) What must be done with the $117?

1. Subtract it from $1170 to find the amount spent.
2. Put the $117 into the bank.
3. Divide it by 312 to find amount saved per day.

(*c*) How much did the man spend?

1. He spent 312×$3.75.
2. He spent $1170−$117, or $1053.
3. He spent $1170+$117, or $1287.

(*d*) What per cent of his earnings did he spend?

1. He earned \$1170 and spent \$1053.

He spent: $\frac{1053}{1170}$ or $1170\overline{)1053.0}$ .9, or 90%, *Ans.*

$$\underline{1053\ 0}$$

2. He spent \$1053 and earned \$1170.

He spent: $\frac{1170}{1053}$ or $1053\overline{)1170.00}$ 1.11, or 111%, *An*

$$\begin{array}{r} 1053 \\ \hline 1170 \\ 1053 \\ \hline 1170 \\ 1053 \\ \hline \end{array}$$

3. Earned \$1170. Spent \$1053. Saved \$117.
117 is $\frac{1}{10}$ or 10% of his earnings, *Ans.*

II. A laborer earns \$2.25 a day for six days a week He works 48 weeks and spends 22% of his wages for rent How much money is left for other uses for the year?

(*a*) How would you find the amount earned in a year?

1. Multiply \$2.25 by 48. **2.** Multiply \$2.25 by 6.

3. Multiply $\$2.25 \times 6 \times 48$.

(*b*) How much did the laborer earn in the year?

1. \$13.50. **2.** \$648. **3.** \$108.

(*c*) What per cent of his earnings did the laborer hav for other expenses than rent?

1. 22%. **2.** 88%. **3.** 78%.

(*d*) How much money is left for "other uses"?

1. \$142.56. **2.** \$505.44. **3.** \$570.24.

III. In a small bake shop, each of 15 gas burners is used 4 hr. a day, 365 da. a year. Each burner consumes 6 cu. ft. of gas per hour. Find the yearly gas bill. Gas costs 80¢ per thousand cubic feet.

(*a*) Must there be any division in this problem?

1. No.
2. We divide by 15 to find how much gas 1 burner uses.
3. We divide the total cu. ft. of gas used by 1000.

(*b*) What unit of measure will appear in the answer?

1. $. 2. Days. 3. Cu. ft.

(*c*) The product of 15×6 will give what fact?

1. No. of cu. ft. of gas the shop uses a day.
2. No. of cu. ft. of gas the shop uses an hour.
3. No. of cu. ft. of gas one burner uses in a year.

(*d*) How would you find the amount of gas used in a year?

1. Find the product of $15 \times 6 \times 4 \times 365$.
2. Multiply 365 by 80¢.
3. Find the sum of $15+6+4+365$.

(*e*) The cost of the gas for a year is:

1. $312.00. 2. $105,120.00. 3. $105.12.

To think clearly about problems form these habits:

I. Read the problem carefully. Read it several times if necessary to understand it.

II. Answer these questions before starting to work:

1. What facts are given that are important?
2. What does the problem ask for?
3. What are the proper steps in the solution?

PROBLEM SCALE NO. XVI

210. Problems of the Farm. (WRITTEN WORK.) You can earn 100 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

Mrs. Nelson, the wife of a southern Minnesota farmer, kept an account of all the eggs, chickens, and cream which she sold during the year 1927. The following table shows how Mrs. Nelson's accounts totaled for each month during the year.

MONTH	DOZ. EGGS SOLD	EGGS	CHICKENS	CREAM
Jan.	110	$26.91	$ .00	$14.53
Feb.	205	$40.38	$ 8.72	$12.10
March	172	$48.24	$ .00	$13.98
April	177	$50.19	$ .00	$14.76
May	162	$56.77	$ .00	$17.55
June	155	$63.29	$24.61	$18.72
July	144	$60.99	$26.35	$13.08
Aug.	135	$56.45	$ 7.97	$11.64
Sept.	121	$43.54	$ .00	$20.39
Oct.	121	$46.02	$19.58	$18.24
Nov.	114	$41.76	$ .00	$15.00
Dec.	201	$36.00	$ .00	$15.65

1. What was the total amount received during the year for the chickens? (4)

2. How many dozen eggs did Mrs. Nelson sell during the year? (4)

3. What was the total amount Mrs. Nelson received for the cream? (5)

4. What was the total amount received for eggs? (5)

5. What was the average price per dozen received for ıe eggs? (7)

6. What was the total amount received for the eggs, ickens, and cream sold during the year? (7)

7. What per cent of this total income came from the gs? (8)

8. What per cent of the total income came from the ickens sold? (8)

9. What per cent of the total income came from the eam? (8)

10. Do these three per cents total 100%? If not, explain hy. (8)

11. What was the average monthly income from the le of chickens? (8)

12. What was the average monthly income from both ickens and eggs? (8)

13. What per cent of the total income was the amount ceived during the three summer months of June, July, d August? (10)

14. What per cent was taken in during the three winter onths of December, January, and February? (10)

STANDARDS

Superior work:	29–100 points	Slow work:	5–9 points
Good work:	17– 28 points	Inferior work:	0–4 points
Average work:	10– 16 points		

SUMMARY OF ADDITION

211. Different Kinds of Addition. (WRITTEN WORK.) In the exercise below you will find different kinds of addition examples. You should be able to do each example easily to be ready for the seventh grade.

Many pupils find these suggestions helpful:

1. Write figures plainly.
2. Keep columns and decimal points straight.
3. Carry the correct number.
4. Check by adding downward
5. Watch common denominators in fractions.
6. Reduce answers whenever necessary.
7. Name the answer when necessary, as: $, cu. ft., tons, etc.

1.

3086
6303
2756
9690
657
5475

2.

$4.66
.87
8.67
4.95
9.35

3.

$21\frac{7}{10}$
$93\frac{3}{5}$
$61\frac{1}{2}$
$65\frac{5}{6}$
34
$56\frac{11}{15}$

4.

hr.	*min.*
3	25
6	30
	55
2	10
4	

5.

$1\frac{1}{2}$
2
$\frac{2}{3}$
$7\frac{5}{6}$
$6\frac{7}{8}$
$8\frac{5}{12}$

6.

.98+8.25+1.21+2.45+2.98

7.

4% of 150+6% of 85+12% of 96

8.

ft.	*in.*
9	4
8	11
6	
5	7

9.

Find the average:

167
350
445
749
412
449

SUMMARY OF SUBTRACTION

212. Different Kinds of Subtraction. (WRITTEN WORK.) In the exercise below you will find different kinds of subtraction examples. You should do them all easily.

Many pupils find these suggestions helpful:

1. Write figures plainly.
2. Keep columns straight and decimal points in line.
3. When any figure in the minuend is too small, remember what you must do to subtract.
4. Check by adding the difference and subtrahend.

Thus: 600←*Minuend* *Check:* 247
−247←*Subtrahend* +353
353←*Difference* 600

5. Reduce fractions in answers when necessary.
6. Name answers when necessary, as: $, sq. ft., lb., etc.

1.	2.	3.	4.
80328	$ 80.56	.8596	124.84 − 34.48
−27096	−56.79	−.0258	

5.	6.	7.	8.
$8\frac{1}{2}$	4 ft. 6 in.	$78\frac{1}{3}$	8 lb. 7 oz.
$-3\frac{3}{7}$	−2 ft. 11 in.	$-8\frac{4}{5}$	−3 lb. 13 oz.

9.
119129 mi.
−94679 mi.

10. The difference between the whole and 23% of the whole equals what?

11. The total length of a certain road is 45 miles. 29 mi. of the road are paved. How much of it is unpaved?

SUMMARY OF MULTIPLICATION

213. Different Kinds of Multiplication. (WRITTEN WORK.) In the exercise below are different kinds of multiplication examples you have learned. You should do them easily.

Many pupils find these suggestions helpful:

1. Write all figures plainly.
2. Take care to handle zeros correctly.
3. Watch out for decimal points.
4. Change small mixed numbers to improper fractions.
5. Check by multiplying.

Thus: 672×306 *Check:* 306×672

6. Name answers when necessary, thus: bushels, miles, cubic feet, etc.

1. 4093×647

2. $\$85.67 \times 38$

3. 4 ft. 9 in. $\times 12$

4. 7.39×5.09

5. $6 \times 1\frac{5}{12}$

6. 6258×169

7. 98% of 4021

8. $2\frac{4}{5} \times 3\frac{4}{7}$

9. 6 lb. 5 oz. $\times 75$

10. 74% of 287

11. 87×1000

12. $5\frac{1}{2}\%$ of $\$685$

13. The average weight of the seven cows in this picture is 1108 lb. What is the total weight of these cows?

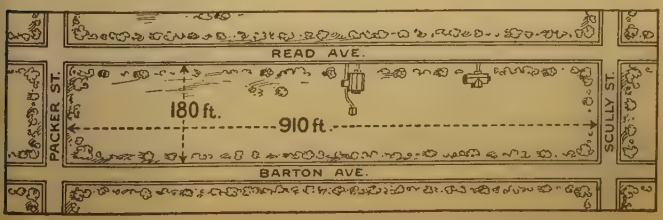
SUMMARY OF DIVISION

214. Different Kinds of Division. (WRITTEN WORK.) In the exercise below are different kinds of division examples. You should be able to do them all easily.

Many pupils have found these suggestions helpful:

1. Write all figures plainly.
2. Be careful to get the correct figure in the quotient.
3. Multiply and subtract with care.
4. Look for remainders every time.
5. Handle decimal points first when necessary.
6. Reduce fractions in answers when necessary.
7. Check by multiplying quotient and divisor.
8. Name answers when needed, as: $, hours, pounds, etc.

1. $472\overline{)145848}$
2. $464102 \div 506$
3. $6\overline{)22 \text{ yd. } 2 \text{ ft.}}$
4. $\$199.88 \div 38$
5. 155.48 is what % of 197?
6. $5\frac{1}{4} \div \frac{7}{12}$
7. 3810 is 46% of what number?
8. $5\frac{2}{5} \div 9$
9. $520\overline{)35464}$
10. If the land in the picture below is divided into 18 equal lots, how many square feet will be in each lot?



LARGE NUMBERS

215. Reading Large Numbers. (Oral Work.) Most of the arithmetic work you will have to do when you are grown men and women will use small numbers. But sometimes you must be able to read large numbers in order to understand what is written in newspapers, magazines, etc.

Study the Number Chart given below, and then practice reading the numbers in the exercise on the next page.

Number Chart

4,000,000,000.	four billion
400,000,000.	four hundred million
40,000,000.	forty million
4,000,000.	four million
400,000.	four hundred thousand
40,000.	forty thousand
4000.	four thousand
400.	four hundred
40.	forty
4.	four
.4	four tenths
.04	four hundredths
.004	four thousandths
.0004	four ten thousandths

5,250,017.27 is read "5 million, 250 thousand, seventeen **and** 27 hundredths."

6,942,314,264.17 is read "6 billion, 942 million, 314 thousand, 264 **and** 17 hundredths."

2,027,000.0009 is read "2 million, 27 thousand, **and** 9 ten thousandths."

4,444,444.4444 is read 4 million, 444 thousand, 444 **and** 4 thousand 444 **ten thousandths.**

EXERCISE

Read these numbers:

1.	$25,094,577.00.	Total cost of the famous Brooklyn Bridge.
2.	$548,810,639.	Value of property lost by fire in the United States during the year 1924.
3.	$2,529,027,661.	Amount received by the U. S. Treasury from Jan. 1 to March 15, 1924.
4.	105,710,620.	Number of the people in the United States according to the 1920 census.
5.	302,110,000.	Estimated population of China (18 provinces).
6.	3,729,665.	Area of Canada in square miles.
7.	3,026,789.	Area of the United States in square miles.
8.	239,068,295.	Weight of silver in ounces (fine) produced in the world in 1924.
9.	24,288,808.	Pupils enrolled in the public schools in the United States in 1924.
10.	$7,977,617,000.	Amount of money on deposit in savings banks in the United States in 1925.
11.	14,657,000.	Area of the moon in sq. mi. (about 4 times that of Europe).
12.	92,897,400.	Distance in miles between the sun and earth.
13.	$1,205,271,945.	Money paid by insurance companies in the U. S. to policy-holders in 1924.
14.	$83,007,435.36.	Amount of salaries paid to New York City teachers during the school year, 1924-1925.

Bring to class newspaper and magazine clippings showing large numbers. Be ready to read any number you find in the clippings.

PROBLEM SCALE NO. XVII

216. Problems from Geography. (WRITTEN WORK.) You can earn 94 points by working every problem correctly. Find your score and rate yourself.

AREA AND POPULATION OF THE EARTH BY CONTINENTS

CONTINENTAL DIVISIONS	AREA IN SQUARE MILES	INHABITANTS (Approximate)	INHABITANTS PER SQUARE MILE*
Africa	11,622,619	142,000,000	12.2
America, N.	8,589,257	136,000,000	15.8
America, S.	7,570,015	64,000,000	8.4
Asia	17,206,000	921,000,000	53.5
Australia	3,312,613	9,000,000	2.7
Europe	3,872,561	476,000,000	122.9
Polar Regions	5,081,935	Unknown	.....
Totals	57,255,000	1,748,000,000	*Average* 30.8

1. Write in words the area of Asia. (5)

2. Write in figures the total population of the Earth. (5)

3. What continent is about 50% as thickly settled as North America? (5)

4. How many more people live in Asia than in Europe? (5)

5. Is North America the first, second, third, or fourth most thickly settled continent? (5)

6. Is the per-sq.-mile population as given for South America about correct according to the other figures? (6)

*"Inhabitants per square mile" means the average number of people living on each square mile of the continent. One could find the total population of a continent by multiplying the area in square miles by the number of inhabitants per square mile.

7. Is the per-sq.-mile population for Africa as reported in the table about right according to the other figures? (6)

8. Which continent is most thickly settled? (6)

9. How many more people live in Asia than on the rest of the Earth as reported in the table? (7)

10. If a city's area is 8 sq. mi. and its population is 14,156, what is the density of population (inhabitants per square mile)? (7)

11. Check the figure for the number of inhabitants in North America by area $\times$ per-square-mile population. (7)

12. The area of London is about 693 sq. mi. The population of London in 1921 was 7,476,168. What was the population of London per square mile in 1921? (7)

13. Is Europe about 400%, 600%, 800%, or 1000% as thickly settled as North America? (7)

14. Check the figure for number of inhabitants in Europe by area times per-square-mile population. (8)

If you know the area and the number of inhabitants, you can find the density of population or number of people per square mile by dividing the total population by the area.

15. The total area of New York City is about 299 sq. mi. The population in 1921 was 5,751,859. What was New York City's population per square mile in 1921? (8)

STANDARDS

Superior work:	40–94 points	Slow work:	11–16 points
Good work:	28–39 points	Inferior work:	0–10 points
Average work:	17–27 points		

SUMMARY OF FRACTIONS

217. Addition and Subtraction Review. (WRITTEN WORK.) Study the exercise below. It may interest you to know how the examples were chosen. A large number of sixth-grade pupils were given a series of examples in fractions. When the papers were corrected, it was found that some examples were "missed" much oftener than others. The harder examples on the papers were taken out and put into this exercise.

Pupils getting ready for the seventh grade ought to work any fraction example correctly. Be sure you can work every one of these before you leave the exercise.

EXERCISE

1.
$4\frac{5}{6}$
$15\frac{2}{3}$
20
$32\frac{1}{8}$

2. $3\frac{3}{5}+6\frac{5}{6}+17+8\frac{1}{2}=?$

3.
$14\frac{1}{6}$
$-13\frac{5}{8}$

4.
$21\frac{3}{4}$
18
25
16

5.
$30\frac{2}{3}$
$6\frac{1}{4}$
$53\frac{5}{6}$
$29\frac{3}{8}$

6.
36
$-9\frac{6}{11}$

7. $20\frac{2}{3}-20\frac{1}{2}=?$

8. $66\frac{2}{3}-17\frac{1}{11}=?$

9. $7\frac{1}{4}+2\frac{5}{6}=?$

10. $1\frac{4}{9}-\frac{4}{5}=?$

11. $66\frac{2}{3}\%$ = what fraction?

12. Write "point eight seven five" as a fraction.

13. (*a*) Write $\frac{4}{5}$ as a per cent. (*b*) Write $\frac{728}{1000}$ as a decimal. (*c*) Write $33\frac{1}{3}\%$ as a fraction.

14. *Fill the vacant places:* $\frac{2}{3}=\frac{}{6}=\frac{8}{}=\frac{10}{}=\frac{}{18}=\frac{}{21}=\frac{16}{}$.

15. *Reduce to lowest terms:* $\frac{19}{38}$, $\frac{9}{21}$, $\frac{20}{30}$, $\frac{15}{20}$, $\frac{40}{100}$, $\frac{35}{56}$.

218. Multiplication and Division Review. (WRITTEN WORK.) The exercise below is your opportunity to make sure that you know enough about the multiplication and division of fractions to do seventh-grade work in a way to please you.

If you find any examples you cannot work or do not really understand, follow the suggestion below.

Suggestion: Look in the Self-Help Index, find the places where the multiplication or division of fractions is taught and practiced, and study those articles again.

EXERCISE

Many pupils get the addition and multiplication of fractions and mixed numbers confused. Work the next 8 examples as a protection against this danger.

1. $\frac{1}{2}+\frac{1}{2}=$
2. $\frac{1}{2}\times\frac{1}{2}=$
3. $\frac{2}{3}\times\frac{4}{5}=$
4. $\frac{2}{3}+\frac{4}{5}=$
5. $\frac{3}{4}+7\frac{1}{11}=$
6. $\frac{3}{4}\times7\frac{1}{11}=$
7. $3\frac{3}{8}\times1\frac{5}{9}=$
8. $3\frac{3}{8}+1\frac{5}{9}=$

Find the quotients of these division examples:

9. $4\div\frac{2}{3}=$
10. $\frac{7}{9}\div8=$
11. $7\frac{1}{3}\div5=$
12. $\frac{4}{5}\div\frac{8}{15}=$
13. $\frac{9}{13}\div\frac{7}{26}=$
14. $8\div6\frac{3}{4}=$
15. $2\frac{1}{6}\div3\frac{5}{7}=$
16. $6\frac{2}{3}\div6\frac{3}{5}=$
17. $19\div7\frac{3}{5}=$
18. $4\frac{1}{2}\div\frac{7}{8}=$
19. $\frac{1}{12}\div3\frac{5}{6}=$
20. $\frac{9}{20}\div6=$

In the next examples write the remainders of the quotients as fractions. Leave the answers exactly right.

21. $24\overline{)876}$
22. $9\overline{)4}$
23. $8\div7$
24. $\frac{290}{15}$
25. $33\overline{)6809}$

In the next examples correct or improve any answer you can.

26. $19\times\frac{1}{3}=\frac{19}{3}$
27. $4\frac{1}{2}\times2\frac{1}{3}=10\frac{3}{6}$
28. $27\div\frac{2}{3}=18$
29. $5\frac{1}{3}\div2\frac{2}{5}=1\frac{54}{36}$

AN IMPORTANT PROJECT

219. Division of Decimals. (A Class Project.) You have found some things in arithmetic harder than others. It is always wise to practice on the hard things.

The pupils of a very good class in the sixth grade were asked to decide by vote the topic that seemed hardest to them. They agreed to practice extra on the topic that got the most votes. The topics and votes are recorded below.

Topics	Votes
(*a*) Multiplication of mixed numbers, as: $4\frac{1}{4}\times3\frac{1}{3}$	(3)
(*b*) Division of mixed numbers, as: $4\frac{1}{4}\div3\frac{1}{3}$	(2)
(*c*) Multiplication of decimals, as: 4.72×38.9	(0)
(*d*) Division of decimals, as: $3.72\overline{)42.408}$	(9)
(*e*) Finding a per cent of a number, as: find 4% of 275.	(2)
(*f*) Finding what per cent one number is of another, as: 48 is what per cent of 144?	(7)
(*g*) Multiplying denominate numbers, as: 2 ft. 7 in.$\times$9	(5)

$$\begin{array}{r} 11.4 \\ 3.72\overline{)42.408} \\ 37\,2 \\ \hline 5\,20 \\ 3\,72 \\ \hline 1\,488 \\ 1\,488 \\ \hline \end{array}$$

What topic did the class consider hardest? Here is a sample of (*d*)→

The second most difficult topic was (*f*). That is also division with decimal remainders.

$$\begin{array}{r} .33\frac{1}{3}, \text{ or } 33\frac{1}{3}\% \\ \frac{48}{144}, \text{ or } 144\overline{)48.00} \\ 43\,2 \\ \hline 4\,80 \\ 4\,32 \\ \hline 48 \end{array}$$

Sample: 48 is what per cent of 144?

"What per cent of" means "What fraction of," as: Is 48 a third ($33\frac{1}{3}\%$) of 144?

The third most difficult topic was (*g*), the multiplication of denominate numbers.

2 ft. 7 in. ×9 18 ft. 63 in. 5 ft. 3 in. 23 ft. 3 in.	5 ft. 3 in. 12)63

Remember to reduce answers when working with denominate numbers.

The class just mentioned decided, because of the votes, to work 12 examples in division of decimals and 6 examples using denominate numbers. Why did this class vote to work on the things it found hard? The examples this class used are given below. If you work them carefully, the drill will help you also.

EXERCISE

1. $75.39\overline{)68152.56}$
2. $.462\overline{)259.182}$
3. $80.1\overline{)3029.382}$
4. $358\overline{)590.70}$
5. $.642\overline{)57266.4}$
6. $9.07\overline{)341.032}$
7. 45.82 is what % of 79?
8. 467.29 is what % of 563?
9. 397.62 is what % of 846?
10. 25.2 is what % of 35?
11. 137.6 is what % of 215?
12. 15.39 is what % of 81?
13. 5 ft. 7 in. ×6
14. 3 lb. 5 oz. ×7
15. 7 yd. 2 ft. ×5
16. 2 pk. 3 qt. ×8
17. 1 gal. 2 qt. ×9
18. 4 qt. 1 pt. ×4

Talk over in class this question: "On what kind of work would more drill help us?" Use the Self-Help Index, page 457, in making plans for this drill.

SELF-TESTING DRILL NO. 25

220. Directions. Time allowed: 20 minutes. There is no work in this drill which should trouble pupils who are getting ready for the seventh grade.

1. $53\overline{)1696}$

2. $167.1 - 156.5 = ?$

3.
$$\begin{array}{r} 8440 \\ -3899 \\ \hline \end{array}$$

4. $\frac{2}{5} + \frac{1}{2} = ?$

5.
$$\begin{array}{r} 989 \\ 887 \\ 486 \\ 798 \\ 375 \\ \hline \end{array}$$

6.
$$\begin{array}{r} .290 \\ \times 7.8 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 790 \\ \times 129 \\ \hline \end{array}$$

8. $4\frac{1}{2} + 1\frac{3}{4} = ?$

9. $1\frac{1}{9} \times \frac{2}{3} = ?$

10. $3\frac{1}{5} \div 1\frac{1}{2} = ?$

11. $1\frac{9}{24} \div 3\frac{2}{3} = ?$

12.
$$\begin{array}{r} 938 \\ 995 \\ 645 \\ 773 \\ 728 \\ 584 \\ 859 \\ 188 \\ 918 \\ \hline \end{array}$$

13. $8\frac{2}{9} - 4\frac{2}{3} = ?$

14.
$$\begin{array}{r} 28.8 \\ \times 17.7 \\ \hline \end{array}$$

15. Divide and carry remainder to 2 decimal places: $17\overline{)44}$

16. $.9020 \div 98 = ?$

17. $8\overline{)9 \text{ lb. } 8 \text{ oz.}}$

18. *B* is what % of *A* below?

19. Bar *X* (not shown in the drawing at the left) is 75% as long as Bar *Y*. If Bar *X* is placed alongside Bar *Y*, to what letter on Bar *Y* will it reach?

Go on to the next page.

20. *Write as one number:*

1 million 0 hundreds 9 units 8 hundredths 437 thousands 0 tens 0 tenths

STANDARDS

Number correct....	0–3	4–5	6	7	8	9	10	11	12	13–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Are you remembering to use the Self-Help Index and to practice on drills on which you made a low rating?

SUMMARY OF PERCENTAGE

221. Review of Percentage. (CLASS WORK.) First study the exercise below; then take it up in class discussion. Be sure you understand thoroughly everything in this article.

EXERCISE I

1. The per cent sign is %. 8% means .08, or $\frac{8}{100}$. 225% = 2.25, or $2\frac{1}{4}$.

2. A fraction may be expressed as a %. $\frac{1}{4}$ of an acre = 25% of an acre. $\frac{7}{8}$ of the book = $87\frac{1}{2}$% of the book. "Twice as much" = 200% of. "Half of it" = 50% of it. $3\frac{1}{3}$ times = $333\frac{1}{3}$% of.

3. A per cent may be expressed as a fraction.

20% of = $\frac{20}{100}\times$ = $\frac{1}{5}\times$ 87% of = $\frac{87}{100}\times$

$16\frac{2}{3}$% of = $\frac{1}{6}\times$ $33\frac{1}{3}$% of = $\frac{1}{3}\times$ 25% of = $\frac{1}{4}\times$

4. A decimal may be expressed as a per cent.

.04 = 4% .4 = 40% .19 = 19% 2.5 times = 250% of

5. A per cent may be expressed as a decimal.

27% = .27 675% of = 6.75×

6. Draw this table on the blackboard or on your paper. Fill in every vacant place.

Fraction Form	Per cent Form	Decimal Form
$\frac{1}{2}$		
	75%	
		.25
		$.66\frac{2}{3}$
	$33\frac{1}{3}\%$	
$\frac{9}{100}$		
	125%	
	.1% or $\frac{1}{10}\%$ or $\frac{1}{10}$ of 1%	.001
$\frac{8}{1000}$		.008

There are several uses of percentage. Two of these we have learned this year. Keep in mind that both of these uses of percentage are often applied in finding interest, discounts, commissions, etc.

First Use: **To find a certain per cent of a number.**

\$600
$\times .04$
\$24.00

Sample. Find 4% of \$600. 4% of \$600 means $\$600 \times .04$.

Since "4% of" means $\frac{4}{100}\times$, which is a pretty small part, is \$24 a reasonable answer?

EXERCISE II

1. Find 6% of \$925.

2. What is 28% of 324 acres?

3. $12\frac{1}{2}\%$ of 1244 = ?

4. 250% of 850 = ?

5. 94% of \$34.62 = ?

6. 1% of 87432 = ?

7. How much is a 7% discount on \$1250?

8. $66\frac{2}{3}\%$ of 216 = ?

9. The interest on \$847 for 1 yr. at 4% = ?

10. The discount on a bill of \$362 at 2% = ?

11. The 40% commission on a sale of \$5000 = ?

12. Would 2% of any amount be a very small part of it? Almost all of it? Or much greater than the amount?

13. Would 40% of any number be more or less than $\frac{1}{2}$ of it?

14. Would 101% of any number be greater or less than the number itself?

15. Compute the discount and find the reduced price:

	Regular Price	Discount	Reduced Price		List Price	Dealer's Discount	Price to Dealer
(*a*)	\$24.00	50%	.....	(*f*)	\$ 540.00	40%	.....
(*b*)	\$75.00	2%	.....	(*g*)	\$ 6.50	10%	.....
(*c*)	\$ 8.50	40%	.....	(*h*)	\$ 18.00	25%	.....
(*d*)	\$54.00	5%	.....	(*i*)	\$1750.00	28%	.....
(*e*)	\$ 2.00	30%	.....	(*j*)	\$ 375.00	15%	.....

Second Use: **To find what per cent one number is of another.** Think what fraction is made by the two numbers.

Sample (*a*): Eighteen hits out of 24 shots means what per cent of hits?

```
   .75
24)18.00
  16 8
  1 20
  1 20
```

Think "18 out of 24" as $\frac{18}{24}$ or $24\overline{)18.}$ The per cent is found by getting the quotient in decimal form and then changing the decimal to a per cent, thus: .75 = 75%. 18 hits out of 24 shots means that 75% of the shots were hits.

It is a good plan to look at an answer and say to yourself, "Is that a reasonable answer?"

Thus in Sample (*a*), $75\% = \frac{3}{4}$. Is 18 about $\frac{3}{4}$ of 24?

SAMPLE (*b*): A class of 30 pupils equals what per cent of a class of 24 pupils?

$\frac{30}{24}$ or $24\overline{)30}$

$$24\overline{)30.00}\quad 1.25$$

$$\begin{array}{r} 24 \\ \hline 60 \\ 48 \\ \hline 120 \\ 120 \\ \hline \end{array}$$

Think "what per cent of" is "what fraction of."

$1.25 = 125\%$. A class of 30 pupils equals 125% of a class of 24 pupils.

Exercise III gives practice in finding what per cent one number is of another.

EXERCISE III

1. 12 is what % of 24?
2. 12 is what % of 48?
3. 12 is what % of 96?
4. 12 is what % of 12?
5. 12 is what % of 6?
6. 12 is what % of 2?
7. 19 is what % of 76?
8. 38 is what % of 190?
9. 360 is what per cent of 4220?
10. $\frac{17}{20}$ of A is what per cent of A?
11. $\frac{37}{20}$ of B is what per cent of B?
12. Change $\frac{11}{12}$ to a decimal and then to a per cent.
13. Joseph earns \$1500. He saves \$500. He saves what per cent of his earnings?
14. Mr. Clark made a 2150-mile trip. What per cent of his trip was completed when he had gone 430 miles?
15. In a certain town there are 1284 families. 214 of these own their own homes. What per cent of the families own their own homes?

16. A truck can carry 2 tons. (Capacity, 2 T.) Its load is 1500 lb. What per cent of its carrying capacity is the load?

FIG. 1

FIG. 2

FIG. 3

17. About what per cent of Fig. 1 is black?

18. About what per cent of Fig. 2 is black?

19. (*a*) In Fig. 3, *B* is about what per cent of *A*?

(*b*) *A* is about what per cent of *B*?

20. In Fig. 4, is the weight of *B* more than, less than, or equal to 100% of the weight of *A*?

FIG. 4

21. In Fig. 5, is *B's* weight more or less than 100% of *A's* weight?

FIG. 5

22. Sales in 1926 were 216 cars. Sales in 1927 were 324 cars. The 1927 sales were what per cent of the 1926 sales?

23. What numbers should be on the blanks?

16 = ... % of 32; ..... % of 64; ..... % of 8.

19 = ... % of 19; ..... % of 38; ..... % of $9\frac{1}{2}$.

22 = ... % of 11; ..... % of 44; ..... % of 33.

32 = ... % of 64; ..... % of 16; ..... % of 96.

VOCABULARY PRACTICE

222. One-Minute Talks about Arithmetic. (CLASS WORK.) On page 336 is a list of words and phrases which are frequently used in arithmetic. Study these to get the important facts of each one in mind. When you are called

upon to explain and discuss any one of them, give the main facts in clear well-spoken English. Use the blackboard to illustrate your talk if you want to do so.

1. Area
2. A checking account
3. Place value of numbers
4. Altitude
5. A right triangle
6. Cancellation
7. Dimensions
8. A check stub
9. Common denominators
10. An acute angle
11. Best form of answers
12. Denominate numbers
13. Checking for accuracy
14. Decimal form of remainders
15. "Per cent of" means "fraction of"
16. Cubic measure
17. Rectangular solid
18. $\frac{1}{2}b \times a$
19. Parallel lines
20. Kinds of discount
21. A rectangle
22. A proper fraction
23. A square foot
24. Multiplier
25. Graphs
26. Estimating answers
27. Interest
28. A numerator
29. Parallelogram
30. Perimeter
31. The average
32. A reciprocal
33. Volume
34. Quotients
35. Units of measure
36. A savings bank
37. Minuend
38. Subtrahend
39. Difference
40. Multiplicand
41. Dividend
42. Divisor
43. Product

PROBLEM SCALE NO. XVIII

223. Problems from History. (WRITTEN WORK.) You can earn 110 points by working every problem correctly. Find your score and rate yourself.

I. Problems about "Jefferson's Bargain," or the Louisiana Purchase.

FACTS TO USE IN SOLVING PROBLEMS

Date of Purchase, 1803.
Land was bought from France by United States.
Price paid, $15,000,000.
Approximate area of Purchase, 1,000,000 sq. mi.
France (in Europe) contained in 1803 about 209,000 sq. mi.
Area of United States before Purchase, about 828,000 sq. mi.
There are 640 acres to the square mile.
Wisconsin contains about 56,000 sq. mi.

1. How many years ago did President Jefferson buy the land? (3)

2. How large was the United States after adding to itself the Louisiana Purchase? (5)

3. What was the price per square mile? (6)

4. About how many acres of land were obtained by the Purchase? (7)

5. About how many times as great as France itself was the area of the land sold by France? (7)

6. What was the price per acre? (8)

7. The area of Wisconsin is what per cent of the area of the Louisiana Purchase? (8)

8. The land in Wisconsin today is worth how many times as much as the total Louisiana Purchase in 1803? (Figure land today worth $100 per acre on the average.) (10)

II. Problems showing increase in speed of travel.

FACTS TO USE IN SOLVING PROBLEMS

The first stagecoach between New York and Philadelphia traveled at the rate of 3 miles an hour.

It is 90 miles from New York to Philadelphia.

A train in 1823 made an average speed of 15 miles an hour.

A modern express train averages 45 miles an hour.

An auto can average 25 miles an hour on trips in the country.

9. How many days did it take a stagecoach traveling 12 hr. a day to go from New York to Philadelphia? (8)

10. The speed of a train in 1823 was what per cent of the speed of a modern express train? (8)

11. An automobile averaging 25 mi. an hour travels how many times as fast as the first stagecoach between New York and Philadelphia? (8)

12. How many round trips could an automobile make between New York and Philadelphia while a stagecoach was going one way? (8)

13. The speed of a modern express train is what per cent of the speed of a train in 1823? (8)

14. The average speed of the automobile is what per cent of the speed of a train in 1823? (8)

15. The speed of a train in 1823 is what per cent of the speed of an automobile averaging 25 mi. an hour? (8)

STANDARDS

Superior work:	22–110 points	Slow work:	4–6 points
Good work:	14– 21 points	Inferior work:	0–3 points
Average work:	7– 13 points		

A USEFUL PROBLEM-SOLVING HABIT

224. Estimating Answers. (CLASS WORK.) Before working a problem, stop and think what answer would be reasonable. Then, when you have worked the problem, compare your answer with your estimate. This habit often prevents foolish mistakes.

The exercise below is to help you form the habit of estimating reasonable answers before you begin work.

Do not work these examples now, but tell which of the answers on the right is the best estimate for each example on the left. The estimates are not in order. The best estimate for Example 1 is (*g*). Give as your answer the letter that stands beside the best estimate for each example

EXAMPLES	ESTIMATED ANSWERS
1. 61×1194	(*a*) 3000
2. $8.47\overline{)7.80087}$	(*b*) 940
3. $87246 - 34182$	(*c*) 10
4. 94 is 11% of what number?	(*d*) 97
5. Dimensions, $5'9'' \times 8'2'' \times 10''$; $V = ?$	(*e*) 1
6. $8.27\overline{)78.0087}$	(*f*) .094
7. 573×462	(*g*) 72,000
8. $0.94 \div 11$	(*h*) 53,000
9. 9% of 7324	(*i*) $16\frac{2}{3}$
10. Dimensions, $15'11'' \times 32'$; area = ?	(*j*) 200,000
11. $4.28 + 9.90 + 15.5 + 4.8 =$	(*k*) 63
12. 84 is what % of 500?	(*l*) 48 cu. ft.
13. 5.73×4.62	(*m*) 480 sq. ft.
14. $8.47\overline{)780.087}$	(*n*) 73.24
15. $428 + 990 + 1550 + 48$	(*o*) 35.00
16. $9\frac{1}{3} \times 6\frac{3}{4}$	(*p*) 25

PROBLEM SCALE NO. XIX

225. Problems for Review. (WRITTEN WORK.) You can earn 98 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. The lengths in feet of 8 of the battleships of the United States Navy are given in the table below:

NAME	LENGTH
California	624
Colorado	624
Idaho	624
New York	573
Pennsylvania	608
Texas	573
Utah	521
Wyoming	562

If these 8 vessels stood in a straight line with their ends touching, how far would they reach? (5)

2. How many more feet than .5 of a mile is your answer to Problem 1? (7)

3. What is the average length of the 8 battleships listed in Problem 1? (7)

4. The total wealth of a small village is $891,152.75. There are 467 people in this village. What is the per capita wealth? (How much money per person?) (6)

5. The per capita wealth in a certain town is $2407. There are 3009 inhabitants in this town. Find the total wealth. (6)

6. At the right are the milk sales of the Sidwell Dairy Co. for 5 days.

What was the sale of milk for the 5 days? (6)

GAL.	QT.	PT.
16	3	1
24	0	1
19	2	1
21	1	0
17	2	1

7. Mr. Brown earned $84.67 during 14 days of harvesting. What was his average earning per day? (7)

8. The Overland Express traveled 165.7 miles in 3.6 hours. What was the average speed per hour? (7)

9. If Mrs. Jones buys $3\frac{1}{4}$ yd. of cloth at 17¢ a yard, how much change should she receive from a $5.00 bill given to the clerk? (7)

10. Divide 26.8 by 3.1417. Carry quotient to 2 decimal places. (7)

11. Frank Jones bought his home for $6580. He paid $2961 at the time of purchase. What per cent of the cost did he pay at the time of the purchase? (7)

12. If $x = 4.25$, what does $x \times 61$ equal? (8)

13. The coffee we use in our homes has been roasted. When green coffee is roasted, it loses 18% of its weight. Find the loss by the roasting of 125 lb. of coffee. (8)

14. 800 lb. of green coffee were roasted before being delivered to a store. What did the roasted coffee weigh? (10)

STANDARDS

Superior work:	33–98 points	Slow work:	6–8 points
Good work:	21–32 points	Inferior work:	0–5 points
Average work:	9–20 points		

SELF-TESTING DRILL NO. 26

226. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Correct mistakes and use the Self-Help Index if you need to.**

1. 941×40

2. $37.43 - 25.89 = ?$

3. 607×19

4.
20.439
70.905
17.008
19.019

5. $7\frac{3}{16} + 3\frac{5}{16}$

6. $82330 - 18443$

7. $\frac{1}{2} \div \frac{2}{3} = ?$

8. $4\frac{1}{6} + 7\frac{1}{2} = ?$

9. $7\frac{1}{12} - \frac{7}{12}$

10.
99
89
281
172
294
24
57
87
19

11. $62\overline{)4853}$

12. *Reduce to lowest terms:* $\frac{16}{10} = ?$

13. $11\frac{3}{4} \div \frac{2}{3} = ?$

14. $4.5\overline{).4005}$

15. *Add:*
3 lb. 12 oz.
2 lb. 5 oz.
9 lb.
2 lb. 4 oz.

16. 87.9×765

17. $\frac{8}{9} \times \frac{3}{20} = ?$

18. In the drawing, M is what % of N?

19. 76 is what per cent of 56?

20. Principal, $376; Rate, $7\frac{1}{2}\%$; Time, 1 yr. 6 mo. Find the interest.

STANDARDS

Number correct..	0–3	4–6	7	8	9	10–11	12	13	14	15–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

SUMMARY OF MENSURATION

227. Areas and Volumes. (STUDY PERIOD.) The next statements are important facts about mensuration. They are worth remembering.

FACTS ABOUT MENSURATION

1. Remember the three kinds of measurements:

First. **Length** or distance. Units of measure of length or distance are the inch (″), the foot (′), the yard, and the mile.

Second. **Area** or surface. Units of measure of area or surface are the square inch (sq. in.), the square foot (sq. ft.), the square yard (sq. yd.), the acre (A), and the square mile (sq. mi.).

Third. **Volume.** Units of measure of volume are the cubic inch (cu. in.), the cubic foot (cu. ft.), and the cubic yard (cu. yd.).

☛ The Tables of Measure on page 456 should be thoroughly understood.

2. The area of a rectangle is found by multiplying length by width, when both dimensions are expressed in the same unit. Units of measurement of area are: sq. in., sq. ft., sq. yd., etc.

3. The volume of a rectangular solid is found by multiplying length by width by height, when the dimensions are expressed in the same unit. Units of measurement of volume are: cu. in., cu. ft., cu. yd., etc.

4. The area of a right triangle equals $\frac{1}{2}$ of the base times the altitude.

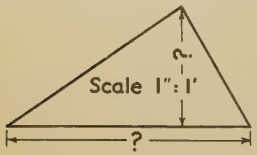
5. The area of a parallelogram equals base $\times$ altitude. The altitude is not a side unless the parallelogram is also a rectangle.

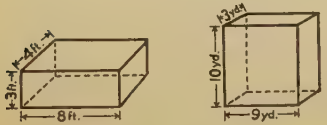
6. The area of **any** triangle equals $\frac{1}{2}$ base $\times$ altitude. The altitude in triangles other than right triangles must be drawn.

7. To draw to scale means letting given lengths represent a distance, longer or shorter than the lengths in the drawing.

PROBLEMS

1. Find the dimensions, using the scales indicated.





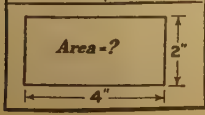
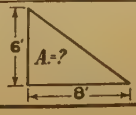
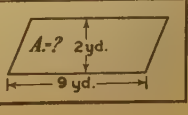
2. Find the volumes of the two rectangular solids shown at the left.

3. Find the area of a triangle whose base is 4 ft. and whose altitude is 7 ft.

4. Find the area of a parallelogram whose base is 4 yd. and whose altitude is 2 yd.

5. Tell the kind of angle each of these is:

228. Rapid Drill. (ORAL WORK.) There is no example or question below which should trouble a pupil who is well prepared to begin seventh-grade arithmetic. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$1.6+.8+.9$	200% of 450 = ?	*Best form of answer?* 4 ft. 16 in.	What is a checking account?	$.01\times100$	*Read:* XXIX
2.	$\frac{2}{3}+\frac{1}{4}$	250% = what decimal?	$600-426$	$\frac{1}{3}$ = what %?	$\frac{5}{4}$ = what %?	$2.00-1.47$
3.	$8\div10$	*Reduce:* $\frac{20}{4}$	18×0	Meaning of altitude.	.06 = what %?	$\frac{4}{5}\div3\frac{3}{5}=\frac{4}{5}\times\ldots$
4.	$4+0$	*Best form of answer?* $6\frac{24}{10}$	438×100	$304-59$	*Right ans.?* $46\overline{)276}$	What is "drawing to scale"?
5.	Find $\frac{1}{10}$ of 3.87	7.9×7	What question in % is asked? $\$74.24\times.045$	$3\frac{1}{2}\%$ = what decimal?	What is an acute angle?	Change dimensions to ft. 3 yd. 2 ft. $\times3\frac{1}{2}$ ft.
6.	.065 = what %?	Give some uses of discounts.	$\frac{1}{8}$ = what %?	$6\frac{2}{5}=\frac{?}{5}$	*What is wrong?* $.46\times.8=368$	$9-0$
7.	$\frac{1}{4}\times\frac{1}{4}$	$27+8+9$	$8-3\frac{3}{5}$	18×9	$48\overline{)472}$	$231-68$
8.						

SELF-TESTING DRILL NO. 27

229. Directions. Time allowed: 20 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Try to beat all your past records.**

1.
$$\begin{array}{r} 94501 \\ -16410 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 341490134 \\ 80008 \\ \hline 558159631 \end{array}$$

3. $49\overline{)1225}$

4. $.98\overline{).9310}$

5.
$$\begin{array}{r} 460 \\ \times 730 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 73 \\ 655 \\ 39 \\ 69 \\ 59 \\ 39 \\ 875 \\ 259 \\ 43 \\ \hline \end{array}$$

7. $9\frac{1}{2} - 3\frac{3}{10} = ?$

8. $1\frac{3}{5} \div 1\frac{1}{15} = ?$

9.
$$\begin{array}{r} 267 \\ \times 37 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 85.4 \\ \times 9.28 \\ \hline \end{array}$$

11. $11\frac{5}{6} + 5 + 3\frac{3}{4} = ?$

12. $\frac{3}{28}$ of $\frac{4}{15} = ?$

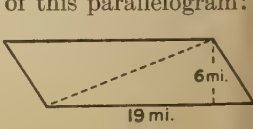
13. $42 - 21.138 = ?$

14. Reduce 1 yd. 2 ft. 7 in. to inches.

15.
$$\begin{array}{r} 4 \text{ ft. } 9 \text{ in.} \\ -3 \text{ ft. } 11 \text{ in.} \\ \hline \end{array}$$

16. $1\frac{1}{6} \div \frac{3}{5} = ?$

17. Find $\frac{1}{2}$ of the area of this parallelogram:



18. Find 4% of \$705.

19. 32 is what % of 192?

20. The Roman numeral XXXIX is the number.....

STANDARDS

Number correct..	0–3	4–5	6	7–8	9	10	11	12	13	14–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

PROBLEM SCALE NO. XX

230. Problems from U. S. History. (WRITTEN WORK.) You can earn 45 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. The State of Michigan contains about 36,000,000 acres of land. The State of Nebraska contains about 50,000,000 acres of land. About how many more acres does Nebraska have than Michigan? (3)

2. The Lincoln Highway runs from New York to San Francisco, through Chicago and Cheyenne. The approximate distances are as follows: New York to Chicago, 836 miles; Chicago to Cheyenne, 1085 miles; Cheyenne to San Francisco, 1386 miles. Find the total length. (3)

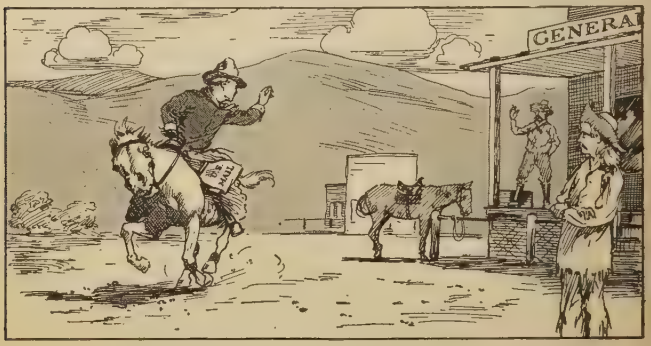
3. When Lincoln was hired out by his father, he earned about 31 cents a day. If there were 26 working days in a month, how much would his month's wages have been? (4)

4. The original area of the United States was 892,135 sq. mi. By 1900 it had increased to 3,742,870 sq. mi. How many square miles had we added to the original area? (4)

5. In 1925 the Governor of Iowa received an annual salary of $5000, an allowance of $600 for house rent, and $1200 for services as a member of the Executive Council. How much did he receive for his service to the State? (5)

6. The Fourteenth Census shows that Nebraska produced over 57,000,000 bushels of wheat in 1919. At $.95 per bushel what would the entire crop be worth? (6)

7. During the early days of our great Northwest, Doctor McLaughlin, the leader of the Oregon Country, ruled a territory 956 miles wide and 800 miles long. How many square miles did the territory contain? (6)



8. In 1861 President Lincoln's inaugural address was carried westward by the Pony Express. The entire trip of 2000 miles from St. Joseph, Missouri, to Sacramento, California, was made in 7 days (24 hr. per day) and 17 hours. About how many miles did the Pony Express average per hour? (7)

9. During 1919-1920, 30,198 dressed sealskins valued at $1,509,600 were shipped from the Prebyloff Islands to the United States. What did these skins bring on the average? (Figure to the nearest dollar.) (7)

STANDARDS

Superior work:	28–45 points	Slow work	10–14 points
Good work:	21–27 points	Inferior work:	0– 9 points
Average work:	15–20 points		

SELF-TESTING DRILL NO. 28

231. Directions. Time allowed: 20 minutes. Rate yourself. **Are you checking your work?**

1. 246×7

2. $9788 - 9169$

3. 647×94

4. $6.132 \div 4.2 = ?$

5. $5.8 \times 3.33 = ?$

6.
67
452
626
728
2
469
463
647
259

7. $.3635 \times 7.06$

8. $89.37 - 24.68$

9. $\frac{1}{7} \times \frac{1}{2} \times \frac{1}{2} = ?$

10. $5 \div \frac{1}{10} = ?$

11. $1\frac{1}{18} \div \frac{7}{9} = ?$

12. $58)2837$

13.
99
98
85
60
70
21
5084
78
17

14. $4 - 1\frac{2}{5} = ?$

15. $\frac{10}{13} \times 2\frac{1}{3} \times \frac{11}{15} = ?$

16.
2 qt. 1 pt.
$\times 9$

17.
$\frac{1}{8}$
$\frac{1}{3}$
$\frac{5}{12}$

18. What per cent of the area is dotted?

19. Write $\frac{5}{3}$ in the form of per cent.

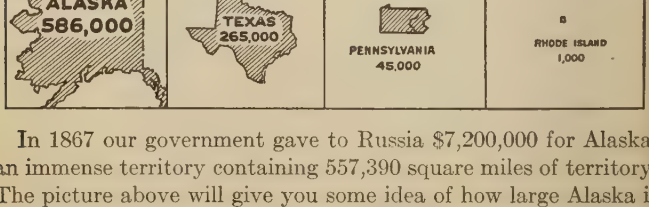
20. If 74 were increased by 10%, what would be the resulting number?

STANDARDS

Number correct...	0–3	4–5	6–7	8	9	10	11 12	13	14	15–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

PROBLEM SCALE NO. XXI

232. Problems about the Purchase of Alaska. (WRITTEN WORK.) You can earn 55 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.



In 1867 our government gave to Russia $7,200,000 for Alaska, an immense territory containing 557,390 square miles of territory. The picture above will give you some idea of how large Alaska is in comparison with certain states. The figures in the drawing represent approximate areas in square miles.

1. Is Alaska about 1, 2, 3, or 4 times as large as Texas, which contains 265,896 square miles? (3)

2. The United States by 1923 had taken $79,000,000 in furs, $400,000,000 in fish, and $450,000,000 in minerals from Alaska. What was the total value of these products? (4)

3. In 1900 the Government gave $25,000 to the reindeer stations in Alaska. Only $19,330 was used. How much was left? (4)

4. By 1924 how long had Alaska belonged to U. S.? (5)

5. Up to January, 1921, Alaska had produced $320,000,000 of gold since the discovery of gold at Juneau in 1880. How many more dollars in gold alone had Alaska produced from 1880 to 1921 than the amount the United States paid Russia for the territory? (5)

6. How much more have we gained from Alaska in furs, fish, and minerals, than the amount we paid for it? (6)

7. About how much per square mile did Alaska cost? (6)

8. Give the area of Alaska in acres. (1 sq. mi. = 640 A.) (7)

9. What is the average amount we have made each year from its products? (7)

10. What per cent of the 1920 reindeer fund was spent? (See Problem 3.) (8)

STANDARDS

Superior work:	30–55 points	Slow work:	8–11 points
Good work:	23–29 points	Inferior work:	0– 7 points
Average work:	12–22 points		

Pupils find problems more interesting when they know a good deal about the topics mentioned in the problems. Read over the problems about Alaska again. Can you think of interesting investigations that might be made concerning some of the things mentioned in these problems?

SELF-TESTING DRILL NO. 29

233. Directions. Time allowed: 20 minutes.

1. $284606255377 + 5424316$

2. $47603 - 45278$

3. 385×29

4. $.7094 - .5058 = ?$

5. 214×2.3

6. $1\frac{3}{2} \div \frac{1}{2} = ?$

7. 924×84

8.
316
216
316
329
357

9.
$6\frac{7}{8}$
$\frac{1}{3}$
$5\frac{3}{4}$
3
$\frac{1}{6}$
$7\frac{5}{12}$

10. $47\overline{)4367}$

11. $\frac{1}{2} \div 15 = ?$

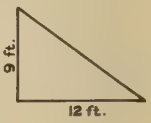
12. $89\overline{)730.5}$

Go on to the next page.

13. Find the interest on \$950 for 2 years @ 3%.

14. $1\frac{13}{50}$ minus $\frac{4}{25}$ = ?

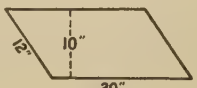
15. Find area of this right triangle:



16. $\frac{5}{16} \times \frac{8}{15}$ = ?

17. $6\overline{)9 \text{ ft. } 8 \text{ in.}}$

18. What is the area of this parallelogram?



19.
$$\begin{array}{r} \frac{9}{25} \\ \frac{1}{3} \\ \frac{7}{15} \\ \frac{2}{5} \\ \hline \end{array}$$

20. What amount is a 2% discount on \$79.50?

STANDARDS

Number correct.....	0–2	3	4	5–6	7	8	9	10	11	12–19	20
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Many pupils like to make a copy of their Progress Charts to keep in their "memory books." Do you think your parents would be interested in looking at your record of the year's work as shown on your Chart?

A BIRD'S-EYE VIEW OF ARITHMETIC

234. Processes Needed in Seventh-Grade Arithmetic. (Class Work.) On the next pages is a list of the processes or topics you will need to know in order to begin seventh-grade arithmetic. The list is divided into six parts. Do not be easy on yourself just because it is near the end of the school year. If you are uncertain about any process, now is the time to study it.

PART I. CAN YOU WORK WITH WHOLE NUMBERS?

Addition

1. 48931
   7062
   3008
   2140
   76309

In addition remember to carry the correct number, to add all the numbers, and to check your work. What is the sum of Example 1?

What articles in this book give good practice for those who wish to improve in addition?

Subtraction

2. 763508
   −28699

In subtraction, remember that numbers sometimes have to be changed in order to subtract. How is subtraction checked for accuracy? What is the difference of Example 2? Which is the minuend? Which is the subtrahend?

What articles in this book would you use for extra study and practice on subtraction if you were a little uncertain about this topic?

Multiplication

3. 2789
   ×608

Would a zero bother you in multiplication? Why is it a good plan to check examples in multiplication? What is the product of Example 3?

If a pupil needed some extra work in multiplication, what articles would you advise him to use?

Division

4. $49\overline{)295127}$

Quotient figures must be carefully studied in division. Do you understand how to write a remainder in decimal form? Can you depend on your ability to check your own work in division? What is the quotient of Example 4?

Suppose a friend of yours needed to "brush up" on division. What articles would you suggest for study and practice?

PART II. CAN YOU WORK WITH FRACTIONS?

Addition

5. $21\frac{1}{4}$
7
$24\frac{5}{6}$
$13\frac{2}{3}$

Can you find common denominators easily? Can you change fractions to equivalent ones having a common denominator? Have you a clear idea of how to add the sum of the fractions to the whole numbers? Are you careful about leaving fractions in answers in lowest form? What is the sum of Example 5?

Do you know the Self-Help Index well enough to use it to find articles for studying addition of fractions?

Subtraction

6. $21\frac{1}{6}$
$-13\frac{3}{8}$

In subtraction, fractions often have to be changed to forms having common denominators. Can you make the fraction $\frac{1}{6}$ in Example 6 larger in order to subtract? What is the difference of Example 6?

Make a list of the articles in this book which you think would be good ones to use in reviewing subtraction of fractions.

Multiplication

7. $4\frac{1}{2} \times 3\frac{1}{3} =$

Do you feel sure of your ability to change mixed numbers to improper fractions? Can you cancel easily? Should fractions in answers be reduced to good forms? How would you work this example? What is the product of Example 7?

What references to multiplication of fractions are listed in the Self-Help Index?

Division

8. $3\frac{1}{5} \div \frac{8}{15}$

What does "invert" mean? How do you invert a divisor which is printed as a mixed number? Is canceling ever used in division? What is the quotient of Example 8?

If your teacher asked you to help another pupil in division, what articles would you plan to use in giving this help?

PART III. CAN YOU WORK WITH DECIMALS?

Place Value

9. 288.76

In the number at the left, do the two 8's mean the same thing? Which is larger? How much larger is it? How would you multiply any number by 10? Could you divide by 100 without using pencil and paper?

If a pupil were uncertain about the place value of numbers, what articles in this book should he study?

Addition

10.
3.17+.06+4.7

What is the one thing to remember about the decimal point in addition? Can you copy and find the sum of Example 10 and be sure you are correct?

Subtraction

11. 17.
−8.008

Can subtraction be checked when the numbers used are decimals? Can you read the number 8.008 correctly? What is the difference of Example 11?

Multiplication

12. 36.7
×2.08

What is the rule for "pointing off" the number of decimals in the product? Can you check multiplication of decimals? What is the product of Example 12?

What articles in this book explain how to find the number of decimal places in a product?

Division

13. $.37\overline{)1.5096}$

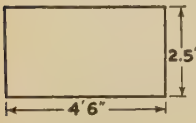
A sixth-grade pupil said that he remembered how to handle the decimal points in division by this sentence: "Shift, shift, place your point." What does that mean?

What is meant by the decimal form of remainder? How is division of decimals checked for accuracy? What is the quotient of Example 13?

Study the article references for division of decimals in the Self-Help Index to see if the references are to the right articles.

PART IV. CAN YOU FIND AREAS AND VOLUMES?

14. ***Areas of Rectangles***

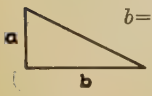


What is meant by the sentence: "Dimensions must be expressed in the same unit before multiplying"?

What kind of units of measure are used in expressing areas? What is the area of the rectangle at the left?

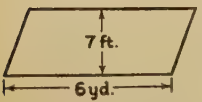
15. ***Areas of Rt.* △.**

$a = 3$ in.
$b = 6$ in.



Which two sides of a rt. △ are used as dimensions? What is the rule for finding the area of a rt. △? What is the area of the right triangle in Example 15?

16. ***Areas of Parallelograms***



What is meant by "altitude" of a parallelogram? What are parallel lines?

Could you show on the blackboard why a parallelogram has the same area as a rectangle which has the same base and altitude?

What is the area of the parallelogram in Example 16?

What articles in this book show you how to find the area of parallelograms?

17. ***Area of Any Triangle***

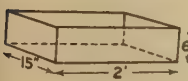
Why must we find the altitude of a triangle which is not a right triangle?

Can you show on the blackboard why any triangle can be thought of as half of some parallelogram?

What is the area of the triangle in Example 17?

What references to this topic are given in the Self-Help Index?

18. ***Volumes of Rectangular Solids***



How do we find the volume of a rectangular solid?

Can we multiply dimensions that are expressed in different units?

What units express volume?

What is the volume of the solid shown in Example 18?

PART V. CAN YOU WORK WITH DENOMINATE NUMBERS?

Addition

19.

2 ft. 6 in.
9 in.
4 ft. 2 in.

Do you know the Tables of Measure well enough to change the form of a denominate number correctly and easily?

What denominate number found by working the example at the left should be changed in form? What is the sum of Example 19?

Subtraction

20.

4 hr. 15 min.
−2 hr. 40 min.

In the example at the left what denominate number is changed in form but not in value in order to subtract?

What is the difference of Example 20?

Multiplication

21.

4 qt. 1 pt.
×7

What is the product of Example 21?

What articles in this book would you use for review if multiplication of denominate numbers troubled you?

Division

22.

$4\overline{)7 \text{ ft. } 4 \text{ in.}}$

What fact about the relation of feet and inches is it important to know in working the division example at the left?

What is the quotient of Example 22?

PART VI. CAN YOU WORK WITH PER CENTS?

Changing the Form of Per Cents

23. (*a*) $\frac{1}{2}$ = what %?
(*b*) $66\frac{2}{3}\%$ = what fraction?
(*c*) .045 = what %?
(*d*) $2 \times N$ = what % of N?
(*e*) 125% = what decimal?

Is the per cent form of a number ever used in actually working an example?

What do these sentences mean?

In arithmetic we use three languages. We often translate or change from one language to another. Thus, we translate per cents to equivalent decimals or fractions, and sometimes we change a number from a decimal form to a per cent form.

Finding a Per Cent of a Number

24. Find 4% of 936.

25. What is $7\frac{1}{2}$ % of 450?

26. 125% of 860 = ?

In working examples like those at the left, why do we sometimes change the per cent to a fraction and sometimes to a decimal?

Per cent of means $\times$ **decimal.**

What are the answers of Exs. 24, 25, and 26?

Finding What Per Cent One Number Is of Another

27. 6 is what % of 9?

28. 188.5 is what per cent of 725?

"Per cent of" means "fraction of."

What question in percentage is answered by this example: $\frac{634}{806}$ or 806)634?

Do you remember to change the quotient of examples like 27 and 28 to per cent form? Find the answers of Exs. 27 and 28.

Finding Interest

29. What is the interest on \$650 at 6% for one year?

30. Interest on \$500 at $4\frac{1}{2}\%$ for 1 yr. 6 mo. = ?

Is interest reckoned by the day, month, or year?

6% interest on \$100 for a year is what amount? For two years? For 6 mo.? What are the answers of Examples 29 and 30?

SELF-HELP STUDY AND PRACTICE LESSONS

235. How to Use the Self-Help Study and Practice Lessons. (CLASS WORK.) While you are doing sixth-grade work, you will often need to review some of the things you learned in the fourth and fifth grades. It will not be very often though, that all of the pupils in your class will need to review these things at the same time. Sometimes you will need to study and practice on some of these lessons while other pupils are working on different lessons. That is why these lessons are called **Self-Help Lessons.** You must learn how to help **yourself.**

Look at Article 237. What does it tell you about? Notice that it is for self-help **study.** You can study everything in this article without the help of your teacher.

Now look at Articles 243 and 249. They are also articles to be **studied.**

What is the name of Article 244? Do you see that it is for self-help **practice?** It gives you some good practice on what you **studied** in Article 243.

Important: Remember that all of these Self-Help Study and Practice Lessons are to be used by you to **help yourself.** You can't **help yourself** if you look at the answers in the exercises and in the Answer Key before you have written the answers to the examples in any article. Be fair in playing this "game" with **yourself.** If you are fair, you will find that you can teach yourself many things that you may have forgotten.

SELF-HELP LESSONS ON ADDITION

236. How to Use the Lessons on Addition. (SILENT READING.) Articles 236 to 240 are for pupils who want to help themselves improve in addition. The little chart below can be used in finding the kind of work you need.

> Art. 237 shows you six places in addition which often cause trouble. Study Art. 237 if you do not understand addition very well.
>
> Art. 238 gives you useful practice on "little" addition examples which are needed for accurate work. Use this article if you are troubled with mistakes when you add columns of figures.
>
> Arts. 239 and 240 give practice on harder examples. These articles give good practice for those who understand addition and can do very easy work but need more practice on harder kinds of work.

237. Six "Trouble Spots" in Addition. (FOR SELF-HELP STUDY.) You will probably read this article if you are having trouble with addition. Study it carefully, because it explains the "spots" where mistakes in addition are most likely to be made.

I.

Keep your place in the column.

3
4
4
4
7
7

In a long column it is easy either to skip a figure or to add a figure twice.

Some pupils find it a help to put their pencil point by the side of each figure as it is added.

II.

Carry the correct number.

36
25
85
93
64

3

What number is carried to the tens' column? Many mistakes are made by carrying a 1 when a 2 or even a 3 should be carried.

Always be careful to carry the correct number.

III.

Write the correct figure.

In Example II on page 360, the sum of the units' column is 23. This means you should write the 3 under the 4 and carry the 2.

What would you say to a pupil who kept doing the wrong thing, such as writing a 2 instead of the 3, or even forgetting to write any number at all?

IV.

Stay in the proper column.

36
25
85
93
64

3

After you have carried 2 to the 6, think 8 and then add 9; think 17 and add 8 to the 17; etc.

Some pupils let their eyes wander into the wrong column, and add numbers in the wrong column.

V.

Add all columns.

473
241
26
70
25
32

Some pupils will add the units' and tens' columns correctly, and then forget to add the hundreds', because there are no numbers near the bottom.

Always look all the way up the column farthest to the left in any example before leaving it. By doing this you can always make sure that you have not forgotten to add some numbers at the top of the last column.

VI.

When numbers are left out, don't be puzzled.

(*a*)	(*b*)	(*c*)
272	272	272
6	116	006
72	172	072
493	493	493

Look at Examples (*a*) and (*b*) above. Most pupils would find Example (*a*) the harder. When there is a gap, it means 0.

We do not use 0 at the left of whole numbers. But Example (*a*) above can be thought of as it is shown in Example (*c*).

238. "Little" Addition Facts. (For Self-Help Practice.) The addition facts in this article are harder ones that you find "hidden" in addition examples. You may have been having trouble with addition because you do not know these facts well enough. Practice on them makes

them easy. Put a slip of paper over the answers when you want to test yourself.

	(*A*)	(*B*)	(*C*)	(*D*)
1.	23+8=**31**	34+9=**43**	28+7=**35**	49+7=**56**
2.	11+9=**20**	18+4=**22**	38+2=**40**	29+2=**31**
3.	39+4=**43**	26+8=**34**	58+8=**66**	48+5=**53**
4.	16+0=**16**	49+9=**58**	27+3=**30**	14+6=**20**
5.	36+7=**43**	16+6=**22**	48+3=**51**	28+8=**36**
6.	72+9=**81**	29+1=**30**	16+9=**25**	37+9=**46**
7.	64+7=**71**	49+8=**57**	26+4=**30**	46+5=**51**
8.	65+9=**74**	28+6=**34**	15+7=**22**	25+8=**33**
9.	37+8=**45**	39+6=**45**	24+8=**32**	25+6=**31**
10.	17+4=**21**	43+9=**52**	33+7=**40**	22+8=**30**
11.	47+6=**53**	29+3=**32**	27+7=**34**	35+0=**35**
12.	27+5=**32**	39+5=**44**	38+9=**47**	87+9=**96**

239. Addition Examples with Answers. (For Self-Help Practice.) Every sixth-grade boy and girl should be able to find the sums of the addition examples in this article without help and without mistakes. **Cover the answers while working.** Let the answers really help you by using them as a check **after** the work is finished.

1.	2.	3.	4.	5.	6.
2					16
9	66	53	36	78	45
8	55	83	57	31	44
0	78	54	47	34	86
3	97	29	80	65	77
22	**296**	**219**	**220**	**208**	**268**

7.	8.	9.	10.	11.	12.
					873
516	87	20	290	62	305
136	29	899	752	210	879
135	92	541	630	709	334
179	97	164	919	110	827
825	18	768	759	537	269
1791	**323**	**2392**	**3350**	**1628**	**3487**

240. Helping Yourself in Addition. (For Self-Help Practice.) Can you work and check these examples? Art. 263, page 388, explains how to check addition.

1.	2.	3.	4.	5.
98	67	87	18	8865
72	524	6	30	1547
39	499	57	85	6978
48	747	88	69	2991
45		48	79	3368

6.	7.	8.
$89.79	7965	$8.56
4.95	6349	.37
45.55	1896	9.37
32.46	8167	2.84
34.26	2958	7.95
		5.66

☛ When you are through, compare your answers with those in the Answer Key on page 431. Correct all your mistakes.

An important question to ask yourself: Can I add well enough to do good sixth-grade work?

If you can work all of the examples in Art. 240 correctly in five minutes addition will not trouble you much in your sixth-grade arithmetic.

SELF-HELP LESSONS ON SUBTRACTION

241. How to Use the Lessons on Subtraction. (SILENT READING.) Articles 241-246 are for pupils who want to help themselves improve in subtraction. The little chart below can be used in finding the kind of work you need.

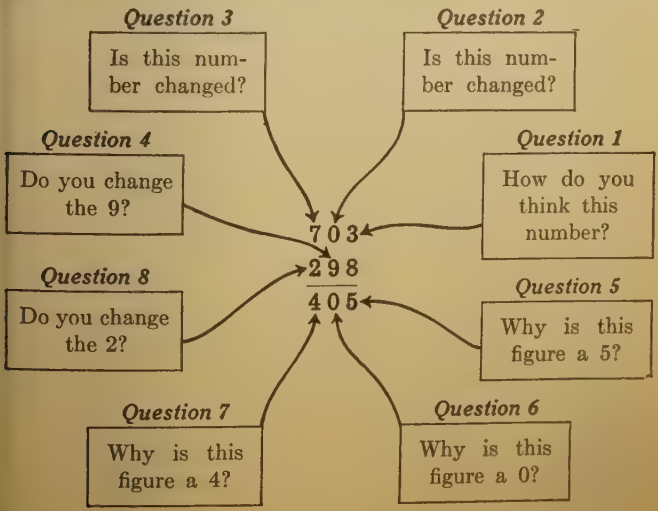
> Art. 242 gives practice on "little" subtraction examples needed in accurate work in harder examples.
>
> Art. 243 asks questions about things to do in hard examples.
>
> Arts. 244 to 246 give practice on harder examples.

242. "Little" Subtraction Examples. (FOR SELF-HELP PRACTICE.) If you make little mistakes in subtraction, study the examples below until each one is very easy for you. Cover answers with a slip of paper while practicing.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)	(*G*)	(*H*)
1.	19	26	31	20	27	30	30	27
	10	19	22	9	17	14	11	18
	9	**7**	**9**	**11**	**10**	**16**	**19**	**9**
2.	32	30	41	22	25	21	83	31
	14	16	25	9	19	14	48	23
	18	**14**	**16**	**13**	**6**	**7**	**35**	**8**
3.	35	40	31	32	26	78	38	32
	18	22	12	15	18	10	29	28
	17	**18**	**19**	**17**	**8**	**68**	**9**	**4**
4.	36	23	63	45	94	31	34	43
	36	15	59	30	27	17	19	27
	0	**8**	**4**	**15**	**67**	**14**	**15**	**16**

5.	35	54	30	32	62	34	51	45
	17	39	13	13	47	18	40	36
	18	**15**	**17**	**19**	**15**	**16**	**11**	**9**
6.	30	61	34	54	70	33	61	56
	20	48	25	26	58	24	39	37
	10	**13**	**9**	**28**	**12**	**9**	**22**	**19**
7.	63	70	59	62	31	30	47	47
	46	47	49	46	16	5	29	0
	17	**23**	**10**	**16**	**15**	**25**	**18**	**47**

243. Studying the "Hard Spots" in Subtraction. (For Self-Help Study.) Study these questions carefully, then ask your teacher to let you explain subtraction to her.



Explain how you get each figure in each difference below. The differences are all correct.

(*a*)

607
389
218

(*b*)

426
198
228

(*c*)

3002
1935
1067

244. Subtraction Examples with Answers. (For Self-Help Practice.) Notice that the answers are given in the examples below. Here is a way to use the answers so that they will really help you:

1. Study until you see **why** each answer is correct.
2. Then cover the answers and work the examples.
3. Compare your own answers with the printed ones. Be sure to make yourself correct any mistakes.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	124	$6.73	113	$3.05	128	856
	75	1.04	39	2.18	69	279
	49	**$5.69**	**74**	**$.87**	**59**	**577**
2.	100	723	$1.11	941	134	671
	61	318	.63	304	58	384
	39	**405**	**$.48**	**637**	**76**	**287**
3.	$8.63	883	$8.93	925	264	631
	6.79	785	8.08	479	186	555
	$1.84	**98**	**$.85**	**446**	**78**	**76**
4.	806	693	934	958	625	703
	429	197	679	203	466	403
	377	**496**	**255**	**755**	**159**	**300**

245. Further Subtraction Practice. (For Self-Help Practice.) Good work in the sixth grade requires that you work examples like the ones below correctly and easily.

Do not copy the examples, but put your paper under each row of four examples so that it will cover the answers. After you have finished your work, compare your answers with the printed ones.

1.	2.	3.	4.
76942	54952	53864	37870
73567	13870	25749	24928
3375	**41082**	**28115**	**12942**
5.	**6.**	**7.**	**8.**
8589	98202	45926	47977
1335	43456	19297	16119
7254	**54746**	**26629**	**31858**
9.	**10.**	**11.**	**12.**
23021	9586	81000	17547
20090	7546	54426	9865
2931	**2040**	**26574**	**7682**
13.	**14.**	**15.**	**16**
13970	11470	22256	157455
4859	7507	16858	378
9111	**3963**	**5398**	**157077**
17.	**18.**	**19.**	**20.**
11914	31052	94933	82647
2037	15048	90639	45088
9877	**16004**	**4294**	**37559**

246. Final Practice on Subtraction. (For Self-Help Practice.) Every sixth-grade pupil should be able to work these examples accurately and easily. The way to check subtraction is explained in Art. 263, page 388. Be careful to name answers when necessary.

1. 10580 5946	2. 8129 5590	3. 59650 24560	4. 5058 4701
5. 15332 13480	6. 5090 2338	7. 7217 1398	8. 57745 1328
9. 10632 2923	10. 15551 7962	11. 11313 1627	12. $121.85 76.01
13. 5236 1530	14. 17352 7108	15. $137.32 46.01	16. 11003 4699
17. 8774 4675	18. 6598 5729	19. 9624 8208	20. 12868 7648
21. 17879 9721	22. $59.41 35.54	23. 11946 9368	24. 10120 7785

☞ After you have finished, check your work by comparing your answers with those given on page 431.

An important question to ask yourself: Can I subtract well enough to do good sixth-grade work?

If you can work Art. 246 correctly in 7 minutes, subtraction will not trouble you much in sixth grade.

If you want to measure your ability to subtract, ask your teacher to let you take the Subtraction Test on page 5 again.

SELF-HELP LESSONS ON MULTIPLICATION

247. How to Use the Lessons on Multiplication. (SILENT READING.) Arts. 247 to 253 are given here for pupils who want to help themselves improve in multiplication. The little chart below will be useful in finding the kind of work that is needed.

> Art. 248 gives practice on the "little" examples which are needed for accurate work in harder multiplication.
>
> Arts. 249 and 250 explain how to work harder multiplication examples. Pupils who do not understand multiplication should study these articles carefully.
>
> Arts. 251-253 give good practice for those who understand how to multiply, but need more practice.

248. "Little" Examples Used in Multiplication. (FOR SELF-HELP PRACTICE.) The examples below and on the next page are "little" facts which are very helpful in "big" multiplication.

To work the "big" example at the right you must know the "little" facts beside it.

4×6	296
$4\times9+2$	$\times84$
$4\times2+3$	1184
8×6	2368
$8\times9+4$	24864
$8\times2+7$	

It will save time this year if you will practice on the "little" facts below and on the next page until each one is as easy for you as $2+2=4$. Cover the answers. The answers will help you most if you use them as a check of your own work.

(*A*)	(*B*)	(*C*)	(*D*)
$7\times7+6=\mathbf{55}$	$6\times3+5=\mathbf{23}$	$9\times2+7=\mathbf{25}$	$9\times8+7=\mathbf{79}$
$9\times4+6=\mathbf{42}$	$9\times3+7=\mathbf{34}$	$9\times8+6=\mathbf{78}$	$9\times6+6=\mathbf{60}$

(A)	(B)	(C)	(D)
$8\times7+3=59$	$9\times7+8=71$	$7\times6+5=47$	$8\times0+7=7$
$8\times9+5=77$	$9\times0+6=6$	$8\times0+4=4$	$9\times7+6=69$
$9\times7+3=66$	$9\times4+7=43$	$9\times8+8=80$	$9\times5+7=52$
$6\times9+4=58$	$9\times7+4=67$	$9\times1+8=17$	$8\times6+3=51$
$7\times6+4=46$	$8\times9+6=78$	$8\times9+7=79$	$9\times6+8=62$
$8\times7+5=61$	$9\times6+7=61$	$7\times5+6=41$	$9\times9+7=88$
$7\times8+6=62$	$8\times6+6=54$	$8\times7+4=60$	$9\times5+8=53$
$9\times9+5=86$	$9\times4+8=44$	$9\times9+6=87$	$8\times6+7=55$
$9\times0+4=4$	$5\times3+4=19$	$8\times7+6=62$	$9\times9+8=89$
$6\times2+5=17$	$9\times7+7=70$	$9\times3+8=35$	$6\times7+4=46$
$9\times5+6=51$	$8\times8+7=71$	$7\times0+5=5$	$6\times9+5=59$
$7\times6+3=45$	$7\times9+5=68$	$8\times1+6=14$	$9\times6+3=57$
$8\times8+5=69$	$9\times2+8=26$	$9\times3+4=31$	$8\times2+5=21$
$7\times7+4=53$	$7\times8+4=60$	$6\times3+4=22$	$6\times6+4=40$
$6\times8+5=53$	$9\times4+5=41$	$7\times4+5=33$	$5\times8+4=44$
$9\times9+4=85$	$8\times8+6=70$	$8\times2+6=22$	$9\times9+3=84$
$7\times4+6=34$	$8\times3+7=31$	$9\times5+5=50$	$8\times8+4=68$
$9\times8+4=76$	$9\times3+6=33$	$9\times8+3=75$	$8\times4+7=39$
$9\times6+5=59$	$9\times2+6=24$	$6\times8+4=52$	$8\times2+7=23$
$7\times7+5=54$	$7\times9+4=67$	$7\times9+6=69$	$4\times9+3=39$

249. Understanding Multiplication. (For Self-Help Study.) This article explains just how two numbers are multiplied to get the correct answer. Several examples are worked for you. Study these examples and notice which step or steps you have been doing incorrectly.

Example A below shows how to find the product of 78 and 36.

Multiply:	
(*A*)	78
	36
Add	468
	234
	2808

(*a*) First multiply 78 by 6. Think: Six 8's are 48. Write the 8 under the 6 and carry the 4. Then think: Six 7's are 42. 42 and 4 (carried) are 46. Write the 46.

(*b*) Now multiply 78 by 3. Think: Three 8's are 24. Write the 4 **under the 3** of the 36. Carry the 2. Then think: Three 7's are 21. 21 and 2 (carried) are 23. Write the 23.

(*c*) Add 468 and 234. The product is 2808.

Example B explains how to multiply 147 by 26.

(*a*) First multiply 147 by 6. Think: $6\times7=42$. Write the 2 under the 6 and carry the 4. $6\times4+4=28$. Write the 8 and carry the 2. $6\times1+2=8$. Write the 8.

(*b*) Now multiply 147 by 2. Think: $2\times7=14$. Write the 4 **under the 2** of the 26. Carry the 1. $2\times4+1=9$. Write the 9. $2\times1=2$. Write the 2.

(*c*) Add 882 and 294. The product is 3822.

Multiply:	
(*B*)	147
	26
Add	882
	294
	3822

Zero sometimes causes trouble. Example C shows how to handle zeros.

Multiply:	
(*C*)	201
	25
Add	1005
	402
	5025

(*a*) First multiply 201 by 5. Think: $5\times1=5$. Write this 5 below the 5 of the 25. $5\times0=0$. Be sure to **write the 0.** $5\times2=10$. Write the 10.

(*b*) Now multiply 201 by 2. Think: $2\times1=2$. Write this 2 **under the 2** of the 25. $2\times0=0$. Be sure to **write the 0.** $2\times2=4$. Write the 4.

(*c*) Add 1005 and 402. The product is 5025.

Always keep your work in **straight columns** when you multiply. Notice the dotted lines showing the straight columns in Example D. Don't let the zeros in Example D bother you.

Multiply:

(D)					
		4	2	6	
		3	7	5	
	2	1	3	0	
2	9	8	2		
1 2	7	8			
1	5	9	7	5	0

(*a*) Think: $5 \times 6 = 30$. Write the 0 **under the 5** of the 375 and carry the 3. Is the rest of the multiplication with the 5 correct?

(*b*) Think: $7 \times 6 = 42$. Write the 2 **under the 7** of the 375 and carry the 4. Is the rest of the multiplication with the 7 correct?

(*c*) Think: $3 \times 6 = 18$. Write the 8 **under the 3** of the 375 and carry the 1. Is the rest of the multiplication with the 3 correct?

(*d*) In this example you add three numbers (2130, 2982, and 1278) to get the product. Is the addition correct? What is the product?

When you multiply, always write the right-hand number of your product exactly *under* the number that you are multiplying by.

This is wrong.	*Why?*	*This is correct.*	*Why?*
27		27	
13		13	
81		81	
→27		27	
108		351	*Correct Answer.*

You have now reviewed the main "ideas" in multiplication. On the next page are some "stumbling blocks" that often make trouble for pupils. The examples explained on the next page may be just what you need to strengthen your "weak spots" in multiplication.

250. Studying Some "Hard Spots" in Multiplication. (FOR SELF-HELP STUDY.) Pupils are sometimes puzzled by examples like the ones in this article. Study the examples; then others like these will not puzzle you.

Example A has a zero in the multiplier.

	Long Way	*Short Way*
(*A*)	784	784
	206	206
	4704	4704
	000	15680
	1568	161504
	161504	

Multiply with the 6 as in any other example.

Long Way. Think: $0\times4=0$. Write 0 under the zero in 206. Think: $0\times8=0$. Write the 0. Think: $0\times7=0$. Write the 0. Then multiply by the 2 as in any other example.

Short Way. Write 0 under the zero of the 206. Then think: $2\times4=8$. Write the 8 under the 2 of the 206. Continue multiplying with the 2 as in any other example.

Add as you would do in any other example.

You multiply with zero again in Example B. This example is worked by the "short way" only.

Multiply:	
(*B*)	239
	420
	4780
	956
	100380

(*a*) Write 0 under the zero of the 420. Then think: $2\times9=18$. Write the 8 under the 2 of the 420 and carry the 1. Think: $2\times3=6$. $6+1$ (carried) $=7$. Write the 7. Think: $2\times2=4$. Write the 4. Notice that you have multiplied with the 2 in this example just as you would do in any other example.

(*b*) Multiply with the 4 just as you would do in any other example.

(*c*) Add as in any other example. What is the product?

Learn the "short way" of multiplying with zero. It is quicker and easier than the "long way."

251. Easy Multiplication with Answers. (FOR SELF-HELP PRACTICE.) You should practice on the work below until it is very easy for you. Cover the answers with a slip of paper when you are working the examples.

Of course, this practice will not help you much if you use the answers while you are working. **After** your work is finished, compare your answers with the printed ones.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	314	501	415	916	527	72¢
	8	7	9	6	8	6
	2512	**3507**	**3735**	**5496**	**4216**	**$4.32***
2.	308	$ 8.37	709	326	207	470
	8	6	5	7	9	7
	2464	**$50.22**	**3545**	**2282**	**1863**	**3290**
3.	609	405	936	$ 2.89	709	708
	8	6	9	7	7	9
	4872	**2430**	**8424**	**$20.23**	**4963**	**6372**

*$6\times72¢=432¢$, but $4.32 is a better form for the answer.

252. Multiplication Examples with Answers. (FOR SELF-HELP PRACTICE.) The answers printed in this exercise will help you to find your mistakes if you make any.

When you practice, place your paper over the answers or copy some examples carefully and close your book.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	346	$5.79	389	217	380
	8	7	14	32	67
	2768	**$40.53**	**5446**	**6944**	**25460**

	(A)	(B)	(C)	(D)	(E)
2.	120	608	436	9728	857
	71	95	321	906	158
	8520	**57760**	**139956**	**8813568**	**135406**
3.	8905	291	1364	564	$71.09
	203	659	507	496	804
	1807715	**191769**	**691548**	**279744**	**$57156.36**

253. Final Practice in Multiplication. (For Self-Help Practice.) Every sixth-grade pupil should be able to work the examples below accurately and easily. The way to check multiplication is explained in Art. 263, page 388.

1. 683 247	2. 902 151	3. 147 328	4. 289 140	5. 547 496
6. 863 158	7. 289 579	8. 702 134	9. 145 605	10. 360 229
11. 291 369	12. $8.25 582	13. 459 748	14. 607 478	15. 461 407
16. 608 136	17. 91 823	18. 76¢ 350	19. 387 279	20. $3.54 307

☛ After you have worked and checked every example, compare your answers with those given in the Answer Key on page 432.

An important question to ask yourself: Can I multiply well enough to do good work in the sixth grade?

If you can work 11 examples in Art. 253 correctly in 10 minutes, multiplication will not bother you in sixth grade.

SELF-HELP LESSONS ON DIVISION

254. How to Use the Lessons on Division. (SILENT READING.) Arts. 254-262 contain review work on division. The chart below will help you to find the lessons on division.

> Arts. 255 and 256 review short division.
>
> Art. 257 explains how to use the four steps in long division to solve a division example.
>
> Arts. 258 and 259 explain how to find quotient figures and give practice.
>
> Art. 260 explains zeros in the quotient and gives practice.
>
> Arts. 261 and 262 give general practice on division for those who understand how to divide but need more practice.

255. Finding Errors in Uneven Divisions. (FOR SELF-HELP PRACTICE.) In this article **some of the remainders are wrong.** It would seem easy so to subtract that each remainder would be found correctly. But some pupils make mistakes in the subtracting. Can you find **every incorrect remainder?** Compare your list with the list on page 432.

The remainders are the small figures in the quotients. List the wrong ones by their column letters and row numbers.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)	(*G*)
1.	9^{2} 4)38	9^{4} 5)49	6^{3} 4)27	2^{3} 6)15	3 6)18	3^{4} 7)25	3^{6} 9)33
2.	4 7)28	3^{2} 6)20	5^{4} 9)50	4^{5} 7)34	3^{4} 6)22	7^{6} 8)62	5^{5} 7)41
3.	8^{6} 8)70	4^{3} 7)31	5^{5} 6)35	3^{8} 9)35	4^{6} 7)33	7^{5} 6)47	6^{3} 8)51

	(A)	(B)	(C)	(D)	(E)	(F)	(G)
4.	8^{3} 6)51	7 7)50	7^{3} 8)60	9^{3} 6)57	5^{7} 9)52	7^{5} 7)54	7 6)42
5.	2^{6} 8)22	3^{7} 9)34	9^{5} 6)59	4^{8} 9)44	7 7)51	8 8)64	8^{6} 7)62
6.	6^{7} 9)61	9^{7} 8)79	7^{8} 7)55	5^{6} 9)51	6^{7} 8)55	8^{5} 7)61	8^{8} 9)80
7.	5^{8} 9)53	7^{2} 8)61	9 7)64	9^{5} 9)86	6^{5} 8)53	7^{9} 9)71	7^{1} 8)56

256. Easy Division Practice. (For Self-Help Practice.) Lay the edge of your paper **above** the examples to cover the quotients, and write the answers. Practice until every example here is **very** easy.

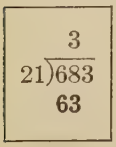
	(A)	(B)	(C)	(D)	(E)	(F)
1.	30 6)180	323 3)969	51 5)255	71 4)284	\$1.11 7)\$7.77	31 9)279
2.	52 2)104	\$.30 5)\$1.50	41 6)246	81 7)567	\$.80 8)\$6.40	52 4)208
3.	\$ .41 8)\$3.28	70 7)490	71 9)639	91 6)546	51 7)357	71 6)426
4.	69 6)414	\$1.01 5)\$5.05	79 4)316	\$.49 7)\$3.43	89 8)712	58 7)406
5.	35 9)315	\$2.00 6)\$12.00	38 5)190	49 8)392	78 7)546	203 3)609

257. How to Work Long Division. (For Self-Help Study.) The example $683 \div 21$ is worked in this article.

Step 1. Divide.

(*a*) *What do you divide?* See how many 21's there are in 68. Since there are three 2's in 6, there are three 21's in 68.

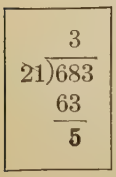
(*b*) You write the 3 over the 8, since it is **68** that is divided by 21.



Step 2. Multiply.

(*a*) *What do you multiply?* 3×21, because 3 shows how many 21's to take. Write the product 63 as shown.

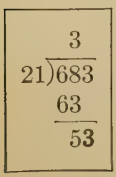
(*b*) *Judge.* Is the product (63) small enough? You must be able to subtract 63 from 68, or your work is wrong.



Step 3. Subtract.

(*a*) *What do you subtract?* 63 from $68 = 5$; write the 5 as shown.

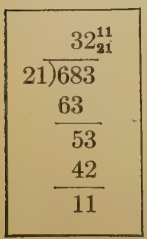
(*b*) *Compare.* Is the remainder (5) smaller than the divisor? The remainder must be smaller than 21, or the work is wrong.



Step 4. Bring Down.

(*a*) *What do you bring down?* **3**, the next figure in the number you are dividing (683), must be written after the 5.

(*b*) *Begin again.* Now you must divide 53 by 21. This is done below.



Steps 1-3 are like Steps 1-3 above.

Step 1. $5 \div 2$. Quotient figure is 2.

Step 2. $2 \times 21 = 42$.

Step 3. $53 - 42 = 11$.

Step 4. There is no figure to bring down. Express the remainder (11) in fractional form in the quotient by writing it over the divisor (21).

FINDING QUOTIENT FIGURES

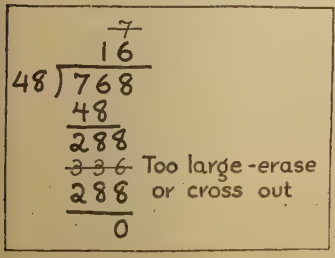
258. How to Find Quotient Figures. (For Self-Help Study.) Some quotient figures are not so easy to find as those you have studied. You probably will find this is true when you try to get the quotient figures in Sample (*A*). See if you can tell how to finish Sample (*A*).

Sample (A)

```
   1?
48)768
   48
  288
```

The first figure in the quotient is easy, but you must then find how many 48's there are in 288. It looks as if there might be 7, for there are seven 4's in 28.

But $7 \times 48 = 336$, which cannot be subtracted from 288. If, then, 7×48 is too large, try 6×48; this gives you 288, which can be subtracted from 288. Use 6, then, as the correct quotient figure, and multiply.



Notice the sample at the left. It shows a good way to arrange your work.

When you try 7 as the quotient figure, write it small and very lightly above the correct place; also write the 336 which you get by multiplying, very lightly, as shown.

When you see that the 7 and the 336 are too large, cross them out or erase them. Then write 6 in the quotient and the 288 under the 288.

Later you will be able to find difficult quotient figures by thinking correctly, without writing them lightly. But until you know well how to find quotient figures, it is helpful to write the **trial** figure in the quotient. When you find the

correct quotient figure to use, it is called the **true** or **correct quotient figure.**

Study Samples (*B*), (*C*), and (*D*) below, and see if you understand every step shown in the work. Check the answer to each sample.

SAMPLE (*B*)

$$\begin{array}{r} 3 \\ 72\tfrac{43}{54} \\ 54\overline{)3931} \\ 378 \\ \hline 151 \\ 162 \quad \text{Too large, Erase} \\ 108 \\ \hline 43 \end{array}$$

SAMPLE (*C*)

$$\begin{array}{r} 4 \\ 39\tfrac{12}{65} \\ 65\overline{)2547} \\ 260 \quad \text{Too large, Erase} \\ 195 \\ \hline 597 \\ 585 \\ \hline 12 \end{array}$$

SAMPLE (*D*)

$$\begin{array}{r} 69 \\ 58\tfrac{29}{78} \\ 78\overline{)4553} \\ 468 \quad \text{Too large, Erase} \\ 390 \\ \hline 653 \\ 702 \quad \text{Too large, Erase} \\ 624 \\ \hline 29 \end{array}$$

In Sample (*B*) 3 was a trial quotient figure because 5 into 15 goes 3 times. But 3 is too large, because $3 \times 54 = 162$. 162 is larger than 151.

In Sample (*C*) you know that the trial quotient figure 4 is too large, because $4 \times 65 = 260$. 260 is larger than 254.

In Sample (*D*) both trial figures were too large.

☛Often you will write a figure in the quotient above the place where it belongs, and then, when you multiply, you will find that the figure you have written is correct.

You should then write it in its proper place and cross out or erase the first figure that you wrote.

This sentence is worth remembering:

In finding quotients, if the trial quotient figure which you use is too large, try the next smaller figure.

259. Further Study of Hard Quotient Figures. (For Self-Help Study and Practice.) The samples in this article give help in finding the right quotient figures to use.

Remember:

1. First try the quotient figure which seems right.
2. If it is too large, try the next smaller one.
3. Keep coming down 1 at a time until you get the correct figure.
4. Do as much of the work as you can without writing. You will save erasing.

Study these examples:

Sample (*A*)

7	Think, but
6	do not
5	write:
29)145	7×29=203
145	6×29=174

Sample (*B*)

9	Think, but
8	do not
7	write:
38)277	9×38=342
266	8×38=304

Sample (*C*)

7	Think, but
6	do not
5	write:
4	7×17=119
17)71	6×17=102
68	5×17= 85

See if you understand every part of these samples:

Sample (*D*)

9
58
$47\frac{11}{38}$
38)1797
152
277
266
11

Sample (*E*)

7
6
53
42
17)714
68
34
34
0

Sample (*F*)

8 7
726
615
29)17835
174
43
29
145
145
0

If you understand the steps in division and how to get the correct quotient, the drill in this article will make quotient-finding easier. Practice finding the correct quotients.

EXERCISE (*A*)

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	13)68	44)174	83)496	61)487	23)178
2.	93)370	43)168	82)572	14)157	52)362
3.	67)364	91)726	42)251	53)316	94)843
4.	63)501	51)407	72)646	41)284	73)359
5.	31)247	71)492	64)190	33)229	43)171
6.	59)358	48)291	26)177	67)481	49)347
7.	28)118	39)239	77)632	68)485	79)637
8.	37)278	28)174	18)58	39)278	29)178
9.	19)78	17)86	19)98	16)94	47)348

☛ You will find the correct quotients on page 432.

EXERCISE (*B*)

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	59)538	28)260	86)808	19)179	77)718
2.	47)431	69)628	17)154	88)806	39)356
3.	58)418	49)394	37)297	26)157	66)557
4.	76)635	68)558	79)633	77)635	17)69
5.	39)278	29)176	18)78	16)88	26)191
6.	17)98	88)806	49)444	37)357	19)96

7.	$67\overline{)610}$	$36\overline{)321}$	$57\overline{)511}$	$23\overline{)205}$	$46\overline{)413}$
8.	$16\overline{)142}$	$29\overline{)238}$	$28\overline{)236}$	$19\overline{)158}$	$39\overline{)317}$
9.	$62\overline{)430}$	$84\overline{)754}$	$14\overline{)57}$	$54\overline{)482}$	$86\overline{)573}$

☛ You will find the correct quotients on pages 432 and 433.

260. Zero in the Quotient. (For Self-Help Study and Practice.) Zero (0) in the quotient sometimes troubles pupils. Do you really understand the work below?

PART 1

Study Examples (*A*) to (*E*) in this article, and see if you can decide what to do. Watch zeros closely here.

Example (A)

```
        6
    46)27738
New  276
Dividend→13
```

```
        ↓
       60
    46)27738
      276
       13
```

```
       603
    46)27738
      276
      138
      138
        0
```

In Example (*A*), after 3 is brought down from the dividend, we have 13 as a **new dividend.**

But 46 will not go into 13 even once. We must show this fact in the quotient. Notice the use of 0 in the quotient.

We do not need to multiply by the 0, but we bring down the 8 next. 46 will go into **138;** so you can finish the example as shown.

Copy Example (*B*) on a piece of paper and see if you can finish it by yourself. Check your answer to see if it is right.

Example (B)

```
        8
    73)58692
New  584
Dividend→ 29
```

Study these three rules. Then study the work in Examples (*C*), (*D*), and (*E*) below. Notice how each example follows the rules.

1. If the divisor will not go into the new dividend, be sure to write 0 in the quotient.
2. Then bring down the next figure of the dividend.
3. The last figure of the quotient (not counting the remainder) must always be above the last figure of the dividend.

(*C*) $72\overline{)65232}$ — 906; 648; 432; 432; 0

(*D*) $84\overline{)25284}$ — 301; 252; 84; 84; 0

(*E*) $58\overline{)23497}$ — $405\frac{7}{58}$; 232; 297; 290; 7

When you understand every step in the examples you have just studied, work the next three examples. There are zeros in the quotients. Watch for them. Use the answers below **after** you have finished your work.

1. $43\overline{)34486}$ 2. $65\overline{)32955}$ 3. $97\overline{)87995}$

Answers: 1. 802 2. 507 3. $907\frac{16}{97}$

Practice with Zero in the Quotient

If you think carefully, you will have no trouble with these examples and problems. Some have remainders.

1. $67\overline{)40334}$ 2. $73\overline{)15045}$ 3. $92\overline{)46828}$ 4. $89\overline{)53700}$

5. Mr. Harvey hauled to town some corn that weighed 11,540 pounds. A bushel of corn weighs 56 pounds. How many bushels of corn did Mr. Harvey haul? How many pounds over?

6. The Martin Motor Sales Company sold 82 cars last month. These cars cost them $66,010. If all the cars were alike, what did each one cost?

☛ The answers are given on page 433.

PART 2

Final Zero in Quotient

Examples (*A*), (*B*), and (*C*) on this page and the next need careful study. They show you a new thing to watch for in long division. Make your later work easy by understanding these examples now.

(*A*)

$$\begin{array}{r} 370 \\ 74\overline{)27380} \\ 222 \\ \hline 518 \\ 518 \\ \hline 0 \end{array}$$

In Sample (*A*), after we subtract 518 from 518, notice that there is still a number to be brought down. In this example it is 0.

74 will not go into 0. Notice the **0** in the quotient to show this. The last figure of the quotient is always over the last figure of the dividend.

(*B*)

$$\begin{array}{r} 370\frac{15}{74} \\ 74\overline{)27395} \\ 222 \\ \hline 519 \\ 518 \\ \hline 15 \end{array}$$

In Sample (*B*) we put 0 in the quotient to show that 74 will not go into 15. 15 is the **last new dividend.** Remember what to do with the remainder, 15.

Be sure to write 0 in the quotient before you write the remainder.

(C) $370\frac{5}{74}$

$74\overline{)27385}$

222

518

518

5

Sample (*C*) also has 0 in the quotient. It shows that 74 will not go into 5. 5 is the **last new dividend.**

Remember that you must write 0 above the last figure in the dividend **before** you write the fraction part of the quotient.

Study Samples (*A*), (*B*), and (*C*) again to see if the rules below would be good ones.

When the divisor will not go into the last new dividend:

1. Write 0 in the quotient.

2. Write the remainder, if any, in the quotient.

3. See that the last figure of the quotient (not counting the remainder) is above the last figure of the dividend.

4. Take time to check your work.

Use the rules above and work the following examples. Use the answers below **after** you have finished your work.

1. $62\overline{)35960}$ 2. $83\overline{)79687}$ 3. $57\overline{)23961}$

Answers: 1. 580 2. $960\frac{7}{83}$ 3. $420\frac{7}{19}$

Practice with Final Zeros in the Quotient

1. $54\overline{)44820}$ 2. $93\overline{)26992}$ 3. $89\overline{)55180}$ 4. $66\overline{)47525}$

5. An army airplane made a flight of 2730 miles in 21 hours. How many miles per hour did the plane average?

6. A book store sold 36 geographies for $50.40. The books were the same price. What did each one sell for?

☛ The answers are given on page 433.

261. Helping Yourself in Division. (For Self-Help Practice.) Remember to check examples. How to check division is explained in Art. 263.

1. $67\overline{)5561}$	6. $23\overline{)1331}$	11. $85\overline{)65690}$	16. $36\overline{)27655}$
2. $46\overline{)1597}$	7. $59\overline{)3427}$	12. $79\overline{)55930}$	17. $18\overline{)4554}$
3. $38\overline{)2774}$	8. $35\overline{)2021}$	13. $39\overline{)1440}$	18. $569\overline{)42675}$
4. $10\overline{)269}$	9. $57\overline{)4798}$	14. $27\overline{)1819}$	19. $15\overline{)6958}$
5. $46\overline{)2438}$	10. $14\overline{)7196}$	15. $92\overline{)82064}$	20. $47\overline{)343489}$

☛ Compare your answers with those on page 433.

262. Final Practice in Division. (For Self-Help Practice.) Can you work these examples correctly and easily?

1. $17\overline{)819}$	7. $84\overline{)5208}$	13. $38\overline{)3382}$	19. $12\overline{)276}$
2. $52\overline{)3848}$	8. $59\overline{)3599}$	14. $70\overline{)5040}$	20. $60\overline{)9600}$
3. $60\overline{)2400}$	9. $40\overline{)3800}$	15. $31\overline{)1557}$	21. $14\overline{)994}$
4. $69\overline{)6003}$	10. $51\overline{)4912}$	16. $63\overline{)1638}$	22. $40\overline{)3320}$
5. $59\overline{)1369}$	11. $72\overline{)43272}$	17. $95\overline{)38760}$	23. $52\overline{)4420}$
6. $38\overline{)2774}$	12. $83\overline{)3403}$	18. $27\overline{)24313}$	24. $96\overline{)5671}$

☛ The answers to the examples are given on page 433.

An important question to ask yourself: Can I divide well enough to do good work in the sixth grade?

If you can work 12 of the examples in Art. 262 correctly in 20 minutes, division will not trouble you much in the sixth grade.

CHECKING FOR ACCURACY

263. Checking. (For Self-Help Study.) This article shows you how to make fairly sure that your arithmetic work is done correctly. Pupils who are careful workers take time to check their work.

Check your addition like this:

*1872** (*Check answer*)
237
89
634
912
1872* (*First answer*)

First: Add **upward** and write answer below the bottom line.

Second: Add **downward** and write the answer above the columns. When the two numbers marked with stars (*) are the same, you may be pretty sure you have made no mistakes.

Check your subtraction like this:

Subtract: 3274*

327 } Check by
2947 } addition.

*3274**

When the two starred (*) numbers are the same, you may be pretty sure that you have the correct answer.

Check your multiplication like this:

The Work	*The Check*
47	24
24	47
188	168
94	96
1128*	*1128**

When the two starred (*) numbers are the same, you may be pretty sure the work is correct.

Check your division like this:

The Work	*The Check*
$41\frac{2}{27}$	$41\frac{2}{27}$
$27\overline{)1109^*}$	$\times 27$
108	287
29	82
27	1107
2	$+2$
	*1109**

FIRST: Multiply the whole number part of the quotient by the divisor.

SECOND: To this product, add the numerator of the unreduced fractional part of the quotient.

WORDS USED IN ARITHMETIC

264. Review of Vocabulary. (FOR SELF-HELP STUDY.) It is worth while to review now some words that are often used in arithmetic. Study these words as they are shown below. Then answer the questions on the next page.

1.

248
72
316
636←***Sum*** (*Answer*)

2.

4367←***Minuend***
−2136←***Subtrahend***
2231←***Difference*** (*Ans.*)

3.

476←***Multiplicand***
×24←***Multiplier***
1904
952
11424←***Product*** (*Answer*)

4.

$206\ \frac{19}{24}$←***Quotient*** (*Ans.*)
Divisor→$24\overline{)4963}$←***Dividend***
48
163
144
19←***Remainder***

EXERCISE

Answer the 12 questions that are asked below these examples:

1. $8-2=6$ 2. $14\div7=2$ 3. $4\times5=20$ 4. $9+7=16$

5.
$$\begin{array}{r} 64\tfrac{1}{32} \\ 32\overline{)2049} \\ 192 \\ \hline 129 \\ 128 \\ \hline 1 \end{array}$$

6.
$$\begin{array}{r} 37 \\ \times 13 \\ \hline 111 \\ 37 \\ \hline 481 \end{array}$$

7.
$$\begin{array}{r} 16 \\ 21 \\ \hline 50 \\ 8 \\ \hline 95 \end{array}$$

8.
$$\begin{array}{r} 8937 \\ -4016 \\ \hline 4921 \end{array}$$

QUESTIONS

1. What is the name of the 6 in Example 1?
2. What is the 2049 in Example 5 called?
3. Which number is the minuend in Example 8?
4. What name is given to the 20 in Example 3?
5. The 37 in Example 6 is called what?
6. What is the 4016 in Example 8 called?
7. What is the 32 in Example 5 called?
8. Which number is the multiplier in Example 6?
9. The 95 in Example 7 is given what name?
10. What is the 4921 in Example 8 called?
11. What name is given to the $64\frac{1}{32}$ in Example 5?
12. In Example 5, the last 1 (below the 128) is called what?

☞ Compare your answers to these questions with the answers printed on page 433.

ADDITION OF FRACTIONS AND SMALL MIXED NUMBERS

265. How to Use the Lessons on Addition of Fractions and Small Mixed Numbers. (SILENT READING.) The following chart tells where to find the different Self-Help Lessons on addition of fractions and small mixed numbers.

Arts. 266 and 267 explain common denominators.
Arts. 268 and 276 give practice in finding common denominators.
Arts. 269 and 271 explain the reducing of fractions in answers.
Arts. 270 and 272 give practice in reducing fractions in answers.
Art. 273 explains how to add mixed numbers.
Art. 274 gives practice in adding mixed numbers.
Art. 275 gives practice on changing the form of fractions.

266. Working with Common Denominators. (FOR SELF-HELP STUDY.) Here are two examples in fractions where you have to find common denominators.

(*A*) $\frac{1}{2}+\frac{1}{4}=?$

In working Example (*A*) we change $\frac{1}{2}$ to $\frac{?}{4}$s. $\frac{1}{2}=\frac{2}{4}$. The example then becomes $\frac{2}{4}+\frac{1}{4}=?$ The answer is $\frac{3}{4}$. The denominator of one of the fractions in the example ($\frac{1}{4}$) is the common denominator.

(*B*) $\frac{1}{2}+\frac{1}{3}=?$

We cannot use either $\frac{?}{2}$s or $\frac{?}{3}$s for the common denominator in working Example (*B*) because we cannot change $\frac{1}{2}$ to $\frac{?}{3}$s or $\frac{1}{3}$ to $\frac{?}{2}$s. We can change both $\frac{1}{2}$ and $\frac{1}{3}$ to $\frac{?}{6}$s. This is the best common denominator to use. $\frac{1}{2}=\frac{3}{6}$. $\frac{1}{3}=\frac{2}{6}$. $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

In adding fractions with different denominators, sometimes **both** denominators have to be changed to some

denominator that is not given. Such a denominator is also called a **common** denominator.

267. The Rule for Finding Common Denominators. (For Self-Help Study.) Sometimes it is hard to see just what the common denominator ought to be in an example. In such cases use the rule below.

> *Rule:* **To find a common denominator multiply the denominators. The product will always be a common denominator.**

What is the common denominator that you would use in this example? ⟶ $\frac{1}{8}+\frac{2}{3}=?$

What does the rule tell you to do? Can $\frac{1}{8}$ and $\frac{2}{3}$ be changed to $\frac{}{24}$? See if the following is done correctly:

$$\frac{1}{8}+\frac{2}{3}=?$$

$$\frac{1}{8}=\frac{1\times 3}{8\times 3}=\frac{3}{24} \qquad \frac{2}{3}=\frac{2\times 8}{3\times 8}=\frac{16}{24} \qquad \frac{3}{24}+\frac{16}{24}=\frac{19}{24}$$

$$\text{So, } \frac{1}{8}+\frac{2}{3}=\frac{19}{24}$$

268. Practice in Finding Common Denominators. (For Self-Help Practice.) If you can tell what denominators to use in Examples 1 to 10, common denominators will not trouble you much after this. When you are through, compare your answers with those on page 434.

1.	2.	3.	4.	5.
$\frac{}{2}+\frac{}{3}+\frac{}{5}$	$\frac{}{4}-\frac{}{16}$	$\frac{}{7}+\frac{}{2}+\frac{}{14}$	$\frac{}{6}-\frac{}{2}$	$\frac{}{5}+\frac{}{6}+\frac{}{15}$
6.	**7.**	**8.**	**9.**	**10.**
$\frac{2}{9}-\frac{4}{11}$	$\frac{}{12}+\frac{}{16}$	$\frac{}{40}-\frac{}{20}$	$\frac{}{15}+\frac{}{2}+\frac{}{6}$	$\frac{}{9}+\frac{}{7}$

Work the following examples, and then compare your answers with those on page 434. Correct all mistakes.

11. $\frac{1}{2}+\frac{1}{4}=$ 16. $\frac{1}{3}+\frac{1}{6}=$ 21. $\frac{1}{2}+\frac{1}{6}=$

12. $\frac{1}{2}+\frac{1}{8}=$ 17. $\frac{1}{7}+\frac{1}{14}=$ 22. $\frac{1}{3}+\frac{1}{12}=$

13. $\frac{1}{5}+\frac{1}{10}=$ 18. $\frac{1}{4}+\frac{1}{6}=$ 23. $\frac{1}{3}+\frac{1}{8}=$

14. $\frac{1}{4}+\frac{1}{16}=$ 19. $\frac{5}{6}-\frac{3}{4}=$ 24. $\frac{81}{6}-\frac{75}{8}=$

15. $\frac{3}{5}-\frac{1}{3}=$ 20. $\frac{4}{9}+\frac{1}{3}=$ 25. $\frac{2}{3}+\frac{3}{4}=$

269. Reducing Fractions in Answers. (For Self-Help Study.) In adding fractions watch **forms of answers.** Answers should always be reduced to simplest, or lowest, terms.

1. $\frac{1}{2}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$, *Answer.* Let us agree to change $\frac{4}{6}$ to $\frac{2}{3}$, for $\frac{2}{3}$ is $\frac{4}{6}$ in lowest terms.

2. $\frac{2}{3}+\frac{5}{6}=\frac{9}{6}=1\frac{3}{6}=1\frac{1}{2}$, the correct form of the answer.

3. $4\frac{1}{12}+1\frac{1}{4}+5\frac{4}{12}=10\frac{8}{12}=10\frac{2}{3}$, *Answer.*

4. $5\frac{1}{2}+2\frac{1}{4}+1\frac{1}{8}+3\frac{1}{8}=11\frac{8}{8}$. $\frac{8}{8}=1$. $11+1=12$. The best form for the answer to this example is 12.

Reducing fractions to lowest terms must not change the **value** of the fractions. Thus, in Example 1 above, $\frac{4}{6}$ has the same value as $\frac{2}{3}$, but $\frac{2}{3}$ is easier to think about than $\frac{4}{6}$. To reduce a fraction to lowest terms means: "express it in simplest form"; that is, so that the numerators and denominators are the **smallest possible whole numbers.**

> **A fraction is in lowest terms when the numerator and denominator cannot be divided without remainder by the same whole number (except 1).**

Caution: Sometimes you may have to divide both terms more than once. Thus, $\frac{12}{30}=\frac{6}{15}=\frac{2}{5}$.

270. Practice in Reducing Answers. (For Self-Help Practice.) Change the examples below to correct forms for final answers. Look on page 434 to check.

1. $\frac{4}{6}$ 3. $8\frac{16}{6}$ 5. $\frac{8}{8}$ 7. $1\frac{4}{10}$ 9. $\frac{9}{3}$ 11. $4\frac{9}{6}$

2. $2\frac{3}{2}$ 4. $6\frac{5}{3}$ 6. $7\frac{2}{4}$ 8. $3\frac{6}{4}$ 10. $5\frac{11}{5}$ 12. $3\frac{3}{6}$

271. Changing Improper Fractions to Good Forms for Answers. (For Self-Help Study.) Changing an improper fraction to a mixed number is useful in these three places:

First Use: When the sum of several fractions is an improper fraction. Thus: ⟶

$$\begin{array}{r} 6\frac{3}{8} \\ 2\frac{7}{8} \\ 4\frac{5}{8}\downarrow \\ \hline 12\frac{15}{8} = 13\frac{7}{8},\ Ans. \end{array}$$

An improper fraction in the sum must always be changed to a mixed number.

Second Use: When an unreduced product is an improper fraction. Thus: ⟶

$$2\tfrac{1}{2}\times 5\tfrac{1}{3} = \frac{5}{\cancel{2}_{1}}\times\frac{\cancel{16}^{8}}{3} = \frac{40}{3} = 13\tfrac{1}{3},\ Ans.$$

Never leave the answer of a multiplication example in the form of an improper fraction.

Third Use: When an unreduced quotient is an improper fraction. Thus: ⟶

$$\frac{3}{5}\div\frac{2}{7} = \frac{3}{5}\times\frac{7}{2} = \frac{21}{10} = 2\tfrac{1}{10},\ Ans.$$

Always change an improper fraction in a quotient to a mixed number.

Rule: **Improper fractions can be reduced to mixed numbers by dividing the numerator by the denominator.**

Thus:

$\frac{7}{2} = 3\frac{1}{2}$ because $2\overline{)7}\ 3\frac{1}{2}$ $\qquad$ $\frac{14}{3} = 4\frac{2}{3}$ because $3\overline{)14}\ 4\frac{2}{3}$

If the fraction in the mixed number can be reduced, **reduce it without fail.** Thus:

$\frac{14}{4} = 3\frac{2}{4} = 3\frac{1}{2}$ $\qquad$ $\frac{20}{6} = 3\frac{2}{6} = 3\frac{1}{3}$ $\qquad$ $\frac{15}{10} = 1\frac{5}{10} = 1\frac{1}{2}$

Always reduce improper fractions in answers.

Change the following improper fractions to good forms for answers. Use the answers on page 434 as a check.

1. $\frac{22}{3}$ 2. $\frac{15}{8}$ 3. $\frac{27}{5}$ 4. $\frac{16}{2}$ 5. $\frac{18}{4}$ 6. $\frac{21}{6}$ 7. $\frac{30}{12}$

272. Supplementary Practice in Reducing Answers. (For Self-Help Practice.) Change the examples below to good forms for answers. When you have finished, compare your work with the good forms listed on page 434.

1. $\frac{4}{6}$ 2. $3\frac{2}{4}$ 3. $4\frac{6}{8}$ 4. $\frac{11}{5}$ 5. $\frac{9}{12}$ 6. $\frac{9}{3}$

7. $1\frac{10}{8}$ 8. $\frac{12}{16}$ 9. $6\frac{6}{4}$ 10. $2\frac{4}{6}$ 11. $\frac{3}{2}$ 12. $11\frac{11}{10}$

273. How to Add Mixed Numbers. (For Self-Help Study.) If you are having trouble with the addition of mixed numbers this article will help you.

1. The work below is the correct way to do this example:

$$3\tfrac{3}{4} + 4\tfrac{7}{8} + 4\tfrac{5}{8} = ?$$

$3\frac{3}{4} \rightarrow \frac{6}{8}$
$4\frac{7}{8} \rightarrow \frac{7}{8}$
$4\frac{5}{8} \rightarrow \frac{5}{8}$
$2\frac{1}{4} \qquad \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}$
$13\frac{1}{4}$

First: Add the fractions. Denominators must all be 8; so we get

$$\tfrac{5}{8} + \tfrac{7}{8} + \tfrac{6}{8} = \tfrac{18}{8} = 2\tfrac{2}{8} = 2\tfrac{1}{4}$$

Second: Add $2\frac{1}{4}$ to the whole numbers.

2. Think of the following example as broken up into two addition examples, thus:

$$\left.\begin{matrix}4\frac{1}{2}\\3\frac{3}{8}\\2\frac{1}{4}\end{matrix}\right\}\text{ broken into }\left\{\begin{matrix}4\\3\text{ and}\\2\end{matrix}\right.\quad\left\{\begin{matrix}\frac{1}{2}=\frac{4}{8}\\\frac{3}{8}=\frac{3}{8}\\\frac{1}{4}=\frac{2}{8}\end{matrix}\right.$$

Add the sum of the fractions, properly reduced, to the whole numbers.

$1\frac{1}{8}$ ← $\frac{9}{8}$ or $1\frac{1}{8}$

$10\frac{1}{8}$, the sum of the given mixed numbers.

Remember to reduce improper fractions in answers.

Remember about adding the sum of the fractions to the whole numbers.

3. In adding mixed numbers one thing is very important. What is to be done with the $1\frac{7}{8}$, which is the sum of the fraction part of the example shown here?

$22\frac{1}{2} \rightarrow \frac{4}{8}$
$13\frac{3}{4} \rightarrow \frac{6}{8}$
15
$6\frac{5}{8} \rightarrow \frac{5}{8}$
$1\frac{7}{8}$ $\quad\frac{15}{8}=1\frac{7}{8}$
$57\frac{7}{8}$

The whole number part of the sum of the fractions must be added to the other whole numbers. The fraction part of this sum is then brought down beside the complete whole number.

4. See if this work is right:

$2\frac{2}{3} \rightarrow \frac{8}{12}$
$\frac{1}{4} \rightarrow \frac{3}{12}$
$2\frac{8}{12} \rightarrow \frac{8}{12}$
$4\frac{7}{12}$ $\quad\frac{19}{12}=1\frac{7}{12}$

There is one mistake here. The 1 in the $1\frac{7}{12}$ (sum of the fraction part of the example) was not added to the other whole numbers. The correct answer is $5\frac{7}{12}$.

274. Practice in Adding Mixed Numbers. (For Self-Help Practice.) The examples on the next page contain good practice for you. You can check your own work by using the answers on page 434.

Find the sums. Answers must be left in simplest form.

1. $\frac{5}{8}+\frac{3}{8}$
2. $5\frac{7}{12}+\frac{1}{2}$
3. Three-tenths plus one-half.
4. $\frac{2}{5}+\frac{1}{2}$
5. $4\frac{1}{2}+1\frac{3}{4}$
6. $\frac{2}{3}+\frac{4}{9}$
7. Seven and three twelfths plus three and five twelfths.
8. $\frac{4}{9}+11\frac{5}{6}$
9. $4\frac{2}{3}+7\frac{1}{2}$
10. $\frac{3}{4}+\frac{6}{11}$
11. $\frac{9}{20}+\frac{3}{5}$
12. $\begin{array}{r} \frac{2}{5} \\ \frac{1}{2} \\ \hline \frac{3}{4} \end{array}$
13. $\frac{1}{8}+\frac{1}{3}+\frac{5}{12}$
14. $4+\frac{1}{2}+\frac{1}{4}$
15. Two-sevenths plus one-half plus one-fourteenth
16. $\frac{1}{3}+\frac{4}{5}+\frac{8}{15}$
17. $\begin{array}{r} 3\frac{1}{17} \\ \frac{7}{17} \\ \hline 6\frac{4}{17} \end{array}$
18. $5\frac{5}{16}+\frac{1}{8}+\frac{1}{4}$
19. One-sixth plus seven and three-fourths plus one-third.
20. $\frac{8}{21}+\frac{2}{21}+\frac{4}{21}$
21. $\begin{array}{r} \frac{3}{50} \\ 12\frac{9}{10} \\ \frac{16}{25} \\ \hline \end{array}$
22. $\frac{2}{5}+\frac{3}{35}+\frac{6}{7}$
23. $\frac{1}{4}+8\frac{5}{6}+\frac{1}{3}$
24. *Add:* $\begin{array}{r} \frac{1}{7} \\ \frac{1}{11} \\ \hline \end{array}$
25. *Add:* $\begin{array}{r} 3\frac{2}{15} \\ \frac{3}{5} \\ \hline \end{array}$
26. *Add:* $\begin{array}{r} 9\frac{7}{19} \\ 4\frac{12}{19} \\ \hline \end{array}$
27. $\frac{1}{6}+\frac{7}{18}$
28. $3\frac{1}{2}+4\frac{3}{10}$
29. $2\frac{3}{8}+3\frac{2}{5}$
30. $2\frac{1}{6}+1\frac{4}{5}$

31. *Write the reduced form of each:* $\frac{17}{3}$, $\frac{9}{9}$, $\frac{20}{8}$, $\frac{24}{6}$.

32. What denominator would you use with $\frac{}{4}+\frac{}{3}+\frac{}{6}$?

275. Supplementary Practice on Changing the Form of Fractions. (For Self-Help Practice.) Fractions often must be changed in form without changing their value. Thus: in the example $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$, the $\frac{1}{2}$ had to be changed to $\frac{2}{4}$. All sixth-grade pupils should be able to change the forms of the fractions as suggested below. The answers on page 435 will help with any troublesome examples.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$\frac{1}{3}=\frac{}{9}$	$\frac{1}{2}=\frac{}{6}$	$\frac{1}{5}=\frac{}{30}$	$\frac{3}{5}=\frac{}{10}$	$\frac{5}{9}=\frac{}{18}$	$\frac{11}{12}=\frac{}{24}$
2.	$\frac{1}{2}=\frac{}{14}$	$\frac{1}{3}=\frac{}{15}$	$\frac{1}{3}=\frac{}{12}$	$\frac{5}{6}=\frac{}{24}$	$\frac{1}{6}=\frac{}{18}$	$\frac{3}{8}=\frac{}{24}$
3.	$\frac{1}{4}=\frac{}{8}$	$\frac{1}{4}=\frac{}{12}$	$\frac{1}{6}=\frac{}{12}$	$\frac{2}{9}=\frac{}{36}$	$\frac{5}{8}=\frac{}{24}$	$\frac{5}{6}=\frac{}{30}$
4.	$\frac{1}{3}=\frac{}{6}$	$\frac{1}{2}=\frac{}{10}$	$\frac{3}{4}=\frac{}{12}$	$\frac{2}{7}=\frac{}{14}$	$\frac{3}{7}=\frac{}{14}$	$\frac{7}{12}=\frac{}{36}$

276. Supplementary Practice on Common Denominators. (For Self-Help Practice.) In adding and subtracting fractions we must renew our acquaintance with our old friend, the common denominator. How well do you know common denominators? What common denominators would you use in the examples below? Do not add, but decide what would be the best denominator to use for each example. The correct denominators are given on page 435.

	(A)	(B)	(C)	(D)
1.	$\frac{1}{4}+\frac{1}{8}$	$\frac{2}{7}+\frac{1}{7}$	$\frac{1}{5}+\frac{1}{6}$	$\frac{1}{3}+\frac{1}{4}$
2.	$\frac{1}{2}+\frac{1}{5}$	$\frac{2}{11}+\frac{1}{11}$	$\frac{1}{9}+\frac{1}{12}$	$\frac{1}{6}+\frac{1}{9}$
3.	$\frac{1}{3}+\frac{1}{9}$	$\frac{1}{2}+\frac{1}{3}$	$\frac{1}{7}+\frac{1}{14}$	$\frac{1}{2}+\frac{1}{14}$
4.	$\frac{1}{4}+\frac{1}{12}$	$\frac{1}{4}+\frac{1}{6}$	$\frac{1}{2}+\frac{1}{7}$	$\frac{1}{3}+\frac{1}{15}$
5.	$\frac{1}{2}+\frac{1}{6}$	$\frac{1}{8}+\frac{1}{12}$	$\frac{1}{3}+\frac{1}{12}$	$\frac{1}{4}+\frac{1}{5}$

SUBTRACTION OF FRACTIONS AND SMALL MIXED NUMBERS

277. How to Use the Lessons on Subtraction of Fractions and Small Mixed Numbers. (SILENT READING.) The Self-Help Lessons on subtraction of fractions are for your use whenever you need to restudy this topic. The chart below will help you find the articles you need.

> Art. 278 explains the finding of common denominators.
> Arts. 279 and 280 give practice in using common denominators in easy subtraction.
> Art. 281 explains how to subtract mixed numbers.
> Arts. 282 and 283 give practice in subtracting mixed numbers.

278. The Common Denominator in Subtraction. (FOR SELF-HELP STUDY.) Do you remember that in adding fractions the denominators always have to be the same? The denominators of fractions when you are subtracting must be the same also.

Remember: In adding and subtracting fractions, the denominators must always be the same. This means that you change fractions in form but not in value.

In the example $\frac{3}{4}-\frac{1}{4}$, we do not need to change the form of either fraction, because the fractions have the same denominator.

But in the example $\frac{3}{4}-\frac{1}{2}$, we must change the form (but not the value) of one of the fractions so that both will have the same denominator. Here we would change $\frac{1}{2}$ to $\frac{2}{4}$.

279. Practice in Changing the Form of Fractions. (FOR SELF-HELP PRACTICE.) Every sixth-grade pupil should be

able to see at once what common denominators to use in the subtraction examples below. Practice until you can do so. Art. 267 explains how to find common denominators. The starred (*) examples use smaller denominators than those got by using the rule (Art. 267). The correct denominators are listed on page 435. Use them to check your own work.

1.	2.*	3.*	4.	5.*
$\frac{}{3-5}$	$\frac{}{2-4}$	$\frac{}{4-12}$	$\frac{}{2-5}$	$\frac{}{3-6}$
6.*	**7.***	**8.**	**9.***	**10.***
$\frac{}{6-2}$	$\frac{}{5-10}$	$\frac{}{3-4}$	$\frac{}{4-8}$	$\frac{}{8-2}$
11.*	**12.***	**13.***	**14.***	**15.***
$\frac{}{4-10}$	$\frac{}{10-2}$	$\frac{}{3-12}$	$\frac{}{6-4}$	$\frac{}{5-15}$
16.*	**17.***	**18.***	**19.***	**20.***
$\frac{}{2-16}$	$\frac{}{6-9}$	$\frac{}{8-10}$	$\frac{}{12-6}$	$\frac{}{4-16}$

➡ Before you leave this article, go over it again to make sure that you know each common denominator very well.

280. Practice with Denominators in Easy Subtraction. (For Self-Help Practice.) You should be able to work these examples very easily and quickly. The answers are given on page 435, so you can check your work.

Subtract:

1.	2.	3.	4.	5.	6.	7.
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{5}{6}$	$\frac{5}{6}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{3}{4}$
8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
$\frac{7}{12}$	$\frac{2}{3}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{3}{10}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{10}$	$\frac{7}{10}$

281. How to Subtract Mixed Numbers. (For Self-Help Study.) Below are examples of the **subtraction of mixed numbers.** Are they worked correctly?

(*a*) $17\frac{2}{3}$ / $14\frac{1}{3}$ / $3\frac{1}{3}$, *Ans.*

(*b*) $24\frac{4}{5}$ / $21\frac{2}{5}$ / $3\frac{2}{5}$, *Ans.*

(*c*) $89\frac{9}{13}$ / $74\frac{7}{13}$ / $15\frac{2}{13}$, *Ans.*

➡ In subtracting mixed numbers we first subtract the fractions. Example (*d*) shows the "hard spot" in subtraction.

(*d*) $3\frac{1}{8}$ / $1\frac{7}{8}$

How are we going to take $\frac{7}{8}$ from $\frac{1}{8}$? If the fraction to be subtracted is larger than the fraction to be subtracted from, as in Example (*d*), we have the one "hard spot" in subtraction. Below are three things to study to make the "hard spot" become easy.

THREE FACTS TO USE IN SUBTRACTION

I. Remember that:

$$1 = \tfrac{2}{2},\ \tfrac{3}{3},\ \tfrac{4}{4},\ \tfrac{5}{5},\ \tfrac{6}{6},\ \tfrac{7}{7},\ \tfrac{8}{8},\ \text{etc.}$$

II. In making the minuend fraction larger, add a whole 1 expressed as a fraction. Thus:

$3\frac{1}{8} \rightarrow \frac{8}{8}+\frac{1}{8}=\frac{9}{8}$ / $-1\frac{7}{8}$

$4\frac{1}{4} \rightarrow \frac{5}{4}$ / $-2\frac{3}{4}$

$6\frac{2}{5} \rightarrow \frac{7}{5}$ / $-4\frac{3}{5}$

III. If you use a 1 in fraction form, remember that a whole number must be changed. Thus:

$3\frac{1}{8} \rightarrow \frac{9}{8}$ / $-1\frac{7}{8}$ What change in 3? / -1

$4\frac{1}{4} \rightarrow \frac{5}{4}$ / $-2\frac{3}{4}$ What change in 4? / -2

282. Practice in Subtracting Mixed Numbers. (For Self-Help Practice.) Copy only when you need to. What must you remember about a fraction in an answer? Use the answers on page 435 as a check of your own work.

1.	2.	3.	4.	5.	6.	7.
$4\frac{1}{3}$	$5\frac{1}{4}$	$6\frac{1}{7}$	$7\frac{1}{6}$	$9\frac{3}{8}$	$4\frac{1}{8}$	4
$2\frac{2}{3}$	$2\frac{3}{4}$	$3\frac{4}{7}$	$3\frac{5}{6}$	$6\frac{7}{8}$	$3\frac{3}{8}$	$2\frac{1}{2}$
8.	**9.**	**10.**	**11.**	**12.**	**13.**	**14.**
$10\frac{1}{2}$	7	$4\frac{1}{3}$	$6\frac{1}{4}$	$5\frac{2}{5}$	$4\frac{3}{7}$	$5\frac{5}{16}$
$8\frac{1}{2}$	$6\frac{3}{7}$	$3\frac{2}{3}$	$\frac{3}{4}$	5	$\frac{6}{7}$	$4\frac{9}{16}$
15.	**16.**	**17.**	**18.**	**19**	**20.**	**21.**
$7\frac{1}{4}$	$8\frac{1}{3}$	$1\frac{1}{10}$	$3\frac{1}{4}$	5	$10\frac{1}{6}$	$4\frac{7}{8}$
$3\frac{1}{2}$	$5\frac{5}{6}$	$\frac{4}{5}$	$2\frac{5}{8}$	$2\frac{5}{9}$	$7\frac{1}{2}$	$2\frac{1}{2}$

283. Supplementary Practice on Subtracting Fractions and Mixed Numbers. (For Self-Help Practice.) If you can work 24 of these examples in 15 minutes correctly, you will not have much trouble with subtraction this year. Check your answers with those on page 436.

Only answers in simplest form count as correct.

1. $\frac{5}{8}-\frac{1}{2}$ 2. $\frac{7}{9}-\frac{3}{4}$ 3. $1\frac{3}{5}-\frac{2}{3}$ 4. $9\frac{3}{11}-\frac{3}{11}$ 5. $6\frac{9}{20}$ $2\frac{7}{20}$

6. Ten and two-ninths minus one-sixth.

7. $\frac{7}{18}-\frac{1}{6}$ 8. $4\frac{5}{11}$ 2 9. $\frac{12}{17}-\frac{10}{17}$ 10. $1\frac{3}{4}$ $1\frac{2}{3}$ 11. $3\frac{1}{3}$ $1\frac{3}{7}$

12. $7\frac{3}{8}-6\frac{5}{24}$ 13. $1-\frac{1}{20}$ 14. $5\frac{3}{20}-2\frac{2}{5}$

15. Three and one-sixth minus three-fourths.

16. $7\frac{1}{6} - \frac{1}{2}$ 17. $\frac{2}{3} - \frac{1}{2}$ 18. $9\frac{3}{8} - \frac{3}{4}$ 19. $7\frac{3}{5} - \frac{1}{4}$

20. Nine and one-sixth minus four and two-thirds.

21. $10\frac{5}{10} - \frac{1}{2}$ 22. $5\frac{4}{7}$ 3 23. $6\frac{1}{2} - 6\frac{2}{5}$ 24. $8\frac{9}{50} - \frac{29}{50}$

25. 9 $3\frac{4}{11}$ 26. $\frac{7}{15}$ $\frac{4}{15}$ 27. Eight and two-thirds minus four and one-half.

28. $7\frac{1}{14}$ $\frac{4}{7}$ 29. $9\frac{5}{6}$ $4\frac{1}{4}$ 30. 7 minus $\frac{3}{22}$

MULTIPLICATION OF FRACTIONS AND SMALL MIXED NUMBERS

284. How to Use the Lessons on Multiplication of Fractions and Small Mixed Numbers. (SILENT READING.) The chart below will help you find the articles you may need to use.

Art. 285 explains how to multiply fractions.

Art. 286 gives practice on multiplying easy fractions.

Art. 287 explains that small mixed numbers must be changed to improper fractions.

Art. 288 shows how to change mixed numbers to improper fractions.

Art. 289 gives practice on changing mixed numbers to improper fractions.

Arts. 290 and 291 explain how to cancel.

Art. 292 gives practice in canceling in easy multiplication.

Art. 293 explains how to multiply small mixed numbers.

Arts. 294 and 295 give practice in the multiplication of small mixed numbers.

285. How to Multiply by Fractions. (For Self-Help Study.) If you are having trouble with the multiplication of fractions and small mixed numbers, this article and the next ones will help you.

FIVE HINTS TO USE IN MULTIPLYING FRACTIONS

There are five things to remember in working an example in the multiplication of fractions. These five things are:

1. Cancel whenever you can.

2. Multiply the numerators to obtain the numerator of the product.

3. Multiply the denominators to obtain the denominator of the product.

4. Reduce the product to simplest form.

5. When there is a whole number in the example, it is used as a numerator.

These five points in multiplying fractions are illustrated below:

Point 1. Cancel first: $\frac{\cancel{2}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{14}_{7}}$

Point 2. Multiply numerators: $\frac{\cancel{2}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{14}_{7}} = \frac{2}{\quad}$

Point 3. Multiply denominators: $\frac{\cancel{2}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{14}_{7}} = \frac{2}{21}$

POINT 4. In an example like this, reduce the product:

$$\frac{9}{5}\times\frac{4}{5}=\frac{36}{25}=1\frac{11}{25}.$$

POINT 5. A whole number is used as a numerator:

$$4\times\frac{1}{5}=\frac{4}{1}\times\frac{1}{5}=\frac{4}{5}.$$

If you have forgotten how to cancel, study Arts. 290-291.

SAMPLES FOR STUDY

SAMPLE 1.

$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$, *Ans.*

1. In Sample 1, you get the numerator 8 by multiplying 2×4.

2. You get the denominator 15 by multiplying 3×5.

3. The answer, $\frac{8}{15}$, can not be reduced.

SAMPLE 2.

$\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$, *Ans.*

1. In Sample 2, think $1\times1=1$. (Some pupils write **2.** 2 is **wrong.**)

2. Points 2 and 3 of the Five Points listed above are used in Sample 2.

SAMPLE 3.

$\frac{2}{3}$ of $8=\frac{16}{3}=5\frac{1}{3}$, *Ans.*

1. In Sample 3, remember that "of" means "$\times$" in multiplication.

2. The 8 is used as a numerator.

3. If you really do not understand how $\frac{16}{3}$ is changed to $5\frac{1}{3}$, study Art. 271 now.

SAMPLE 4.

$\overset{2}{\cancel{4}}\times\overset{1}{\cancel{8}}=\frac{2}{12}$, *Ans.*
$\underset{3}{\cancel{9}}\quad\underset{4}{\cancel{8}}$

1. In Sample 4 you get the 2 above the 4 and the 4 below the 8 by canceling with a 2. It would be better to cancel with a 4 instead of a 2 here.

2. The answer should be reduced to simplest form. $\frac{2}{12}=\frac{1}{6}$.

286. Easy Practice on Multiplication of Fractions. (For Self-Help Practice.) Every sixth-grade pupil should be able to work the examples below correctly and easily. After you have finished working the examples, use the answers on page 436 as a check.

1. $\frac{2}{3}\times\frac{4}{5}$	6. $\frac{5}{6}\times\frac{1}{2}$	11. $\frac{4}{7}\times\frac{4}{7}$	16. $\frac{3}{8}$ of 12
2. $\frac{3}{4}$ of $\frac{5}{7}$	7. $\frac{2}{3}$ of 5	12. $\frac{1}{2}$ of $\frac{1}{2}$	17. $\frac{5}{6}$ of 8
3. $\frac{2}{5}\times\frac{1}{5}$	8. $9\times\frac{1}{2}$	13. $\frac{4}{5}$ of 6	18. $\frac{5}{8}\times\frac{3}{10}$
4. $\frac{4}{5}\times\frac{2}{3}$	9. $\frac{1}{4}\times\frac{1}{6}$	14. $\frac{1}{4}\times 12$	19. $\frac{7}{12}$ of 84
5 $\frac{1}{2}$ of $\frac{1}{4}$	10. $\frac{3}{5}\times 7$	15. $4\times\frac{3}{5}$	20. $\frac{2}{3}\times 0$

287. Multiplication of Small Mixed Numbers. (For Self-Help Study.) You will have little if any trouble with the multiplication of mixed numbers if you will understand and **use carefully** the information which follows.

The multiplication of small mixed numbers becomes exactly like the multiplication of fractions if the mixed numbers are first changed to improper fractions.

Study the samples below:

$4\frac{1}{2}\times 8$. Change to $\frac{9}{2}\times 8$.
$3\frac{1}{3}\times 5\frac{1}{2}$. Change to $\frac{10}{3}\times\frac{11}{2}$.
$6\times 7\frac{3}{5}$. Change to $6\times\frac{38}{5}$.
$6\frac{3}{4}\times\frac{2}{7}$. Change to $\frac{27}{4}\times\frac{2}{7}$.

288. How to Change Mixed Numbers to Improper Fractions. (For Self-Help Study.) There are two places in arithmetic where the ability to change mixed numbers to improper fractions is very helpful. These places are:

I. In multiplying small mixed numbers. Thus:

$4\frac{1}{2}\times 3\frac{1}{3}$ becomes $\frac{9}{2}\times\frac{10}{3}$. Or, $7\frac{2}{5}\times 6\frac{3}{7}$ becomes $\frac{37}{5}\times\frac{45}{7}$.

II. In dividing small mixed numbers. Thus:

$10\frac{3}{4} \div 2\frac{5}{6}$ becomes $\frac{43}{4} \div \frac{17}{6}$, becomes $\frac{43}{4} \times \frac{6}{17}$.

Rule: **To change a mixed number to an improper fraction:**

1. Multiply the whole number by the denominator of the fraction.

2. Add this product to the numerator of the given fraction. This sum is the required numerator. The denominator does not change.

Thus: $3\frac{1}{2}^* = \frac{7}{2}$ because: $3 \times 2 = 6$; $6 + 1^* = 7$

$7\frac{3}{5}^* = \frac{38}{5}$ because: $7 \times 5 = 35$; $35 + 3^* = 38$

See if this rule works in the next four examples.

$4\frac{9}{16} = \frac{73}{16}$ $3\frac{1}{7} = \frac{22}{7}$ $8\frac{2}{9} = \frac{74}{9}$ $12\frac{1}{3} = \frac{37}{3}$

Change these mixed numbers to improper fractions:

$1\frac{1}{2}$ $2\frac{1}{3}$ $1\frac{3}{4}$ $3\frac{1}{5}$ $1\frac{1}{16}$ $2\frac{3}{8}$ $1\frac{1}{6}$ $3\frac{1}{10}$ $2\frac{5}{12}$

289. Practice in Changing Mixed Numbers to Improper Fractions. (For Self-Help Practice.) Change the numbers below to improper fractions. When you have finished, use the answers on page 436 to check your work.

1.	2.	3.	4.	5.	6.
$4\frac{1}{2}$	$6\frac{5}{12}$	$6\frac{1}{4}$	$3\frac{4}{7}$	$1\frac{1}{9}$	$3\frac{2}{3}$
7.	**8.**	**9.**	**10.**	**11.**	**12.**
$4\frac{7}{8}$	$2\frac{4}{5}$	$3\frac{3}{10}$	$9\frac{1}{6}$	$7\frac{6}{8}$	$10\frac{11}{16}$
13.	**14.**	**15.**	**16.**	**17.**	**18.**
$19\frac{1}{3}$	$8\frac{6}{9}$	$1\frac{4}{3}$	$11\frac{11}{12}$	$5\frac{3}{5}$	$12\frac{7}{12}$

290. How to Cancel. (For Self-Help Study.) The process of dividing out the same number from a numerator and denominator is called **cancellation.** To be able to cancel quickly and accurately is very useful in multiplying by a fraction. Study the next four examples. They show how canceling is done.

1.

$\frac{2}{5}\times\frac{1}{2}$ is worked:

$$\frac{\overset{1}{\cancel{2}}}{5}\times\frac{1}{\underset{1}{\cancel{2}}}=\frac{1\times1}{5\times1}=\frac{1}{5},\ \textit{Ans.}$$

2.

$\frac{3}{4}\times\frac{2}{3}$ is worked:

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}=\frac{1\times1}{2\times1}=\frac{1}{2},\ \textit{Ans.}$$

In multiplication of fractions a **divisor** may be canceled from any numerator and denominator. If it is canceled from one, it **must** be canceled from the other. Thus:

3. $\frac{\overset{2}{\cancel{4}}}{5}\times\frac{7}{\underset{5}{\cancel{10}}}=\frac{14}{25}$

4. $\frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{9}}}\times\frac{1}{\underset{7}{\cancel{14}}}=\frac{4}{7}$

➡ Always cancel when you can **before you multiply** the numerators; this allows you to work with smaller numbers.

291. Repeated Canceling. (For Self-Help Study.) In multiplying by fractions, you can sometimes cancel more than once. Cancel as many times as you can; it makes the work simpler. Look at the examples below.

1.

$$\frac{3}{5}\times\frac{15}{36}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{15}}}{\underset{4}{\cancel{36}}}=\frac{1\times1}{1\times4}=\frac{1}{4},\ \textit{Ans.}$$

2.

$$\frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{6}}}\times\frac{\overset{4}{\cancel{12}}}{\underset{7}{\cancel{14}}}=\frac{1\times4}{3\times7}=\frac{4}{21}$$

In canceling we can divide out the same number from

the numerator of one fraction and the denominator of another (Example 3). Also, we can use the terms of only one fraction, as we do in reducing a fraction (Example 4).

3. $\frac{\overset{2}{\cancel{6}}}{7}\times\frac{4}{\underset{3}{\cancel{9}}}=\frac{8}{21}$ 4. $\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}\times 7=\frac{14}{3}=4\frac{2}{3}$

292. Practice in Cancellation. (For Self-Help Practice.) Check your answers with those shown on page 436.

1. $\frac{2}{3}\times\frac{3}{4}$	7. $\frac{5}{6}\times\frac{2}{3}$	13. $\frac{2}{5}\times\frac{1}{8}$	19. $\frac{3}{5}\times\frac{20}{21}$
2. $\frac{1}{2}\times\frac{1}{2}$	8. $\frac{7}{8}\times\frac{4}{5}$	14. $\frac{5}{9}\times\frac{3}{4}$	20. $\frac{2}{3}$ of $\frac{9}{10}$
3. $\frac{4}{5}\times\frac{1}{4}$	9. $\frac{1}{2}$ of $\frac{4}{5}$	15. $\frac{25}{36}$ of 18	21. $\frac{3}{4}$ of 24
4. $\frac{10}{12}$ of $\frac{1}{5}$	10. $\frac{9}{10}\times\frac{2}{3}$	16. $\frac{7}{8}\times 12$	22. $\frac{5}{16}\times\frac{3}{10}$
5. $\frac{1}{3}$ of 21	11. $5\times\frac{1}{5}$	17. $\frac{4}{25}\times 50$	23. $\frac{4}{5}\times\frac{5}{12}$
6. $\frac{2}{3}\times\frac{3}{4}$	12. $\frac{3}{8}\times\frac{1}{6}$	18. $\frac{1}{2}$ of 18	24. $\frac{1}{6}$ of $\frac{12}{25}$

293. How to Multiply Small Mixed Numbers. (For Self-Help Study.) Be sure you understand these problems:

Problem 1. I am going away for $2\frac{1}{2}$ months. A boy next door will take care of my hens while I am away. My hens eat $3\frac{3}{5}$ bu. of corn each month. How much corn should I leave?

The work: $2\frac{1}{2}\times 3\frac{3}{5}$. Change to fractions: $\frac{5}{2}\times\frac{18}{5}$.

Cancel and multiply: $\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}}\times\frac{\overset{9}{\cancel{18}}}{\underset{1}{\cancel{5}}}=\frac{9}{1}$, or exactly 9 bu., *Ans.*

Problem 2. My puppy, Blacknose, gains $1\frac{1}{2}$ lb. in weight a month. At this rate how much will he gain in 4 months?

The work: $4\times 1\frac{1}{2}$ or $4\times\frac{3}{2}$ or $\frac{\overset{2}{\cancel{4}}}{1}\times\frac{3}{\underset{1}{\cancel{2}}}=6$ lb., *Ans.*

➡ **Notice that whole numbers are used as numerators.**

$4 \times \frac{3}{2}$ can be thought of as $\frac{4}{1} \times \frac{3}{2}$.
$9 \times \frac{2}{3}$ can be thought of as $\frac{9}{1} \times \frac{2}{3}$.
$\frac{4}{5}$ of 15 can be thought of as $\frac{4}{5} \times \frac{15}{1}$.

Rule: **To multiply small mixed numbers, change to improper fractions and multiply. Remember to leave the product in simplest terms.**

294. Practice in Multiplying Mixed Numbers. (For Self-Help Practice.) Work these examples and then check your answers with those given on page 437.

1. $4\frac{3}{4} \times 6\frac{1}{5}$
2. $\frac{2}{3} \times 20$
3. $13\frac{1}{2} \times 6\frac{1}{3}$
4. $\frac{2}{3} \times 4\frac{5}{8}$
5. $\frac{3}{5}$ of 27
6. $3\frac{5}{6} \times 4\frac{5}{8}$
7. $9 \times 4\frac{1}{5}$
8. $7\frac{3}{4} \times \frac{5}{8}$
9. $\frac{7}{8} \times \frac{4}{5}$
10. $3\frac{1}{2} \times 4\frac{2}{5}$
11. $\frac{5}{9}$ of 8
12. $\frac{2}{7}$ of \$14.49
13. $10 \times \frac{1}{2}$
14. $7\frac{2}{5} \times 4\frac{1}{3}$
15. $\frac{4}{5}$ of 25
16. $2\frac{3}{4} \times 3\frac{1}{3}$
17. $2\frac{2}{15} \times 1\frac{3}{8}$
18. $7\frac{3}{5} \times 4\frac{7}{10}$

295. Supplementary Practice on Multiplication of Fractions and Mixed Numbers. (For Self-Help Practice.) Make sure that you can multiply fractions and small mixed numbers well. Answers are given on page 437.

1. Four and two-fifths times two and three-elevenths.
2. $7 \times \frac{1}{2}$
3. $\frac{2}{3} \times \frac{1}{3}$
4. $\frac{3}{4} \times \frac{2}{7}$
5. $\frac{4}{5} \times \frac{5}{8}$
6. $2\frac{1}{4} \times \frac{16}{27}$
7. $4\frac{1}{2} \times \$3.50$
8. $\frac{7}{9}$ of $\frac{3}{14}$
9. $\frac{1}{2} \times 2\frac{1}{5}$
10. $\frac{5}{12}$ of 4
11. $5\frac{1}{7} \times 1\frac{9}{12}$
12. $5 \times \frac{4}{21}$
13. $8\frac{1}{3} \times \frac{2}{7}$

14. $\frac{5}{11} \times \frac{2}{3}$

15. $\frac{9}{16} \times 8$

16. $\frac{3}{16} \times \frac{4}{15}$

17. $\frac{3}{8} \times \frac{4}{9} \times 1\frac{1}{5}$

18. $1\frac{1}{4} \times \$1.60$

19. $\frac{1}{2} \times \frac{1}{5} \times 7$

20. $\frac{5}{12}$ of $\frac{3}{10} \times \frac{4}{5}$

21. $3\frac{1}{3} \times \frac{1}{4} \times 1\frac{1}{2}$

22. $\frac{1}{2} \times \frac{3}{4} \times \frac{2}{3}$

23. $1\frac{4}{5} \times \frac{1}{4} \times 15$

24. $1\frac{2}{9} \times \frac{3}{5} \times 1\frac{2}{3}$

25. $1\frac{1}{7} \times \frac{1}{6} \times 5\frac{1}{4}$

26. $\frac{1}{2}$ of $\frac{8}{15}$ of $\frac{5}{16}$

27. $7 \times 2\frac{2}{3} \times \frac{3}{10}$

28. $\frac{7}{12} \times \frac{2}{7}$

29. One-half times three twenty-fifths.

DIVISION OF FRACTIONS AND MIXED NUMBERS

296. How to Use the Lessons on Division of Fractions and Small Mixed Numbers. (Silent Reading.) The chart below will help you find the articles you may want.

> Art. 297 explains how to divide by fractions and small mixed numbers.
>
> Art. 298 explains and gives practice in inverting divisors.
>
> Arts. 299 and 300 give practice and some more explanation on dividing by fractions and small mixed numbers.

297. How to Divide by Fractions and Mixed Numbers. (For Self-Help Study.) Be sure that you understand the two "steps" for each sample.

> Sample 1: $\frac{1}{4} \div \frac{2}{3} = ?$

Step I. **Invert the fraction you divide by and change $\div$ to $\times$.**

Thus: $\frac{1}{4} \div \frac{2}{3}$ becomes $\frac{1}{4} \times \frac{3}{2}$. ($\frac{3}{2}$ is $\frac{2}{3}$ inverted.)

Notice: $\frac{2}{3}$ becomes $\frac{3}{2}$ and $\div$ is changed to $\times$.

$\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$.

Step II. Multiply. $\frac{1}{4}\times\frac{3}{2}=\frac{3}{8}$, *Ans.*

SAMPLE 2: $\frac{3}{4}\div\frac{2}{5}=?$

Step I. Invert the fraction you divide by and change $\div$ to $\times$.
Thus: $\frac{3}{4}\div\frac{2}{5}$ becomes $\frac{3}{4}\times\frac{5}{2}$. ($\frac{5}{2}$ is $\frac{2}{5}$ inverted.)

Step II. Multiply. $\frac{3}{4}\times\frac{5}{2}=\frac{15}{8}=1\frac{7}{8}$, *Ans.*

SAMPLE 3: $5\div\frac{3}{4}=?$

Step I. Invert the fraction you divide by and change $\div$ to $\times$.
Thus: $5\div\frac{3}{4}$ becomes $5\times\frac{4}{3}$.

Step II. Multiply. $5\times\frac{4}{3}=\frac{20}{3}=6\frac{2}{3}$, *Ans.*

SAMPLE 4: $2\frac{1}{2}\div3\frac{1}{3}=?$

Step I. Change mixed numbers to improper fractions.
Thus: $2\frac{1}{2}\div3\frac{1}{3}$ becomes $\frac{5}{2}\div\frac{10}{3}$.

Step II. Invert divisor. $\frac{5}{2}\div\frac{10}{3}$ becomes $\frac{5}{2}\times\frac{3}{10}$.

Step III. Multiply. Cancel if you can. $\frac{\overset{1}{\cancel{5}}}{2}\times\frac{3}{\cancel{10}}=\frac{3}{4}$, *Ans.*

Rule: To divide by a fraction, invert the fraction you divide by, change $\div$ to $\times$, and multiply. Mixed numbers are changed to improper fractions.

298. Inverting the Divisor. (For Self-Help Practice.) Examples in the division of fractions are changed to multiplication examples by **inverting** the **divisor** and changing $\div$ to $\times$. Study the two samples below.

(*a*) $\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3} =$ ..... (*b*) $3\frac{1}{2} \div 2\frac{1}{3} = \frac{7}{2} \div \frac{7}{3} = \frac{7}{2} \times \frac{3}{7} =$ .....

Change the divisors below to multipliers. Example 1 is already worked. Use the answers on page 438 as a check.

1. $\div \frac{1}{3} = \times \frac{3}{1}$
2. $\div 2\frac{1}{2} = \times$....
3. $\div 1\frac{4}{5} = \times$....
4. $\div \frac{1}{6} = \times$....
5. $\div 1\frac{1}{2} = \times$....
6. $\div \frac{1}{3} = \times$....
7. $\div 1\frac{1}{8} = \times$....
8. $\div \frac{3}{8} = \times$....
9. $\div 9\frac{1}{8} = \times$....
10. $\div 4\frac{1}{2} = \times$....
11. $\div 1\frac{2}{3} = \times$....
12. $\div 8\frac{1}{2} = \times$....
13. $\div 5\frac{1}{3} = \times$....
14. $\div 2\frac{1}{3} = \times$....
15. $\div 7\frac{1}{3} = \times$....
16. $\div 4\frac{1}{2} = \times$....
17. $\div 3\frac{3}{4} = \times$....
18. $\div 6\frac{1}{4} = \times$ ...
19. $\div \frac{6}{7} = \times$....
20. $\div 2\frac{2}{5} = \times$....
21. $\div 6\frac{1}{2} = \times$....
22. $\div \frac{2}{3} = \times$....
23. $\div 5\frac{1}{8} = \times$....
24. $\div 2\frac{1}{6} = \times$....

299. Practice on Division of Mixed Numbers. (For Self-Help Practice.) Correct answers are on page 438. If the examples below are hard for you, study Art. 297.

1. $\frac{1}{4} \div \frac{1}{3}$
2. $4\frac{1}{2} \div 2\frac{1}{2}$
3. $3\frac{1}{3} \div 1\frac{4}{5}$
4. $\frac{3}{4} \div \frac{1}{6}$
5. $1\frac{1}{5} \div 3$
6. $7 \div 9\frac{1}{8}$
7. $\frac{3}{4} \div 4\frac{1}{2}$
8. $1\frac{1}{5} \div 1\frac{2}{3}$
9. $\frac{5}{6} \div 8\frac{1}{2}$
10. $2\frac{2}{3} \div 5\frac{1}{3}$
11. $2\frac{1}{6} \div 3\frac{3}{4}$
12. $25 \div 6\frac{1}{4}$
13. $11\frac{1}{3} \div \frac{6}{7}$
14. $7\frac{1}{2} \div 2\frac{2}{5}$
15. $3\frac{1}{4} \div 6\frac{1}{2}$
16. $10 \div \frac{1}{3}$
17. $12 \div 1\frac{1}{8}$
18. $11\frac{1}{4} \div \frac{3}{8}$
19. $\frac{14}{15} \div 2\frac{1}{3}$
20. $4\frac{2}{3} \div 7\frac{1}{3}$
21. $2 \div 4\frac{1}{2}$
22. $5 \div \frac{2}{3}$
23. $2\frac{1}{3} \div 5\frac{1}{8}$
24. $26 \div 2\frac{1}{6}$

300. Supplementary Practice on Division of Fractions and Mixed Numbers. (For Self-Help Study and Practice.) If division of fractions and small mixed numbers still bothers you after you have studied Art. 297, you may use the following work for a further study of division.

HINTS ON THE DIVISION OF FRACTIONS

Division of fractions and division of mixed numbers are not hard, if you will take care to have 5 things clearly in mind. These are:

1. If there is a mixed number in the example, change it to an improper fraction. Thus:

$4\frac{2}{3} \div \frac{3}{7}$ becomes $\frac{14}{3} \div \frac{3}{7}$

$\frac{2}{3} \div 4\frac{3}{7}$ becomes $\frac{2}{3} \div \frac{31}{7}$

2. Invert the terms of the **divisor** and change $\div$ to $\times$. Thus:

$\frac{14}{3} \div \frac{3}{7}$ means $\frac{14}{3} \times \frac{7}{3}$

$\frac{2}{3} \div \frac{31}{7}$ means $\frac{2}{3} \times \frac{7}{31}$

3. Make sure which number is the divisor. This is easy in an example like $\frac{14}{3} \div \frac{3}{7}$ because $\div$ is read, "divided by." The divisor is the $\frac{3}{7}$. But you will have to think carefully which number is the divisor when the example is: "Divide $4\frac{2}{3}$ by $\frac{3}{7}$," or "What is the quotient when $4\frac{2}{3}$ is divided by $\frac{3}{7}$?"

Name the divisor in the problem below:

How many catcher's mitts can be made from $16\frac{1}{2}$ sq. ft. of horse hide if it takes $2\frac{3}{4}$ sq. ft. for each mitt?

4. Remember that in the division of fractions a whole number acts like a numerator. Thus:

$\frac{2}{3} \div 4$ means $\frac{2}{3} \div \frac{4}{1}$, or $8 \div \frac{5}{3}$ means $\frac{8}{1} \div \frac{5}{3}$.

5. After you have changed the division example to multiplication form, **multiply numerators to get the numerator of the answer, and multiply denominators to get the denominator of the answer.**

Can you work the examples below correctly? Use the answers on page 438 as a check on your own work.

1. $4 \div \frac{2}{3} =$
2. $\frac{3}{5} \div 3 =$
3. $4\frac{2}{3} \div 7 =$
4. $4\frac{1}{3} \div 3\frac{1}{4} =$
5. $\frac{3}{7} \div \frac{1}{7} =$
6. $2\frac{1}{2} \div 6 =$
7. $8 \div 1\frac{2}{3} =$
8. $3\frac{1}{5} \div 3\frac{2}{3} =$
9. $6\frac{7}{12} \div 3 =$
10. $16 \div \frac{1}{2} =$
11. $\frac{1}{2} \div 2 =$
12. $6\frac{1}{5} \div 2\frac{1}{15} =$
13. $18 \div 4\frac{1}{2} =$
14. $31 \div \frac{1}{4} =$
15. $0 \div 2\frac{1}{5} =$

MIXED PRACTICE

301. Mixed Fundamentals of Fractions. (For Self-Help Practice.) No example in this article should trouble a sixth-grade pupil. Can you do them all correctly without help? Check your answers with those on page 438.

1. *Add:*
$2\frac{1}{2}$
$3\frac{3}{4}$
$6\frac{5}{8}$

2. *Subtract:*
$6\frac{1}{2}$
$2\frac{3}{4}$

3.
$7\frac{1}{4}$
$2\frac{5}{6}$
$6\frac{2}{3}$

4. *Find:*
$\frac{2}{3}$ of $\frac{9}{20}$

5.
$\frac{4}{5} \div 6 = ?$

6. *Subtract:*
$11\frac{1}{5}$
$4\frac{7}{10}$

7.
$2\frac{1}{3} \times 4\frac{1}{5} = ?$

8.
$4\frac{5}{12}$
$7\frac{1}{4}$
$1\frac{3}{8}$
$9\frac{5}{6}$

9.
$3\frac{5}{6} - 1\frac{3}{8} = ?$

10.
$\frac{7}{8} \div 3\frac{1}{3} = ?$

11.
$7\frac{1}{2} \div 1\frac{2}{3} = ?$

12.
$9 \times 4\frac{1}{4} = ?$

302. Supplementary Practice on Fractions. (For Self-Help Practice.) If you find this exercise hard you should do extra work on fractions. References for study are:

Addition	Art. 265	Multiplication	Art. 284
Subtraction	Art. 277	Division	Art. 296

1. $\frac{1}{2}+\frac{1}{4}$	2. $\frac{2}{3}-\frac{1}{6}$	3. $\frac{4}{5}\times\frac{1}{3}$	4. $8\div\frac{1}{5}$
5. $\frac{3}{4}+\frac{5}{8}$	6. $\frac{2}{9}\div\frac{1}{2}$	7. $\frac{9}{10}-\frac{2}{5}$	8. $\frac{7}{8}$ of $\frac{16}{25}$
9. $4\frac{1}{3}+2\frac{3}{4}$	10. $2\frac{2}{5}\div 4$	11. $4\frac{1}{2}-2\frac{2}{3}$	12. $2\frac{1}{3}$ of 6
13. $\frac{8}{9}\times\frac{9}{8}$	14. $\frac{2}{7}+1\frac{1}{3}$	15. $2\frac{9}{16}-1\frac{1}{4}$	16. $3\frac{1}{2}\div 7$

Answers are given on page 438.

303. Finding Errors in Fraction Work. (For Self-Help Practice.) Do you understand fractions well enough to find and correct errors? **There are many mistakes in the work below.** Try to find and correct all of the mistakes. Compare your corrected answers with those on page 439.

1.
$\frac{2}{4}+\frac{3}{5}=\frac{6}{9}$ or $\frac{2}{3}$

2.
$4\frac{1}{2}\quad\frac{4}{8}$
$3\frac{3}{4}\quad\frac{6}{8}$
$7\frac{1}{8}\quad\frac{1}{8}$
$1\frac{1}{2}\quad\frac{12}{8}=1\frac{1}{2}$
$15\frac{1}{2}$

3.
$3\frac{1}{5}$
$8\frac{2}{10}$
$9\frac{3}{5}$
20

4.
8
$-4\frac{1}{2}$
$4\frac{1}{2}$

5.
$7\frac{2}{3}$
-3
$3\frac{1}{3}$

6.
$4\frac{2}{5}$
$-2\frac{1}{2}$
$2\frac{7}{10}$

7.
$\frac{1}{2}\times\frac{1}{2}=\frac{2}{4}$

8.
$4\times\frac{7}{8}=\frac{28}{8}=4\frac{4}{8}=4\frac{1}{2}$

9.
$\frac{\overset{1}{\cancel{4}}}{\cancel{9}_{2}}\times\frac{1}{\cancel{8}_{2}}=\frac{1}{4}$

10.
$\frac{3}{4}\div\frac{5}{6}=\frac{\overset{3}{\cancel{6}}}{4}\times\frac{3}{\underset{2}{\cancel{5}}}=\frac{9}{5}$

11.
$9\div\frac{2}{3}=\frac{1}{9}\times\frac{2}{3}=\frac{2}{27}$

12.
$\frac{4}{5}\div 2=\frac{4}{5}\times\frac{1}{2}=\frac{4}{10}$

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

304. How to Use the Lessons on the Addition and Subtraction of Denominate Numbers. (SILENT READING.) If you are having trouble with the addition and subtraction of denominate numbers, you will find the next articles very helpful. The chart below will show you which articles you may need to study.

Art. 305 deals with changing the form of denominate numbers.
Art. 306 explains and gives practice on changing denominate numbers to good forms for answers.
Art. 307 explains how to add denominate numbers.
Art. 308 gives some hints on changing the form of answers for addition examples.
Art. 309 gives practice on adding denominate numbers.
Art. 310 explains subtracting denominate numbers.
Art. 311 gives practice in subtracting denominate numbers.
Art. 312 gives further practice on subtracting denominate numbers.

305. Changing the Form of Denominate Numbers. (FOR SELF-HELP STUDY.) In sixth-grade arithmetic we sometimes have to change the form of a denominate number. This work is not hard if you can do everything in this article correctly and easily.

IMPORTANT FACT NUMBER 1. In order to change the form of denominate numbers, you must **know** the facts in the Tables of Measure on page 456. Do you really know these facts?

IMPORTANT FACT NUMBER 2. To change the form of denominate numbers you must be able to **use** the facts in the Tables of Measure. The next statements show some changed forms of four denominate numbers.

16 in. = **1 ft. 4 in.**	There are 12 inches in 1 foot.
$\frac{1}{4}$ lb. = **4 oz.**	There are 16 oz. in 1 lb.
$3\frac{1}{2}$ ft. = **42 in.**	1 ft. = 12 in. $\frac{1}{2}$ ft. = 6 in.
$2\frac{1}{2}$ pk. = **20 qt.**	There are 8 qt. in 1 pk.

When you know the Tables of Measure, turn back to page 15 and work Test (A) again.

306. The Form of Answers in the Addition of Denominate Numbers. (FOR SELF-HELP STUDY AND PRACTICE.) In the example at the right, the answer should be reduced, for we do not leave 12 inches or more in answer form. We change amounts of 12 inches or more to feet or to feet and inches.

2 ft.	6 in.
4 ft.	9 in.
4 ft.	2 in.
10 ft.	17 in.

Since 17 in. equals 1 ft. 5 in., the correct form of the answer to this example is 11 ft. 5 in.

Whenever you have 12 inches or more in any part of an answer, change the inches to feet or to feet and inches. Whenever you have two or three denominate numbers in an answer, change the numbers to larger units if possible.

You will see that this has been done in the six answers on the next page. The numbers at the left are answers to addition examples as first found. The correct form for answers as final results are shown at the right. In answer (*b*) two changes are made: in. to ft., then ft. to yd.

First Found Answers	Final Results
(*a*) 4 yd. 7 ft. 3 in.	6 yd. 1 ft. 3 in.
(*b*) 1 yd. 7 ft. 20 in.	3 yd. 2 ft. 8 in.
(*c*) 6 qt. 11 pt.	2 gal. 3 qt. 1 pt.
(*d*) 4 hr. 100 min.	5 hr. 40 min.
(*e*) 3 pk. 14 qt. 8 pt.	5 pk. 2 qt.
(*f*) 6 lb. 40 oz.	8 lb. 8 oz. ($8\frac{1}{2}$ lb.)

EXERCISE

Can you change the following numbers to correct forms for final answers? Check your answers with the correct forms given on page 439.

1. 6 ft. 18 in. **4.** 9 lb. 21 oz. **7.** 6 sq. ft. 150 sq. in.

2. 4 yd. 7 ft. **5.** 4 pk. 7 qt. 8 pt. **8.** 4 sq. yd. 14 sq. ft.

3. 3 qt. 5 pt. **6.** 6 hr. 110 min. **9.** 2 cu. yd. 32 cu. ft.

307. How to Add Denominate Numbers. (For Self-Help Study.) This article tells you how to add denominate numbers. The problems show the best way to do it.

Problem I. One summer Mary's older brother won first prize for raising corn. His reward was a trip to Washington, D. C. On the train to Washington, he kept a record of the time between stations.

Between home and the first station it took 1 hr. and 14 min.; between the first and second stations, 56 min.; between the second and third, 1 hr. and 29 min. How long had he been traveling when he reached the third station?

$$\begin{array}{r} 1\frac{14}{60} \text{ hr.} \\ \frac{56}{60} \text{ hr.} \\ 1\frac{29}{60} \text{ hr.} \\ \hline \end{array}$$

We can solve this problem as shown at the right, but it is easier to do it the new way shown on the next page. Be sure you understand it.

STEP I. Write the numbers which mean hours unde
Hr. Write the numbers which mean minutes under *Mi*

		Hr.	*Min.*
STEP I.		1	14
			56
		1	29
STEP II.	*Add:*	2	99*
		1*	39*
STEP III.	*Add:*	3 hr.	39 min.,
			Ans.

After adding (Step II) the 99 min. must b changed to hr. and mi

99* min. = 1* hr. 39* mi

The 99 min. in reduce form is added to the 2 h Put a line around the 9 min. to be sure that onl 1 hr. 39 min. is adde (Do not add **39** and **99**

PROBLEM II. Yesterday Jack delivered 4 gal. 3 qt. of milk t a bakery, 4 gal. 3 qt. to an ice cream stand, and 5 gal. 1 qt. t a milk depot. How much milk did Jack deliver?

		Gal.	*Qt.*
STEP I.		4	3
		4	3
		5	1
STEP II.	*Add:*	13	7*
		1*	3*
STEP III.	*Add:*	14 gal.	3 qt.,
			Ans.

After adding (Step II), qt. must be changed to ga and qt.

7* qt. = 1* gal. 3* qt.

The 7 qt. in reduced for is added to the 13 gal. Pu a line around the **7** qt. t be sure that only 1 gal. 3 q is added. Do not add th **7** and the **3**.

308. Six Helps in Adding Denominate Numbers. (Fo SELF-HELP STUDY.) Study the following statements:

1. When an answer comes out 67 min., it should b written as 1 hr. 7 min.

2. 15 in. in an answer should be written 1 ft. 3 in.

3. 21 oz. in an answer should be written 1 lb. 5 oz.

4. When there are more feet than 3 in an answer, change the feet to yards and feet.

5. When there are more ounces than 16 in an answer, change the ounces to pounds and ounces.

6. When there are more minutes than 60 in an answer, change the minutes to hours and minutes.

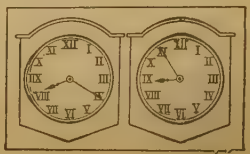
309. Practice in Adding Denominate Numbers. (For Self-Help Practice.) Find the sums. Remember how answers should be written. If you need to, study Art. 307 again. Check your work with the answers on page 439.

1.	2.	3.	4.
2 hr. 30 min.	1 lb. 8 oz.	4 ft. 1 in.	1 pk. 2 qt.
1 hr. 15 min.	2 lb. 3 oz.	2 ft. 9 in.	2 pk. 3 qt.
45 min.	4 lb. 7 oz.	3 ft. 9 in.	1 pk. 1 qt.
1 hr. 15 min.	1 lb. 4 oz.	1 ft. 5 in.	3 pk. 1 qt.

5.	6.	7.	8.
4 ft. 8 in.	9 hr. 27 min.	4 da. 16 hr.	4 gal. 1 qt.
3 ft.	6 hr. 12 min.	3 da.	3 qt.
6 ft. 1 in.	7 hr. 36 min.	5 da. 6 hr.	4 gal. 1 qt.
2 ft. 6 in.	4 hr. 45 min.	4 da. 10 hr.	4 gal. 2 qt.

310. How to Subtract Denominate Numbers. (For Self-Help Study and Practice.) Study Problems I to III to see how to subtract denominate numbers.

Problem I. I started for school at 8:20 and got there at 8:55. How long did it take me to go to school?



8 hr. 55 min.
8 hr. 20 min.
0 hr. 35 min., *Ans.*

PROBLEM II. Mr. Jones is 6 ft. 2 in. tall, and Mr. White is 5 ft. 9 in. tall. How much taller is Mr. Jones?

6 ft. 2 in.		5 ft. 14 in.
5 ft. 9 in.	Think →	5 ft. 9 in.
		0 ft. 5 in., *Ans.*

9 in. cannot be subtracted from 2 in. But the number 6 ft. 2 in. can be changed **in form.** Since 1 ft. = 12 in., we can think of the number 6 ft. 2 in. as 5 ft. 14 in. We can change the form of a number to help us in subtraction.

PROBLEM III. A gasoline tank holds 15 gal. 1 qt. How much gasoline can still be put into the tank if there are 6 gal. 3 qt. already in it?

15 gal. 1 qt. (Tank will hold)
6 gal. 3 qt. (Already in)
8 gal. 2 qt. (Can be put in)

1. Can 3 qt. be subtracted from 1 qt.?

2. 1 gal. = ? qt.

3. How can 15 gal. 1 qt. be changed in form to help in the subtraction?

311. Practice in Subtracting Denominate Numbers. (FOR SELF-HELP PRACTICE.) There are answers on page 439. Use them **only** as a check.

In Example 2, remember that 1 hr. = 60 min.; in Example 3, remember that 1 pk. = 8 qt.; etc.

1.	2.	3.	4.
6 ft. 1 in.	8 hr. 25 min.	14 pk. 2 qt.	9 gal. 1 qt.
4 ft. 8 in.	2 hr. 55 min.	12 pk. 6 qt.	7 gal. 3 qt.
5.	**6.**	**7.**	**8.**
6 lb. 9 oz.	10 T. 1050 lb.	1 hr. 10 min.	8 lb. 4 oz.
2 lb. 14 oz.	2 T. 1650 lb.	55 min.	6 lb. 11 oz.

312. Further Work in Subtraction of Denominate Numbers. (For Self-Help Study and Practice.) The subtraction of denominate numbers is one subject in which you will have to "keep your wits about you." Study the next subtraction examples.

(*A*)	(*B*)
7 ft. 9 in.	3 ft. 4 in.
3 ft. 4 in.	1 ft. 8 in.
4 ft. 5 in.	1 ft. 8 in.
There is no trouble in Example (*A*) above.	Think: 3 ft. 4 in. = 2 ft. 16 in. Then subtract.

(*C*)	(*D*)
5 qt.	55 min. 10 sec.
3 qt. 1 pt.	45 min. 50 sec.
1 qt. 1 pt.	9 min. 20 sec.
Think: 5 qt. = 4 qt. 2 pt. Then subtract.	Think: 55 min. 10 sec. = 54 min. 70 sec. Then subtract.

It will pay you to practice on the subtraction examples below. All of them can be checked by addition. Try checking them. Use the answers on page 439 **after** you have finished checking.

1.	2.	3.
2 ft. 4 in.	5 sq. ft. 82 sq. in.	37 min. 6 sec.
1 ft. 9 in.	3 sq. ft. 105 sq. in.	28 min. 10 sec.
4.	**5.**	**6.**
2 pk. 3 qt.	3 lb. 2 oz.	5 cu. yd. 3 cu. ft.
1 pk. 5 qt.	1 lb. 11 oz.	2 cu. yd. 4 cu. ft.

MENSURATION

313. How to Use the Lessons on Mensuration. (SILENT READING.) The chart below shows you the articles to use in restudying anything about mensuration which you do not now understand very well.

Art. 314 explains the meaning of important words used in mensuration.

Art. 315 explains the finding of areas of rectangles.

Art. 316 gives practice in finding areas.

Art. 317 deals with the changing of dimensions to the same unit before using the dimensions in finding areas.

Art. 318 shows the relation of foot and square foot.

Art. 319 explains and gives practice on the finding of volumes of rectangular solids.

314. Words Used in Mensuration. (FOR SELF-HELP STUDY.) This article shows you the meaning of many important words often used in mensuration.

THE MEANING OF SOME IMPORTANT WORDS

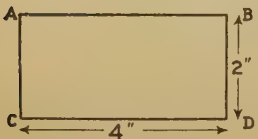


FIG. 1

1. Fig. 1 is called a **rectangle.** A rectangle is a surface or area with opposite sides that are equal and parallel and which has four **right** angles (square corners).

2. Fig. 1 has a perimeter of 12″. **Perimeter** means the **distance around.** Is 12″ correct for the perimeter of Fig. 1?

3. The area of Fig. 1 is 8 sq. in. **Area** is the **amount of surface** covered.

4. The **length** and the **width** of a rectangle are called its **dimensions.**

5. A rectangular **solid** has three dimensions: length, width, and **height** or **depth.** Volume is measured in **cubic** units.

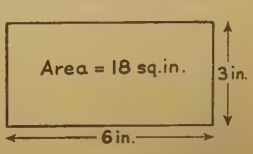
6. **Scale drawing** means drawing an object so that a certain distance on the drawing represents a certain distance on the object. The **scale** shows exactly how much larger or smaller than the object the drawing is. Thus, $1'' = 5$ mi. means that 1 inch on the drawing represents 5 miles of the actual size of the object drawn.

315. How to Find Areas of Rectangles. (For Self-Help Study and Practice.) If you are having trouble with problems which ask you to find areas, you have probably forgotten some facts which you need to know. After you have studied this article, if "area" still troubles you, ask your teacher for more help.

Read carefully the rule given below and then use it to find the area of the rectangles in Article 316.

> *Rule:* **To find the area of a rectangle, express the length and width in the same units, then multiply the length by the width. Area is expressed in square measure.**

1. The **area** is the **amount of space,** or the **surface,** covered by the figure you are using in the problem.



2. The **amount** of area is the product of the dimensions when the dimensions are expressed in the **same unit.**

3. Area is expressed in **square measure.**

316. Finding Areas. (For Self-Help Practice.) The letters "in." mean inch or inches. What do these mean ft., yd., mi.? "×" means "by" or "times." Thus the measurements of a rectangle 4 inches wide and 5 inches long are written "4 in. ×5 in." or "4 in. by 5 in."

Give the area of rectangles with these measurements:

1. 3 in. wide, 5 in. long.
2. 4 in. wide by 8 in. long.
3. 6 in. wide times 9 in. long.
4. 5 in. ×7 in.
5. 9 ft. ×10 ft.
6. 4 yd. ×6 yd.
7. 8 mi. ×11 mi.
8. 7 ft. ×10 ft.

☛ The correct answers are given on page 439.

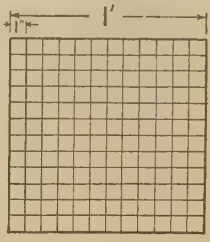
317. Using Dimensions in Finding Areas. (For Self-Help Study.) **In finding areas, dimensions must be in same unit.** When dimensions are in different units, one of the dimensions must be changed so that both will be in the same unit before multiplying to find areas. The table below will help.

Table of Linear (Length) Measure

12 inches (in.) = 1 foot (ft.)
3 ft. = 36 in. = 1 yard (yd.)
$5\frac{1}{2}$ yd. = $16\frac{1}{2}$ ft. = 1 rod (rd.)
320 rd. = 5280 ft. = 1 mile (mi.)

Dimensions	*Area*	
(*a*) 2 ft. ×4 ft.	8 sq. ft.	(Correct)
(*b*) 3 in. ×5 in.	15 sq. in.	(Correct)
(*c*) 4 yd. ×4 yd.	16 sq. yd.	(Correct)
(*d*) 5 mi. ×6 mi.	30 sq. mi.	(Correct)
(*e*) 3 in. ×2 yd.	?	(Make units of length the same)
(*f*) 4 yd. ×7 ft.	?	(Make units of length the same)
(*g*) 7 in. ×3 ft.	?	(What must be done?)

318. The Foot and the Square Foot. It may seem strange that 12 inches equal 1 foot, but 144 square inches equal 1 **square** foot. If you stop to think about the relation between length and area, you will see that these statements **have to be true.**

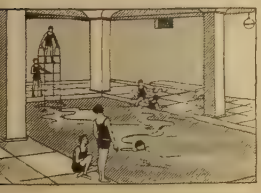


12 inches = 1 foot, but
144 square inches = 1 square foot.

Count the square inches in the drawing to see if an area 1 ft. by 1 ft. **really does** equal 144 sq. in.

VOLUMES

319. How to Find Volumes of Rectangular Solids. (For Self-Help Study and Practice.) In this article you can review how to work problems in the mensuration of volumes.



Problem 1. How much water is there in this diving tank? It is 14 ft. long and 8 ft. wide. The water is 6 ft. deep on the average.

The amount of water in the tank is called the **volume** of water. The unit of measure of the volume of the water is **1 cubic foot** of water. The number of cubic feet of water in the tank is found like this:

14 ft. × 8 ft. × 6 ft.

Length × width × height = volume, or **length × width × depth = volume.**

Express all dimensions in the same unit. Volume is expressed in **cubic** units.

14
×8
112
×6
672 cu. ft. of water.

PROBLEM 2. How many cubic inches ar there in this block of ice, which is 6 in. wide 8 in. high, and 10 in. long?

$$6'' \times 10'' \times 8'' = 480 \text{ cu. in.}$$

Length × width × height = volume.

The ice shown in the picture is in the form of a **rectangular solid.** The word "solid" ha a special meaning in arithmetic. It really means shape Thus air and water can be in the form of a rectangular solid

PRACTICE IN FINDING VOLUMES

Find the volumes of the solids whose dimensions ar given below. See if your volumes are the same as thos on page 439.

1. A box $2'' \times 4'' \times 8''$.

2. A room $18' \times 10' \times 12'$.

3. A book $\frac{1}{2}$ in. $\times$ $4\frac{1}{2}$ in. $\times$ 6 in. (Change to $\frac{1}{2} \times \frac{9}{2} \times 6$ Can you cancel?)

4. A chest $1\frac{1}{2}' \times 1'\ 6'' \times 2'$.

5. A tank 4 ft. $\times$ 8 ft. $\times$ 4 yd

DECIMAL FRACTIONS

320. Self-Help Lessons on Decimal Fractions. (SILENT READING.) Here are the articles to use when you need t review important facts about decimals.

Art. 321 explains the "hard spot" in adding and subtracting decimal numbers.

Art. 322 explains how to read decimal numbers.

Art. 323 explains the meaning of place value of figures in a number.

321. The "Hard Spot" in the Addition and Subtraction of Decimal Fractions. (For Self-Help Study and Practice.) The one thing to remember about the addition and subtraction of decimal fractions is that the **decimal points must be kept in a straight line.**

The next samples show you what it means to keep decimal points in a straight line.

Addition

Correct	Wrong
47.6 2.68 19.3 .45 70.03	47.6 2.68 19.3 .45 9.82

Subtraction

Correct	Wrong
8.720 .236 8.484	8.720 .236 .6360

See if you can copy the next examples and work them correctly. Use the answers on page 439 as a check of the accuracy of your own work.

1. *Add:* 6.34, 17.2, 18.05, .75.
2. $17.367 - 8.072 = ?$
3. $243.6 - 121.43 = ?$
4. $75 - 24.25$
5. $7 + 2.7 + 3.5$
6. $81.004 - 75$

322. How to Read Decimals. (For Self-Help Study.) The **first** place to the **left** of the decimal point is **units** or **ones.** (Thus, 7. means 7 units or 7 **ones.**) The **first** place to the **right** of the decimal point is **tenths.** (Thus, .7 means 7 tenths or $\frac{7}{10}$.) The **second** place to the **left** is tens. (Thus 70. means 7 tens.) The **second** place to the **right** is **hundredths.** (Thus, .07 means 7 hundredths or $\frac{7}{100}$.)

The decimal point divides a number into two parts; the figures on the **left** of the points always represent **whole numbers,** while the figures on the **right** always represent **decimal fractions.**

The number 45.87 is read: Forty-five **and** eighty-seven hundredths.

The number .034 is read: Thirty-four thousandths.

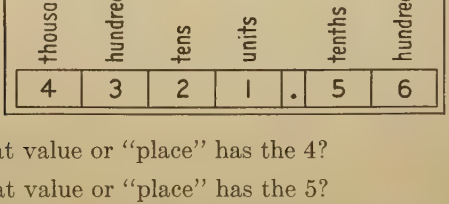
The number 5.09 is read: Five **and** nine hundredths.

Notice that the word "and" is used to represent the decimal point.

See if you can read these numbers:

54.687 678.098 13.826

323. The Value of Place in Numbers. (For Self-Help Study.) The number 4321.56 is shown in the diagram. Answer the questions below the diagram.



1. What value or "place" has the 4?
2. What value or "place" has the 5?
3. Read the facts about "place value" below.

In the numbers below, does the **7** always mean the same thing; that is, does it represent the same amount each time? If not, tell what is the value of **7** each time it occurs.

27 72 720 2.7 2.07 2.007 72,654.

Any figure such as 2, 5, or 7 tells only the **number** of ones, or units. The **place** where the figure stands tells whether the figure means ones or tens, or hundreds, or tenths, or thousands, or hundredths, etc.

Each **place** represents just **ten** times as much as the **place** to its **right.** Thus, 70 is ten times as much as 7; 700 is ten times as much as 70; and 7000 is ten times as much as 700, etc.

Another way to think of place value would be: Each place is just **one tenth** as much as the place to its **left.** Thus, 7 is **one tenth** as much as 70; 80 is **one tenth** as much as 800; and .04 is **one tenth** as much as .4, etc.

ANSWER KEY FOR SELF-HELP LESSONS

These answers are placed here to help you learn how to work different sorts of examples correctly and accurately. They will not do this if you use the answers before you have solved the examples. Be honest with yourself and use the answers **after** you have worked the examples.

Answers for Article 240. (*Helping Yourself in Addition.*)

1. 302	**3.** 286	**5.** 23,749	**7.** 27,335
2. 1837	**4.** 281	**6.** $207.01	**8.** $34.75

Answers for Article 246. (*Final Practice in Subtraction.*)

1. 4634	**7.** 5819	**13.** 3706	**19.** 1416
2. 2539	**8.** 56,417	**14.** 10,244	**20.** 5220
3. 35,090	**9.** 7709	**15.** $91.31	**21.** 8158
4. 357	**10.** 7589	**16.** 6304	**22.** $23.87
5. 1852	**11.** 9686	**17.** 4099	**23.** 2578
6. 2752	**12.** $45.84	**18.** 869	**24.** 2335

Answers for Article 253. (*Final Practice in Multiplication.*)

1. 168,701	**6.** 136,354	**11.** 107,379	**16.** 82,688
2. 136,202	**7.** 167,331	**12.** $4801.50	**17.** 74,893
3. 48,216	**8.** 94,068	**13.** 343,332	**18.** $266 (26600¢
4. 40,460	**9.** 87,725	**14.** 290,146	**19.** 107,973
5. 271,312	**10.** 82,440	**15.** 187,627	**20.** $1086.78

Answers for Article 255. (*Finding Errors in Uneven Division.*)

2(C). 5 rem.	**3(E).** 5 rem.	**5(E).** 2 rem.	**7(C).** 1 rem.
2(D). 6 rem.	**4(B).** 1 rem.	**6(C).** 6 rem.	**7(F).** 8 rem.
2(G). 6 rem.	**4(C).** 4 rem.	**7(B).** 5 rem.	**7(G).** No rem

Answers for Article 259. (*Practice in Finding Quotients.*)

EXERCISE (A)

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	$5\frac{3}{13}$	$3\frac{21}{22}$	$5\frac{81}{83}$	$7\frac{80}{81}$	$7\frac{17}{23}$
2.	$3\frac{91}{93}$	$3\frac{39}{43}$	$6\frac{40}{41}$	$11\frac{3}{14}$	$6\frac{25}{26}$
3.	$5\frac{29}{67}$	$7\frac{89}{91}$	$5\frac{41}{42}$	$5\frac{51}{53}$	$8\frac{91}{94}$
4.	$7\frac{20}{21}$	$7\frac{50}{51}$	$8\frac{35}{36}$	$6\frac{38}{41}$	$4\frac{67}{73}$
5.	$7\frac{30}{31}$	$6\frac{66}{71}$	$2\frac{31}{32}$	$6\frac{31}{33}$	$3\frac{42}{43}$
6.	$6\frac{4}{59}$	$6\frac{1}{16}$	$6\frac{21}{26}$	$7\frac{12}{67}$	$7\frac{4}{49}$
7.	$4\frac{3}{14}$	$6\frac{5}{38}$	$8\frac{16}{77}$	$7\frac{9}{68}$	$8\frac{5}{79}$
8.	$7\frac{19}{37}$	$6\frac{3}{14}$	$3\frac{2}{9}$	$7\frac{5}{39}$	$6\frac{4}{29}$
9.	$4\frac{2}{19}$	$5\frac{1}{17}$	$5\frac{3}{19}$	$5\frac{7}{8}$	$7\frac{19}{47}$

EXERCISE (B)

1.	$9\frac{7}{59}$	$9\frac{2}{7}$	$9\frac{17}{43}$	$9\frac{8}{19}$	$9\frac{25}{77}$
2.	$9\frac{8}{47}$	$9\frac{7}{69}$	$9\frac{1}{17}$	$9\frac{7}{44}$	$9\frac{5}{39}$
3.	$7\frac{6}{29}$	$8\frac{2}{49}$	$8\frac{1}{37}$	$6\frac{1}{26}$	$8\frac{29}{66}$
4.	$8\frac{27}{76}$	$8\frac{7}{34}$	$8\frac{1}{79}$	$8\frac{19}{77}$	$4\frac{1}{17}$

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
5.	$7\frac{5}{39}$	$6\frac{2}{29}$	$4\frac{1}{3}$	$5\frac{1}{2}$	$7\frac{9}{26}$
6.	$5\frac{13}{17}$	$9\frac{7}{44}$	$9\frac{3}{49}$	$9\frac{24}{37}$	$5\frac{1}{19}$
7.	$9\frac{7}{67}$	$8\frac{11}{12}$	$8\frac{55}{57}$	$8\frac{21}{23}$	$8\frac{45}{46}$
8.	$8\frac{7}{8}$	$8\frac{6}{29}$	$8\frac{3}{7}$	$8\frac{6}{19}$	$8\frac{5}{39}$
9.	$6\frac{29}{31}$	$8\frac{41}{42}$	$4\frac{1}{14}$	$8\frac{25}{27}$	$6\frac{57}{86}$

Answers for Article 260. (*Practice with Zero in Quotient, Page 384.*)

1. 602 2. $206\frac{7}{73}$ 3. 509 4. $603\frac{33}{89}$ 5. 206 bu. 4 lb. over 6. $805

Answers for Article 260. (*Practice with Final Zero, Page 386.*)

1. 830 2. $290\frac{22}{63}$ 3. 620 4. $720\frac{5}{66}$ 5. 130 mi. 6. $1.40

Answers for Article 261. (*Helping Yourself in Division.*)

1. 83	5. 53	9. $84\frac{10}{57}$	13. $36\frac{12}{13}$	17. 253
2. $34\frac{33}{46}$	6. $57\frac{20}{23}$	10. 514	14. $67\frac{10}{27}$	18. 75
3. 73	7. $58\frac{5}{59}$	11. $772\frac{14}{17}$	15. 892	19. $463\frac{13}{15}$
4. $26\frac{9}{10}$	8. $57\frac{26}{35}$	12. $707\frac{77}{79}$	16. $768\frac{7}{36}$	20. $7308\frac{13}{47}$

Answers for Article 262. (*Final Practice in Division.*)

1. $48\frac{3}{17}$	7. 62	13. 89	19. 23
2. 74	8. 61	14. 72	20. 160
3. 40	9. 95	15. $50\frac{7}{31}$	21. 71
4. 87	10. $96\frac{18}{51}$	16. 26	22. 83
5. $23\frac{12}{59}$	11. 601	17. 408	23. 85
6. 73	12. 41	18. $900\frac{13}{27}$	24. $59\frac{7}{36}$

Answers for Article 264. (*Review of Vocabulary.*)

1. Difference	4. Product	7. Divisor	10. Difference
2. Dividend	5. Multiplicand	8. 13	11. Quotient
3. 8937	6. Subtrahend	9. Sum	12. Remainder

Answers for Article 268. (*Practice in Finding Common Denominators.*)

1. $\frac{}{30}$	**6.** $\frac{}{99}$	**11.** $\frac{3}{4}$	**16.** $\frac{1}{2}$	**21.** $\frac{2}{3}$
2. $\frac{}{18}$	**7.** $\frac{}{48}$	**12.** $\frac{5}{8}$	**17.** $\frac{3}{14}$	**22.** $\frac{5}{12}$
3. $\frac{}{14}$	**8.** $\frac{}{40}$	**13.** $\frac{3}{10}$	**18.** $\frac{5}{12}$	**23.** $1\frac{1}{24}$
4. $\frac{}{6}$	**9.** $\frac{}{30}$	**14.** $\frac{5}{18}$	**19.** $\frac{1}{12}$	**24.** $4\frac{1}{8}$
5. $\frac{}{30}$	**10.** $\frac{}{63}$	**15.** $\frac{4}{15}$	**20.** $\frac{7}{9}$	**25.** $1\frac{5}{12}$

Answers for Article 270. (*Practice in Reducing Answers.*)

1. $\frac{2}{3}$	**2.** $3\frac{1}{2}$	**3.** $10\frac{2}{3}$	**4.** $7\frac{2}{3}$	**5.** 1	**6.** $7\frac{1}{2}$
7. $1\frac{2}{5}$	**8.** $4\frac{1}{2}$	**9.** 3	**10.** $7\frac{1}{5}$	**11.** $5\frac{1}{2}$	**12.** $3\frac{1}{2}$

Answers for Article 271. (*Changing Mixed Numbers to Good Forms for Answers.*)

1. $7\frac{1}{3}$	**2.** $1\frac{7}{8}$	**3.** $5\frac{2}{5}$	**4.** 8	**5.** $4\frac{1}{2}$	**6.** $3\frac{1}{2}$	**7.** $2\frac{1}{2}$

Answers for Article 272. (*Supplementary Practice in Reducing Answers.*)

1. $\frac{2}{3}$	**2.** $3\frac{1}{2}$	**3.** $4\frac{3}{4}$	**4.** $2\frac{1}{5}$	**5.** $\frac{3}{4}$	**6.** 3
7. $2\frac{1}{4}$	**8.** $\frac{3}{4}$	**9.** $7\frac{1}{2}$	**10.** $2\frac{2}{3}$	**11.** $1\frac{1}{2}$	**12.** $12\frac{1}{10}$

Answers for Article 274. (*Practice in Adding Mixed Numbers.*)

1. 1	**9.** $12\frac{1}{6}$	**17.** $9\frac{12}{17}$	**25.** $3\frac{11}{15}$
2. $6\frac{1}{12}$	**10.** $1\frac{13}{14}$	**18.** $5\frac{11}{16}$	**26.** 14
3. $\frac{4}{5}$	**11.** $1\frac{1}{20}$	**19.** $8\frac{1}{4}$	**27.** $\frac{5}{9}$
4. $\frac{9}{10}$	**12.** $1\frac{13}{20}$	**20.** $\frac{2}{3}$	**28.** $7\frac{4}{5}$
5. $6\frac{1}{4}$	**13.** $\frac{7}{8}$	**21.** $13\frac{3}{5}$	**29.** $5\frac{31}{40}$
6. $1\frac{1}{9}$	**14.** $4\frac{3}{4}$	**22.** $1\frac{12}{35}$	**30.** $3\frac{29}{30}$
7. $10\frac{2}{3}$	**15.** $\frac{6}{7}$	**23.** $9\frac{5}{12}$	**31.** $5\frac{2}{3}$, 1, $2\frac{1}{2}$, 4
8. $12\frac{5}{18}$	**16.** $1\frac{2}{3}$	**24.** $\frac{18}{77}$	**32.** $\frac{}{12}$

Answers for Article 275. (*Supplementary Practice on Changing the Form of Fractions.*)

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$\frac{3}{9}$	$\frac{2}{6}$	$\frac{6}{30}$	$\frac{6}{10}$	$\frac{10}{18}$	$\frac{22}{24}$
2.	$\frac{7}{14}$	$\frac{3}{15}$	$\frac{4}{12}$	$\frac{20}{24}$	$\frac{3}{18}$	$\frac{9}{24}$
3.	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{8}{36}$	$\frac{15}{24}$	$\frac{25}{30}$
4.	$\frac{2}{6}$	$\frac{5}{10}$	$\frac{9}{12}$	$\frac{4}{14}$	$\frac{6}{14}$	$\frac{21}{36}$

Answers for Article 276. (*Supplementary Practice on Common Denominators.*)

	(A)	(B)	(C)	(D)
1.	$\frac{}{8}$	$\frac{}{7}$	$\frac{}{30}$	$\frac{}{12}$
2.	$\frac{}{10}$	$\frac{}{11}$	$\frac{}{36}$	$\frac{}{18}$
3.	$\frac{}{9}$	$\frac{}{6}$	$\frac{}{14}$	$\frac{}{14}$
4.	$\frac{}{12}$	$\frac{}{12}$	$\frac{}{14}$	$\frac{}{15}$
5.	$\frac{}{6}$	$\frac{}{24}$	$\frac{}{12}$	$\frac{}{20}$

Answers for Article 279. (*Practice in Changing the Form of Fractions.*)

1. $\frac{}{15}$	5. $\frac{}{6}$	9. $\frac{}{8}$	13. $\frac{}{12}$	17. $\frac{}{18}$
2. $\frac{}{4}$	6. $\frac{}{6}$	10. $\frac{}{8}$	14. $\frac{}{12}$	18. $\frac{}{40}$
3. $\frac{}{12}$	7. $\frac{}{10}$	11. $\frac{}{20}$	15. $\frac{}{15}$	19. $\frac{}{12}$
4. $\frac{}{10}$	8. $\frac{}{12}$	12. $\frac{}{10}$	16. $\frac{}{16}$	20. $\frac{}{16}$

Answers for Article 280. (*Practice with Denominators in Easy Subtraction.*)

1. $\frac{1}{15}$	3. $\frac{17}{24}$	5. $\frac{13}{20}$	7. $\frac{1}{12}$	9. $\frac{1}{24}$	11. $\frac{17}{40}$	13. $\frac{8}{15}$
2. $\frac{5}{12}$	4. $\frac{3}{10}$	6. $\frac{19}{30}$	8. $\frac{17}{60}$	10. $\frac{1}{30}$	12. $\frac{19}{60}$	14. $\frac{7}{40}$

Answers for Article 282. (*Practice in Subtracting Mixed Numbers.*)

1. $1\frac{2}{3}$	4. $3\frac{1}{3}$	7. $1\frac{1}{2}$	10. $\frac{2}{3}$	13. $3\frac{4}{7}$	16. $2\frac{1}{2}$	19. $2\frac{4}{9}$
2. $2\frac{1}{2}$	5. $2\frac{1}{2}$	8. 2	11. $5\frac{1}{2}$	14. $\frac{3}{4}$	17. $\frac{3}{10}$	20. $2\frac{2}{3}$
3. $2\frac{4}{7}$	6. $\frac{3}{4}$	9. $\frac{4}{7}$	12. $\frac{2}{5}$	15. $3\frac{3}{4}$	18. $\frac{5}{8}$	21. $2\frac{3}{8}$

Answers for Article 283. (*Supplementary Practice on Subtracting Fractions and Mixed Numbers.*)

1. $\frac{1}{8}$
2. $\frac{1}{36}$
3. $1\frac{4}{5}$
4. $4\frac{1}{10}$
5. 9
6. $10\frac{1}{18}$
7. $\frac{2}{9}$
8. $2\frac{5}{11}$
9. $\frac{2}{17}$
10. $\frac{1}{12}$
11. $1\frac{9}{21}$
12. $1\frac{1}{6}$
13. $\frac{19}{20}$
14. $2\frac{3}{4}$
15. $2\frac{5}{12}$
16. $6\frac{2}{3}$
17. $\frac{1}{6}$
18. $8\frac{5}{8}$
19. $7\frac{7}{20}$
20. $4\frac{1}{2}$
21. 10
22. $2\frac{4}{7}$
23. $\frac{1}{10}$
24. $7\frac{3}{5}$
25. $5\frac{7}{11}$
26. $\frac{1}{5}$
27. $4\frac{1}{6}$
28. $6\frac{1}{2}$
29. $5\frac{7}{12}$
30. $6\frac{19}{22}$

Answers for Article 286. (*Easy Practice in Multiplication of Fractions.*)

1. $\frac{8}{15}$
2. $\frac{15}{28}$
3. $\frac{2}{25}$
4. $\frac{8}{15}$
5. $\frac{1}{8}$
6. $\frac{5}{12}$
7. $3\frac{1}{3}$
8. $4\frac{1}{2}$
9. $\frac{1}{24}$
10. $4\frac{1}{5}$
11. $\frac{16}{49}$
12. $\frac{1}{4}$
13. $4\frac{4}{5}$
14. 3
15. $2\frac{2}{5}$
16. $4\frac{1}{2}$
17. $6\frac{2}{3}$
18. $\frac{3}{16}$
19. 49
20. 0

Answers for Article 289. (*Practice in Changing Mixed Numbers to Improper Fractions.*)

1. $\frac{9}{2}$
2. $\frac{77}{12}$
3. $\frac{25}{4}$
4. $\frac{25}{7}$
5. $\frac{10}{9}$
6. $\frac{11}{3}$
7. $\frac{39}{8}$
8. $\frac{14}{5}$
9. $\frac{33}{10}$
10. $\frac{55}{6}$
11. $\frac{62}{8}$
12. $\frac{171}{16}$
13. $\frac{58}{\blacksquare}$
14. $\frac{78}{9}$
15. $\frac{7}{3}$
16. $\frac{143}{12}$
17. $\frac{28}{5}$
18. $\frac{151}{12}$

Answers for Article 292. (*Practice in Cancellation in Multiplication.*)

1. $\frac{\overset{1}{2}}{\underset{1}{3}} \times \frac{\overset{1}{3}}{\underset{2}{4}} = \frac{1}{2}$
2. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
3. $\frac{\overset{1}{4}}{\underset{1}{5}} \times \frac{1}{\underset{1}{4}} = \frac{1}{5}$
4. $\frac{\overset{1}{2}}{\underset{6}{12}} \times \frac{1}{\underset{1}{3}} = \frac{1}{6}$
5. $\frac{1}{\underset{1}{3}} \times \overset{7}{21} = 7$
6. $\frac{\overset{1}{2}}{\underset{1}{3}} \times \frac{\overset{1}{3}}{\underset{2}{4}} = \frac{1}{2}$
7. $\frac{5}{\underset{3}{6}} \times \frac{1}{\underset{1}{2}} = \frac{5}{9}$
8. $\frac{7}{\underset{2}{8}} \times \frac{1}{\underset{1}{4}} = \frac{7}{10}$
9. $\frac{1}{\underset{1}{2}} \times \frac{\overset{2}{4}}{\underset{1}{5}} = \frac{2}{5}$

10. $\frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{15}}} \times \frac{1}{\underset{1}{\cancel{3}}} = \frac{3}{5}$

11. $\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}} \times \frac{1}{\underset{1}{\cancel{5}}} = 1$

12. $\frac{\overset{1}{\cancel{3}}}{8} \times \frac{1}{\underset{2}{\cancel{6}}} = \frac{1}{16}$

13. $\frac{1}{2} \times \frac{1}{\underset{4}{\cancel{8}}} = \frac{1}{20}$

14. $\frac{5}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{4} = \frac{5}{12}$

15. $\frac{25}{\underset{2}{\cancel{50}}} \times \frac{1}{\cancel{18}} = \frac{25}{2} = 12\frac{1}{2}$

16. $\frac{7}{8} \times \frac{3}{\underset{2}{\cancel{12}}} = \frac{21}{2} = 10\frac{1}{2}$

17. $\frac{4}{\underset{1}{\cancel{25}}} \times \frac{\overset{2}{\cancel{50}}}{} = 8$

18. $\frac{1}{2} \times \frac{\overset{9}{\cancel{18}}}{1} = 9$

19. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{4}{\cancel{20}}}{\underset{7}{\cancel{21}}} = \frac{4}{7}$

20. $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{5}{\cancel{10}}} = \frac{3}{5}$

21. $\frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{6}{\cancel{24}}}{} = 18$

22. $\frac{\overset{1}{\cancel{5}}}{16} \times \frac{3}{\underset{2}{\cancel{10}}} = \frac{3}{32}$

23. $\frac{1}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{12}}} = \frac{1}{3}$

24. $\frac{1}{\underset{1}{\cancel{6}}} \times \frac{\overset{2}{\cancel{12}}}{25} = \frac{2}{25}$

Answers for Article 294. (*Practice in Multiplying Mixed Numbers.*)

1. $23\frac{1}{4}$
2. $13\frac{1}{3}$
3. $85\frac{1}{2}$
4. $3\frac{1}{12}$
5. $16\frac{1}{5}$
6. $17\frac{3}{4}\frac{5}{8}$
7. $37\frac{4}{5}$
8. $4\frac{2}{3}\frac{7}{2}$
9. $\frac{7}{10}$
10. $15\frac{2}{5}$
11. $4\frac{1}{9}$
12. $4.14
13. 5
14. $32\frac{1}{15}$
15. 20
16. $9\frac{1}{6}$
17. $2\frac{1}{15}\frac{4}{5}$
18. $35\frac{1}{2}\frac{8}{5}$

Answers for Article 295. (*Supplementary Practice on Multiplication of Mixed Numbers.*)

1. 10
2. $3\frac{1}{2}$
3. $\frac{2}{9}$
4. $\frac{3}{14}$
5. $\frac{1}{2}$
6. $1\frac{1}{3}$
7. $15.75
8. $\frac{1}{6}$
9. $1\frac{1}{10}$
10. $1\frac{2}{3}$
11. 9
12. $\frac{2}{2}\frac{0}{1}$
13. $2\frac{8}{21}$
14. $\frac{10}{33}$
15. $4\frac{1}{2}$
16. $\frac{1}{20}$
17. $\frac{1}{5}$
18. $2.00
19. $\frac{7}{10}$
20. $\frac{1}{10}$
21. $1\frac{1}{4}$
22. $\frac{1}{4}$
23. $6\frac{3}{4}$
24. $1\frac{2}{9}$
25. 1
26. $\frac{1}{12}$
27. $5\frac{3}{5}$
28. $\frac{1}{6}$
29. $\frac{3}{50}$
30. $\frac{5}{12}$

Answers for Article 298. (*Supplementary Practice on Inverting Divisors.*)

1. $\times\frac{3}{1}$	7. $\times\frac{6}{9}$	13. $\times\frac{3}{16}$	19. $\times\frac{7}{6}$
2. $\times\frac{2}{5}$	8. $\times\frac{8}{3}$	14. $\times\frac{3}{7}$	20. $\times\frac{5}{12}$
3. $\times\frac{5}{9}$	9. $\times\frac{8}{73}$	15. $\times\frac{3}{22}$	21. $\times\frac{2}{13}$
4. $\times\frac{6}{1}$	10. $\times\frac{2}{9}$	16. $\times\frac{2}{9}$	22. $\times\frac{3}{2}$
5. $\times\frac{2}{3}$	11. $\times\frac{3}{5}$	17. $\times\frac{4}{15}$	23. $\times\frac{8}{41}$
6. $\times\frac{3}{1}$	12. $\times\frac{2}{17}$	18. $\times\frac{4}{25}$	24. $\times\frac{6}{13}$

Answers for Article 299. (*Practice in Division of Mixed Numbers.*)

1. $\frac{3}{4}$	7. $\frac{1}{6}$	13. $13\frac{2}{9}$	19. $\frac{2}{5}$
2. $1\frac{4}{5}$	8. $\frac{18}{25}$	14. $3\frac{1}{8}$	20. $\frac{7}{11}$
3. $1\frac{23}{27}$	9. $\frac{5}{51}$	15. $\frac{1}{2}$	21. $\frac{4}{9}$
4. $4\frac{1}{2}$	10. $\frac{1}{2}$	16. 30	22. $7\frac{1}{2}$
5. $\frac{2}{5}$	11. $\frac{26}{45}$	17. $10\frac{2}{3}$	23. $\frac{56}{123}$
6. $\frac{56}{73}$	12. 4	18. 30	24. 12

Answers for Article 300. (*Supplementary Practice in Division of Fractions and Mixed Numbers.*)

1. 6	4. $1\frac{1}{3}$	7. $4\frac{4}{5}$	10. 32	13. 4
2. $\frac{1}{5}$	5. 3	8. $\frac{48}{55}$	11. $\frac{1}{4}$	14. 124
3. $\frac{2}{3}$	6. $7\frac{5}{12}$	9. $2\frac{7}{36}$	12. 3	15. 0

Answers for Article 301. (*Mixed Fundamentals of Fractions.*)

1. $12\frac{7}{8}$	3. $16\frac{3}{4}$	5. $\frac{2}{15}$	7. $9\frac{4}{5}$	9. $2\frac{11}{24}$	11. $22\frac{7}{8}$
2. $3\frac{3}{4}$	4. $\frac{3}{10}$	6. $6\frac{1}{2}$	8. $4\frac{1}{2}$	10. $\frac{21}{80}$	12. $38\frac{1}{4}$

Answers for Article 302. (*Supplementary Practice on Fractions.*)

1. $\frac{3}{4}$	5. $1\frac{3}{8}$	9. $7\frac{1}{12}$	13. 1
2. $\frac{1}{2}$	6. $\frac{4}{9}$	10. $\frac{3}{5}$	14. $1\frac{13}{21}$
3. $\frac{4}{15}$	7. $\frac{1}{2}$	11. $1\frac{5}{6}$	15. $1\frac{5}{16}$
4. 40	8. $\frac{14}{25}$	12. 14	16. $\frac{1}{2}$

Answers for Article 303. (*Finding Errors in Fraction Work.*)

1. $1\frac{1}{10}$ **2.** $15\frac{3}{8}$ **3.** 21 **4.** $3\frac{1}{2}$ **5.** $4\frac{2}{3}$ **6.** $1\frac{9}{10}$ **7.** $\frac{1}{4}$ **8.** $3\frac{1}{2}$ **9.** $\frac{1}{6}$ **10.** $\frac{9}{10}$ **11.** $13\frac{1}{2}$ **12.** $\frac{2}{5}$

Answers for Article 306. (*Form of Answers in Addition of Denominate Numbers.*)

1. 7 ft. 6 in. **2.** 6 yd. 1 ft. **3.** 5 qt. 1 pt. **4.** 10 lb. 5 oz. **5.** 5 pk. 3 qt. **6.** 7 hr. 50 min. **7.** 7 sq. ft. 6 sq. in. **8.** 5 sq. yd. 5 sq. ft. **9.** 3 cu. yd. 5 cu. ft.

Answers for Article 309. (*Practice in Adding Denominate Numbers.*)

1. 5 hr. 45 min. **2.** 9 lb. 6 oz. **3.** 12 ft. **4.** 7 pk. 7 qt. **5.** 16 ft. 3 in. **6.** 28 hr. **7.** 17 da. 8 hr. **8.** 13 gal. 3 qt.

Answers for Article 311. (*Practice in Subtracting Denominate Numbers.*)

1. 1 ft. 5 in. **2.** 5 hr. 30 min. **3.** 1 pk. 4 qt. **4.** 1 gal. 2 qt. **5.** 3 lb. 11 oz. **6.** 7 T. 1400 lb. **7.** 15 min. **8.** 1 lb. 9 oz.

Answers for Article 312. (*Further Practice on Subtraction of Denominate Numbers.*)

1. 7 in. **2.** 1 sq. ft. 121 sq. in. **3.** 8 min. 56 sec. **4.** 6 qt. **5.** 1 lb. 7 oz. **6.** 2 cu. yd. 26 cu. ft.

Answers for Article 316. (*Finding Areas.*)

1. 15 sq. in. **2.** 32 sq. in. **3.** 54 sq. in. **4.** 35 sq. in. **5.** 90 sq. ft. **6.** 24 sq. yd. **7.** 88 sq. mi. **8.** 70 sq. ft.

Answers for Article 319. (*Volumes of Rectangular Solids.*)

1. 64 cu. in. **2.** 2160 cu. ft. **3.** $13\frac{1}{2}$ cu. in. **4.** $4\frac{1}{2}$ cu. ft. **5.** 384 cu. ft.

Answers for Article 321. (*The "Hard Spot" in Addition and Subtraction of Decimal Fractions.*)

1. 42.34 **2.** 9.295 **3.** 122.17 **4.** 50.75 **5.** 13.2 **6.** 6.004

EXTRA LESSONS FOR GOOD WORKERS

324. Spending Spare Time. (Optional Work.) Some pupils do so well on their tests that they do not need to spend very much if any time on the Self-Help Lessons. So the question arises: What shall these pupils do while other pupils are bringing their skill in arithmetic up to standard?

There is little reason why ambitious pupils should not work ahead in their arithmetic textbook if they are particularly interested in this school subject. Some pupils like to do extra work in arithmetic, and for such pupils the next articles (325-336) have been prepared. You can use these articles during spare time. Before working on them, however, you should be **very sure** that you do not need to use the Self-Help Lessons as reference is made to them at the end of each test.

325. Practice in Rapid Counting. (Oral Work.) The little examples below are not so easy as they look. You can use this article all by yourself, but it is more fun to use it with another pupil. First do the counting yourself and have another pupil keep time and watch for mistakes. Then have the other pupil do the counting and you keep time and watch for mistakes.

Time allowed for each example in the drill, 45 seconds.

1. Keep multiplying by 2 each time. Start with 1 and see if you can get to a number over 200 without a mistake. Say $2 \times 1 = 2$; $2 \times 2 = 4$; $2 \times 4 = 8$; $2 \times 8 = 16$, etc.

2. Add by 4's. Begin with 5. Think $5+4$ are 9, and 4 are 13, etc. Try to get to 85.

3. Multiply by 2. Start with 3. Think $2\times3=6$; $2\times6=12$; and so on. Try to get to 192 without making a mistake.

4. Subtract by 3. Start with 63. Say $63-3=60$; $60-3=57$; $57-3=54$, etc. Try to get to 0 in the 45 seconds.

5. Multiply by 2, starting with 5. Think $2\times5=10$; $2\times10=20$, and so on. Try to get to 640 without making a mistake.

6. Subtract by 4. Start with 72. Say $72-4=68$; $68-4=64$; $64-4=60$, etc. Try to get to 0 in 45 seconds.

7. Divide by 2, starting with 384. Say $384\div2=192$; $192\div2=\ldots$, etc. Try to get to 3 in 45 seconds.

If you have time you will find it good fun to make up an exercise like the one above and try it on another pupil. In making up an exercise be sure that you do accurate work.

326. Number Relations. (WRITTEN WORK.) In each of the examples below the last number of the series is left out. You are to decide what that number ought to be.

SAMPLE: 1 3 5 7 9 11 .....

In the sample each number is obtained by adding 2 to the number before it. 13 comes next.

What number comes next in each example?

1. $\frac{1}{2}$ 3 $5\frac{1}{2}$ 8 $10\frac{1}{2}$ 13 .....
2. 41 35 29 23 17 11 .....

3. 27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	.....
4. 1	4	16	64	256	1024	.....
5. 16	$15\frac{1}{3}$	$14\frac{2}{3}$	14	$13\frac{1}{3}$	$12\frac{2}{3}$	.....
6. 4A	4B	4C	4D	4E		.....
7. 42160	4216	421.6	42.16	4.216		.....
8. 14	7	$3\frac{1}{2}$	$1\frac{3}{4}$	$\frac{7}{8}$		.....
9. $20a$	$40a^2$	$60a^3$	$80a^4$	$100a^5$		.....
10. 1	3	7	13	21		.....

☛ Correct answers for use after your work is finished are printed on page 452.

Make up some examples like the ones above. Be very careful to work them up correctly; then have another pupil try to solve them.

327. Practice in Estimating. (WRITTEN WORK.) The only written work to be done in this exercise is the writing of the answer you think is correct. Study the answers which are given and select the one which you think is correct.

After your work is done, compare your answers with those printed on page 453.

1. If 54 is divided by 6, the right answer is 4, 8, 9, 18, 28.
2. $126 \div 9 =$ 7 12 14 23 34
3. $312 \div 26 =$ 9 10 12 24 110
4. $3024 \div 7 =$ 4 18 43.2 432 4320
5. $504 \div 63 =$ .8 8 9 17 80

6. $5920 \div 80 =$ 7.4 58 67 74 740
7. $2500 \div 50 =$ 5 7 15 25 50 500
8. $8400 \div 700 =$ 1.2 9 12 120 1200
9. $390 \div 78 =$ 3 4 5 55 60
10. $2418 \div 31 =$ 7.8 78 81 92 780
11. $9486 \div 9 =$ 105 154 1020 1005 1054
12. $6716 \div 46 =$ 25 92 146 209 250

Can you make an exercise like the one you have just worked? Try it. Work out your own examples very carefully and then try them on some other pupil.

328. Number Relations. (WRITTEN WORK.) In each of the number rows there is one number which does not fit or does not belong in the series. Tell in each example what number does not belong. After you have listed these numbers, compare your list with that printed on page 453.

1.	48	40	44	32	24	16	8
2.	$\frac{1}{2}$	$7\frac{1}{2}$	$14\frac{1}{2}$	$21\frac{1}{2}$	$28\frac{1}{2}$	$32\frac{1}{2}$	$35\frac{1}{2}$
3.	2048	1048	512	128	32	8	2
4.	91	78A	65A	52A	39A	26A	13A
5.	1	5	25	125	625	3125	6500
6.	100	88	76	64	52	42	40
7.	6	18	36	54	162	486	1458
8.	3072	1536	768	192	48	12	3
9.	II	IV	VI	VIII	IX	X	XII
10.	30A	27B	24C	21D	18E	15F	12H

329. Finding Errors. (WRITTEN WORK.) Use your pencil in working the examples below if you need to. List your answers carefully. After you have finished your work, compare your list with that printed on page 453.

SAMPLE: If you multiply each product by 5, starting with 3, you get $5 \times 3 = 15$, $15 \times 3 = 45$, $45 \times 3 = 135$, $135 \times 3 = 405$, $405 \times 3 = 1215$, etc.

Below are numbers obtained in similar ways. Check each through carefully, for there are several mistakes. Your list of errors are the answers.

1. $\times 4$, starting with 3: 12, 48, 192, 768, 3072, 12,968
2. $\times 5$, starting with 2: 10, 50, 250, 1250, 6250, 36,250
3. $\div 5$, starting with 15,625: 3125, 625, 125, 35, 7
4. $\div 3$, starting with 729: 243, 81, 27, 9, 3, 1
5. $\div 4$, starting with 12,568: 3072, 768, 192, 48, 12, 4
6. $+89$, starting with 76: 165, 144, 233, 322, 411, 500
7. -47, starting with 601: 554, 507, 460, 413, 366, 319
8. $+4.8$, starting with 18: 22.8, 27.6, 324, 37.2, 42, 46.8
9. $\times 17$, starting with 8: 56, 136, 2312, 39,304, 668,168, 11,358,856
10. -86, starting with 600: 514, 428, 342, 25.6, 170, 94

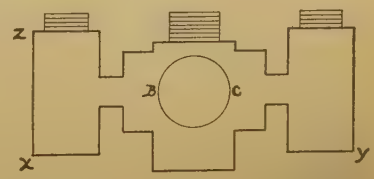
330. Measuring the National Capitol. (GROUP WORK.) On the next page are a picture of the National Capitol at Washington and a diagram showing the ground plan of the building. You will need to study the pictures carefully in working the problems.

NOTE: D is the point directly under A. Think the dimension $D \rightarrow A$ as the distance from the ground to the very top of the statue on the dome. The dimensions of the capitol are as follows: $X \rightarrow Y = 751$ ft.; $X \rightarrow Z = 324$ ft.; $T \rightarrow Z = 108$ ft.; $D \rightarrow A = 288$ ft.; $B \rightarrow C = 116$ ft.

Several pupils should work together as a committee in solving the next problems. Do you think that you could organize these problems into a good report to present to the class?

1. Measure off on the sidewalk or in a field a distance equal to $X \rightarrow Y$ on the map of the National Capitol.

2. Measure off a distance equal to $X \rightarrow Z$.



3. Compare the height of the Senate Chamber ($T \rightarrow Z$) with that of some building near your school.

4. Compare the distance across the dome of the Capitol at its base ($B \rightarrow C$) with the length of your school building.

5. Compare the height of the dome from the ground ($D \rightarrow A$) with the height of some building.

6. Make a drawing to scale of the ground plan of the Capitol, and draw on it, to the same scale, the ground plan of your school building.

331. Practice with Denominate Numbers. (WRITTEN WORK.) There are several answers to each example in the exercise below. After you have finished your work, compare your answers with those printed on page 454. See if you can do the work accurately.

1. Subtract each of these from \$5.00:

(*a*) \$1.25, (*b*) \$4.00, (*c*) \$.25, (*d*) \$1.60, (*e*) \$3.75, (*f*) \$4.37

2. Subtract each of these from 4 miles:

(*a*) .4 mi., (*b*) 2.25 mi., (*c*) 3.00 mi., (*d*) 3.1 mi., (*e*) 2.05 mi., (*f*) .75 mi.

3. Subtract each of these from 2 bushels:

(*a*) 1.05 bu., (*b*) .6 bu., (*c*) 1.4 bu., (*d*) .07 bu., (*e*) .45 bu., (*f*) .35 bu.

4. Subtract each of these from 1 foot:

(*a*) .1 ft., (*b*) .4 ft., (*c*) .3 ft., (*d*) .25 ft., (*e*) .74 ft., (*f*) .08 ft.

5. Add 8 in. to each of the next numbers. Express answers in good form. (See Art. 306.)

(*a*) 2 ft. 3 in., (*b*) 1 ft. 9 in., (*c*) 11 in., (*d*) 3 ft.

6. Add 45 min. to each of the next numbers.

(*a*) 2 hr. 10 min., (*b*) 3 hr. 50 min., (*c*) 45 min., (*d*) 7 hr.

7. Add 2 ft. to each of the next numbers.

(*a*) 1 yd. 1 ft., (*b*) 2 yd., (*c*) 3 yd. 2 ft., (*d*) 1 yd. $1\frac{1}{2}$ ft.

332. Rapid Number Work. (Oral Drill.) Study each of the two parts of this article until you can do them very easily. Use pencils in studying if you need to. Do not mark in the book, for that is unfair. Have another pupil time you and watch for mistakes.

Part I

1. Add by 21's, beginning with 7. Get to 133 in 30 sec.
2. Add by 15's, beginning with 7. Get to 97 in 30 sec.
3. Add by 8's, beginning with 9. Get to 89 in 30 sec.
4. Add by 9's, beginning with 6. Get to 105 in 30 sec.
5. Add by 22's, beginning with 6. Get to 116 in 30 sec.

Part II

1. Count by 6's, subtracting 6 each time. Begin with 74. Think 74, 68, 62, etc. Try to get to 2 in 30 sec.
2. Count by 8's, subtracting 8 each time. Begin with 88. Think 88, 80, 72, etc. Try to get to 0 in 30 sec.
3. Count by 7's, subtracting 7 each time. Begin with 79. Think 79, 72, 65, etc. Try to get to 2 in 30 sec.
4. Count by 9's, subtracting 9 each time. Begin with 67. Think 67, 58, 49, etc. Try to get to 4 in 30 sec.
5. Subtract by 5's, subtracting 5 each time. Begin with 83. Try to get to 3 in 30 sec.
6. Subtract 18 from each of these numbers: 27, 21, 19, 26, 18, 20, 25, 23.
7. Subtract 63 from each of these numbers: 63, 67, 72, 70, 69, 64, 68.

333. Rapid Work with Numbers. (ORAL WORK.) In this exercise you will find there are three different things to do. Study each part carefully before asking another pupil to time you and watch for mistakes. Do not mark in the book for that is not fair, but use paper and pencil while studying if you need to.

PART I

1. Multiply by $\frac{1}{2}$. Begin with 128. Try to get to 1 in 30 sec. Say 128, 64, etc. Think $128 \times \frac{1}{2} = 64$, $64 \times \frac{1}{2} = ?$ etc.

2. Multiply by $\frac{1}{3}$. Begin with 324. Try to get to $1\frac{1}{3}$ in 30 sec. Say 324, 108, etc. Think $324 \times \frac{1}{3} = 108$, then $108 \times \frac{1}{3} = ?$, etc.

3. Multiply by $\frac{1}{4}$. Begin with 256. Try to get to 1 in 30 seconds.

4. Multiply by $\frac{1}{5}$. Begin with 1250. Try to get to 2 in 30 seconds.

5. Multiply by $\frac{1}{3}$. Begin with 1. Try to get to $\frac{1}{2\,4\,3}$ in 30 seconds.

PART II

1. Add .4 each time, beginning with 1.2. Try to get to 6 in 30 seconds.

2. Add 1.1 each time, beginning with 1. Try to get to 8.7 in 30 seconds.

3. Add .09 each time, beginning with .14. Try to get to 1.04 in 30 seconds.

4. Add .15 each time, beginning with 1.25. Try to get to 2.45 in 30 seconds.

5. Add 1.7 each time, beginning with 2.1. Try to get to 12.3 in 30 seconds.

Part III

1. Add by $2\frac{1}{2}$, beginning with 1. Get to 31 in 45 sec.
2. Add by $1\frac{2}{3}$, beginning with $\frac{1}{3}$. Get to 17 in 45 sec.
3. Add by $1\frac{1}{4}$, beginning with $2\frac{3}{4}$. Get to $15\frac{1}{4}$ in 45 sec.
4. Add by $\frac{3}{5}$, beginning with 1. Get to $8\frac{1}{5}$ in 45 sec.
5. Add by $1\frac{1}{6}$, beginning with $1\frac{5}{6}$. Get to $12\frac{1}{3}$ in 45 sec.

334. Practice in Estimating. (Written Work.) For each multiplication below, five answers are given. Without using a pencil, choose the answer you think best in each case. When you have finished your work, compare your list of answers with those printed on page 454.

1. $479 \times 245 =$ 120 1.2 120,000 .012 1200
2. $350 \times 873 =$ 304,500,000 30.45 3045 304,500 304.5
3. $801 \times 520 =$ 416,000 4160 41.60 416,000,000 .0416
4. $396 \times 369 =$ 1480 148,000 148 14.8 1.480
5. $820 \times 706 =$ 574,000,000 5.74 5740 574,000 574
6. $559 \times 132 =$ 7.15 71,500 7,150,000 715 .0715
7. $238 \times 498 =$ 120,000,000 12 1200 1.20 120,000
8. $960 \times 657 =$ 6240 62.4 624,000 6,240,000 .624
9. $747 \times 581 =$ 43.5 4350 435,000 .0435 43,500,000
10. $884 \times 914 =$ 796,500 7965 79.65 7,965,000 7.965

335. Using Judgment. (WRITTEN WORK.) Number your paper for 14 answers. Read each of the statements below. Write the answer which would make each statement true.

Correct answers are on page 454.

1. We cancel when we can in: *addition, subtraction, multiplication of fractions.*

2. Which number is called a denominate number? $4\frac{1}{2}$, $\frac{7}{3}$, 6 ft., $\frac{8}{8}$.

3. We multiply numerators and multiply denominators of fractions when we: *add, subtract, multiply.*

4. We first change mixed numbers to improper fractions when we: *add, subtract, divide.*

5. In the example $24\overline{)247}$, the 24 is: *the divisor, the dividend, the quotient.*

6. If you multiply a number by 1, the number becomes: *larger, smaller, remains the same size.*

7. Numbers which end in 0 or 5 can always be divided by: *2, 3, 5.*

8. Even numbers can always be divided by: *2, 3, 5.*

9. If the sum of the digits of a number is 3, 6, 9, 12, 15, or 18 the number itself can be divided by: *2, 3, 5.*

10. Sum is a word used in: *addition, subtraction, multiplication, division.*

11. If both terms of a fraction are divided by the same number the value of the fraction: *stays the same, grows larger, gets smaller.*

12. Quotient is a word used in: *addition, subtraction, multiplication, division.*

13. Of the three numbers, 2, $\frac{1}{4}$, 1; the $\frac{1}{4}$ is: *the largest, the smallest, the middle one in size.*

14. The numbers smaller than 1 in the following list are . . . . . .

$\frac{8}{9}$ $\frac{1}{4}$ $\frac{7}{14}$ 1 0 $\frac{2}{5}$ 4 $\frac{1}{2}$ $\frac{4}{3}$ 7 $\frac{1}{3}$ $\frac{8}{8}$

336. Arithmetic Puzzles. (WRITTEN WORK.) Follow directions in the four parts of this article. Check your answers with those given on pages 454 and 455.

I. *What number should come next?*

SAMPLE: 1 3 5 7 9 11 ?

Each number in the sample above is obtained by adding 2 to the number before it. 13 comes next.

1.	$\frac{1}{4}$	1	$1\frac{3}{4}$	$2\frac{1}{2}$	$3\frac{1}{4}$	4	?
2.	60	51	42	33	24	15	?
3.	64	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	?
4.	2	6	18	54	162	486	?
5.	10	$9\frac{2}{5}$	$8\frac{1}{5}$	$8\frac{1}{5}$	$7\frac{3}{5}$	7	?

II. *What number should come next?*

1.	81	27	9	3	1	$\frac{1}{3}$	?
2.	0	$33\frac{1}{3}$	$66\frac{2}{3}$	100	$133\frac{1}{3}$	$166\frac{2}{3}$	?
3.	9	$7\frac{1}{2}$	6	$4\frac{1}{2}$	3	$1\frac{1}{2}$	?
4.	1	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{27}{8}$	$\frac{81}{16}$	$\frac{243}{32}$	?
5.	4a	5b	6c	7d	8e	9f	?

III. In each of the number rows below there is a number which does not fit. Six of the numbers in each problem are related in some one way. Find the relationship.

SAMPLE: 2 4 8 16 32 40 64

The number relation in the sample is: multiply a number by 2 to get the next number. 40 does not fit.

Which number does not fit in each of the following?

1.	10	12	22	34	46	58	70
2.	243	81	27	9	3	1	$\frac{2}{3}$
3.	150	143	139	136	129	122	115
4.	1	2	8	32	128	512	2048
5.	$\frac{3}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{4}$	3	$3\frac{3}{4}$	$4\frac{1}{2}$

ANSWER KEY TO EXTRA LESSONS

Answers for Article 326. (*Number Relations.*)

1. $15\frac{1}{2}$ Add $2\frac{1}{2}$ to each number to get the next number.

2. 5 Subtract 6 from each number to get the next number.

3. $\frac{1}{27}$ Divide each number by 3 to get the next number.

4. 4096 Multiply each number by 4 to get the next number.

5. 12 Subtract $\frac{2}{3}$ from each number to get the next number.

6. 4F Use the next letter of the alphabet with the 4 each time to get the next number.

7. .4216 Move the decimal point one place to the left each time to get the next number.

8. $\frac{7}{16}$ Divide each number by two to get the next number.

9. $120a^6$ Add 20 to the number before the "a," and add 1 to the number following the "a" each time to get the next number.

10. 31 Start by adding 2 to the first number to get the second; then increase the number added by 2 each time to get the next number. Thus, 2 is added to the first number, 4 to the second, 6 to the third, etc.

Answers for Article 327. (*Practice in Estimating.*)

1. 9	**3.** 12	**5.** 8	**7.** 50	**9.** 5	**11.** 1054
2. 14	**4.** 432	**6.** 74	**8.** 12	**10.** 78	**12.** 146

Answers for Article 328. (*Number Relations.*)

1. 44 Subtract 8 from each number to get the next number.

2. $32\frac{1}{2}$ Add 7 to each number to get the next number.

3. 1048 Divide each number by 4 to get the next number.

4. 91 Subtract 3 from each number to get the next number.

5. 6500 Multiply each number by 5 to get the next number.

6. 42 Subtract 12 from each number to get the next number.

7. 36 Multiply each number by 3 to get the next number.

8. 1536 Divide each number by 4 to get the next number.

9. IX Add 2 to each number to get the next number.

10. 12H Subtract 3 from each number and attach the next letter of the alphabet to get the next number.

Answers for Article 329. (*Finding Errors.*)

1. 12,968 should be 12,288.

2. 36,250 should be 31,250.

3. 35 should be 25.

4. Correct.

5. 3072 should be 3142.

6. 144 should be 254.

7. Correct.

8. 324 should be 32.4. (Decimal point is omitted.)

9. 56 should be 136.

10. 25.6 should be 256. (The decimal point is incorrect.)

Answers for Article 331. (*Practice with Denominate Numbers.*)

1. (*a*) \$3.75 (*b*) \$1.00 (*c*) \$4.75 (*d*) \$3.40 (*e*) \$1.25 (*f*) \$.63

2. (*a*) 3.6 mi. (*b*) 1.75 mi. (*c*) 1 mi. (*d*) .9 mi. (*e*) 1.95 mi. (*f*) 3.25 mi.

3. (*a*) .95 bu. (*b*) 1.4 bu. (*c*) .6 bu. (*d*) 1.93 bu. (*e*) 1.55 bu. (*f*) 1.65 bu.

4. (*a*) .9 ft. (*b*) .6 ft. (*c*) .7 ft. (*d*) .75 ft. (*e*) .26 ft. (*f*) .92 ft.

5. (*a*) 2 ft. 11 in. (*b*) 2 ft. 5 in. (*c*) 1 ft. 7 in. (*d*) 3 ft. 8 in.

6. (*a*) 2 hr. 55 min. (*b*) 4 hr. 35 min. (*c*) 1 hr. 30 min. (*d*) 7 hr. 45 min.

7. (*a*) 2 yd. (*b*) 2 yd. 2 ft. (*c*) 4 yd. 1 ft. (*d*) 2 yd. $\frac{1}{2}$ ft.

Answers for Article 334. (*Practice in Estimating.*)

1. 120,000	**2.** 304,500	**3.** 416,000	**4.** 148,000	**5.** 574,000
6. 71,500	**7.** 120,000	**8.** 624,000	**9.** 435,000	**10.** 796,500

Answers for Article 335. (*Using Judgment.*)

1. Multiplication of fractions

2. 6 ft.

3. Multiply

4. Divide

5. The divisor

6. Remains the same size

7. 5

8. 2

9. 3

10. Addition

11. Stays the same

12. Division

13. The smallest

14. $\frac{8}{9}$, $\frac{1}{4}$, $\frac{7}{11}$, 0, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{1}{3}$

Answers for Article 336. (*Arithmetic Puzzles.*)

I. **1.** $4\frac{3}{4}$ Add $\frac{3}{4}$ to each number to get the next number.

2. 6 Subtract 9 from each number to get the next number.

3. $6\frac{1}{4}$ Divide each number by 4 to get the next number.

4. 1458 Multiply each number by 3 to get the next number.

5. $6\frac{2}{5}$ Subtract $\frac{3}{5}$ from each number to get the next number.

II. **1.** $\frac{1}{9}$ Divide each number by 3 to get the next number.

2. 200 Add $33\frac{1}{3}$ to each number to get the next number.

3. 0 Subtract $1\frac{1}{2}$ from each number to get the next number.

4. $\frac{7}{6}\frac{2}{4}\frac{9}{}$ Multiply each number by $\frac{3}{2}$ to get the next number.

5. 10g Add 1 to each number and follow it with the next letter of the alphabet to get the next number.

III. **1.** 12 Add 12 to each number to get the next number.

2. $\frac{2}{3}$ Divide each number by 3 to get the next number.

3. 139 Subtract 7 from each number to get the next number.

4. 1 Multiply each number by 4 to get the next number.

5. 2 Add $\frac{3}{2}$ to each number to get the next number.

TABLES OF MEASURE

LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 36 inches = 1 yard (yd.)
$5\frac{1}{2}$ yards = $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods = 5280 feet = 1 mile (mi.)

SQUARE MEASURE (AREA)

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

CUBIC MEASURE (VOLUME)

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord

WEIGHT

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

PUPIL'S SELF-HELP INDEX

337. How to Direct Your Own Review Work. (For Reference.) In order to do good work in arithmetic you must make sure really to understand each new topic as you study it for the first time. You must also keep from forgetting how to do many things which you may not have studied for a long time.

An important question for all of us is: When I have forgotten how to work some kind of example or problem, how can I direct my own review work so that I can restudy and practice? The Pupil's Self-Help Index on pages 457-462 will direct you to articles for study and practice.

The article numbers in heavy type (Art. **237**) refer to articles **to study** in order to understand **how to do something.** The other article numbers refer to good material **for practice.** Think of the Self-Help Index as a friend which is always ready to help you.

ESSENTIALS OF WHOLE NUMBERS

Addition

341
76
841
700

Review Tests	Arts. 2, 123
Six Trouble Spots	Art. **237**
Easy Practice	Art. 238
Harder Practice	Arts. 239, 240
How to Check	Art. **263**

Subtraction

Subtract:
37609
14245

Review Tests	Arts. 4, 124
Hard Spots in Subtraction	Art. **243**
Easy Practice	Arts. 242, 244
Harder Practice	Arts. 245, 246
How to Check	Art. **263**

Multiplication

Multiply:
4381
78

Review Tests — Arts. 6, 125
Understanding Multiplication — Arts. **249, 250**
Easy Practice — Arts. 248, 251
Harder Practice — Arts. 252, 253
How to Check — Art. **263**

Division

$58\overline{)52673}$

Review Tests — Arts. 8, 126
How to Work Long Division — Art. **257**
Finding Quotient Figures — Arts. **258, 259**
Zero in Quotients — Art. **260**
Easy Practice — Arts. 255, 256
Harder Practice — Arts. 259, 261, 262
How to Check — Art. **263**

FRACTIONS AND MIXED NUMBERS

Addition

$2\frac{1}{8}$
$\frac{5}{1\,2}$
$7\frac{1}{3}$

Review Tests — Arts. 11, 127, 217
Finding Common Denominators — Arts. **266, 267,** 268, 276
Reducing Answers — Arts. **269,** 270
Improper Fractions in Answers — Arts. **271,** 272
Adding Mixed Numbers — Arts. **273,** 274
Changing Forms of Fractions — Art. 275

Subtraction

$6\frac{3}{4}-2\frac{1}{6}=$

Review Tests — Arts. 13, 70, 127, 217
Common Denominators — Arts. **278,** 279, 280
Subtracting Mixed Numbers — Arts. **281,** 282, 283

Multiplication

$3\frac{1}{2}\times 1\frac{1}{2}=$

Review Tests — Arts. 15, 92, 128, 218
With Fractions — Arts. **285,** 286
With Small Mixed Numbers — Arts. **287, 293,** 294, 295
Changing Mixed Numbers to Fractions — Arts. **288,** 289

Cancellation Arts. **290, 291,** 292
Improper Fractions in Answers Arts. **271,** 272
With Large Mixed Numbers Art. **112**

Division

$3\frac{3}{4} \div \frac{3}{8} =$

Review Tests Arts. 16, 128, 149, 218
Dividing by Fractions and Mixed Numbers Arts. **297,** 298, 299, **300**

Mixed Practice on Fractions Arts. 301, 302, 303

DECIMAL NUMBERS

Review Tests Arts. 21(*A*), 133
Place Value of Numbers Arts. **322, 323**

Addition

3.14+9.87+ .67 =

Review Tests Arts. 21(*A*), 133
Hard Spot in Addition Art. **321**

Subtraction

4.07−2.18=

Review Tests Arts. 21(*A*), 133
Hard Spot in Subtracting Art. **321**

Multiplication

3.45
×1.08

Review Tests Art. 134
How to Multiply Art. **40**
General Practice Arts. 41, 42, 43
Problems Arts. 44, 51
Multiplying by 10, 100, and 1000 Arts. **46,** 47, **48, 49,** 50, 51

Division

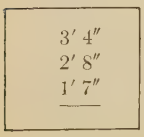
2.14)41.38

Review Tests Art. 134
How to Divide Arts. **64, 65, 67**
General Practice Arts. 66, 68, 76, 219
Application to Problems Arts. 58, 75
Dividing by 10, 100, and 1000 Arts. **55,** 56, 57
Decimal Form of Remainders Arts. **72, 73,** 74

DENOMINATE NUMBERS

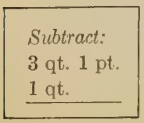
Changing Forms — Arts. 17, 129, 148, **305, 306**
Tables of Measure — Page 456

Addition



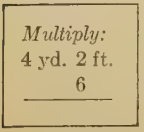
Review Tests — Arts. 18, 130
How to Add — Arts. **307, 308**
Practice — Art. 309

Subtraction



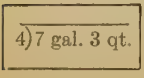
Review Tests — Arts. 19, 130
How to Subtract — Art. **310**
Practice — Arts. 311, 312

Multiplication



Review Tests — Art. 131
How to Multiply — Art. **32**
General Practice — Art. 33
Problems — Art. 34

Division



Review Tests — Art. 131
How to Divide — Arts. **36,** 37
Problems — Art. 38

PERCENTAGE

Introduction to Percentage

Aliquot Parts of 100 ($\frac{1}{3}$ of 100 = ?) — Art. 91
Meaning of Per Cents — Arts. **99,** 100, 101, 102, **104,** 107, 108, **164**
Informal Test on Changing Forms — Art. 136, Tests (*A*) and (*B*)

Finding a Per Cent of a Number (First Use)

Find 14% of 900.

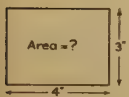
Explanation	Art. **110**
Using Per Cents over 100%	Arts. **109,** 113
Percentage in Discount	Arts. **138-141**
Percentage as Interest	Arts. **187, 188,** 189
General Practice	Arts. 111, 116, 144
Problems	Arts. 103, 120
Informal Tests	Art. 136, Test (*C*)

Finding What Per Cent One Number is of Another (Second Use)

56 is what per cent of 168?

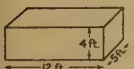
Explanation	Arts. **152, 154**
General Practice	Arts. 153, 154, 155, 156, 176, 190, 221
Problems	Arts. 103, 157, 161, 162, 177, 191

MEASUREMENT



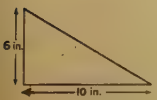
Areas of Rectangles

Review Tests	Arts. 20(*B*), 132(*B*)
Finding Areas	Arts. **227, 315,** 316, **317, 318**
Problems	Arts. 60, 61, 168, 171
Scale Drawing	Art. **59**

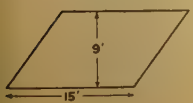


Volume

Review Tests	Arts. 20(*C*), 132(*C*)
Finding Volumes	Arts. **227, 319**
Problems	Arts. 170, 171



Areas of Triangles and Parallelograms



The Right Triangle	Arts. **195, 197**
The Parallelogram	Arts. **199,** 200
Any Triangle	Art. **203**
Problems	Arts. 204, 205

PROBLEM SOLVING

How to Solve Problems Arts. **28, 117, 119, 182, 209, 224**

General Practice. These articles contain the Problem Scales by which you can measure your ability to solve problems.

Arts. 30, 38, 44, 61, 77, 82, 95, 120, 145, 162, 171, 177, 183, 191, 206, 210, 216, 223. 225, 230, 232.

PRACTICE TO MAINTAIN SKILL

Self-Testing Drills. Arts. 26, 39, 45, 53, 63, 71, 79, 84, 90, 97, 106, 115, 118, 143, 147, 151, 160, 165, 173, 178, 186, 193, 202, 208, 220, 226, 229, 231, 233

The examples and problems in the Self-Testing Drills use important facts and abilities in arithmetic. They are good review.

Rapid Oral Drills. Arts. 35, 52, 62, 78, 89, 105, 142, 172, 185, 192, 201, 228.

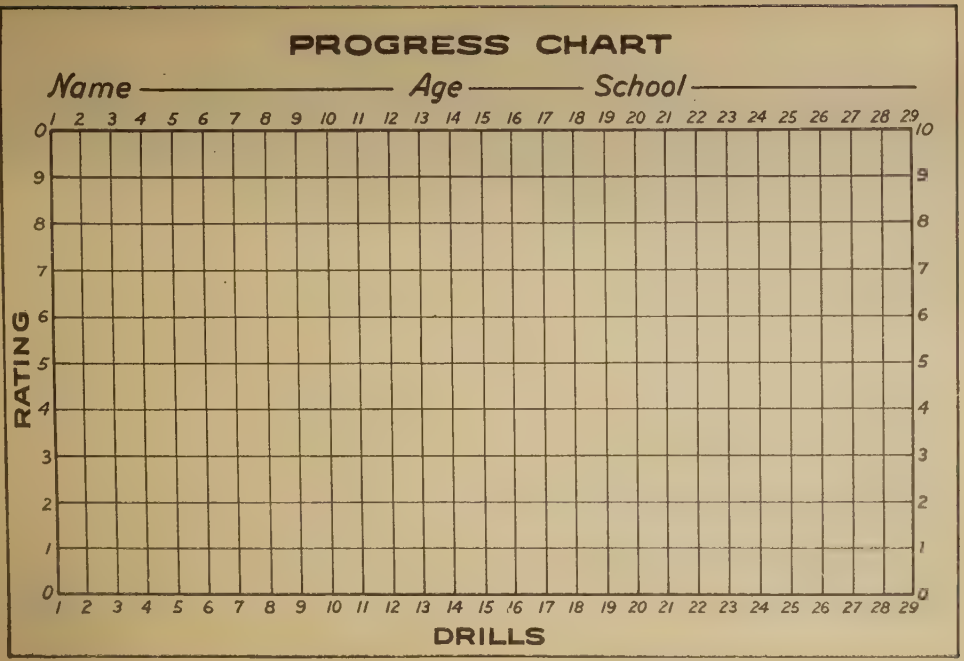
Your teacher or a visitor may ask you to work oral drills without notice; so you had better practice them every once in a while.

Inventory Drills on Useful Facts and Ideas. Arts. 21, 31, 54, 69, 80, 98, 114, 121, 166, 167, 179, 194, 211, 212, 213, 214.

If you can do the work in the Inventory Drills easily and correctly, you understand the main ideas of sixth-grade arithmetic.

MISCELLANEOUS

The Graph	Arts. **81,** 83, 158
Bank Accounts and the Bank Check	Arts. **85, 86,** 87, 88, 137
The Savings Bank	Art. **94**
The Uses of Discounts	Arts. **138, 139, 140, 141,** 145
The Personal Account	Arts. **180,** 181, 184
The Meaning of Interest	Arts. **187, 188,** 189
Reading Large Numbers	Art. 215
Checking Answers	Arts. 21 (*B*), 135, **263**
Important Words	Arts. 222, **264**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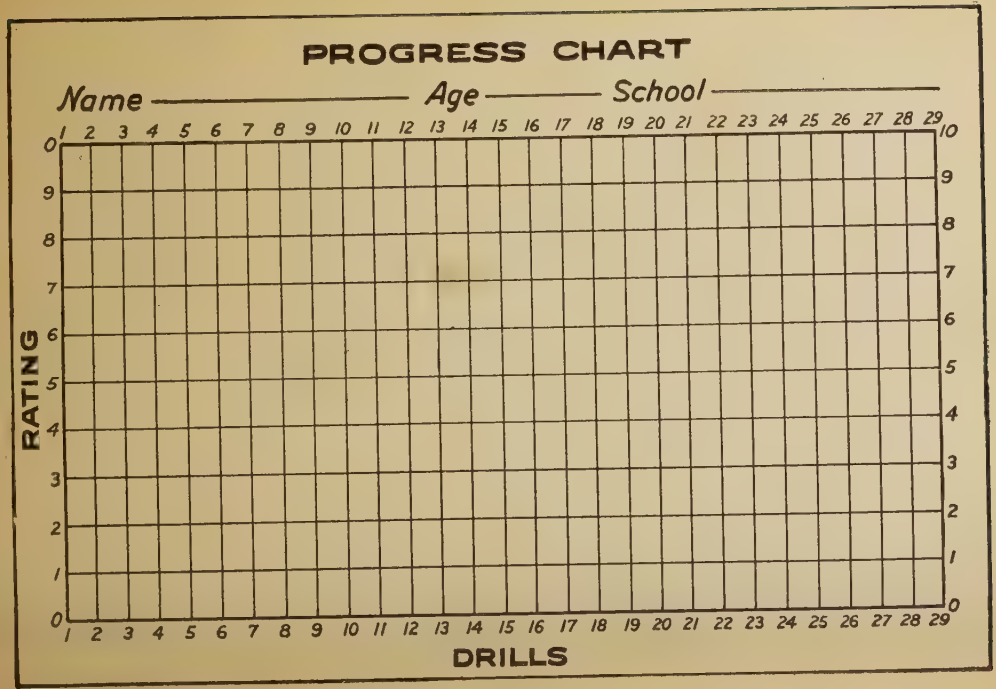
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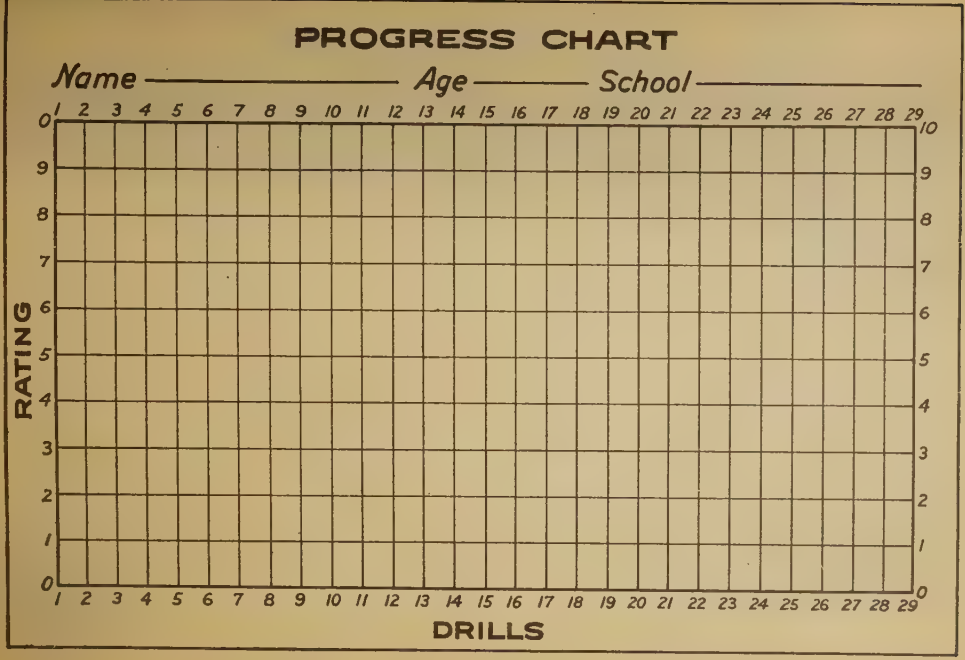
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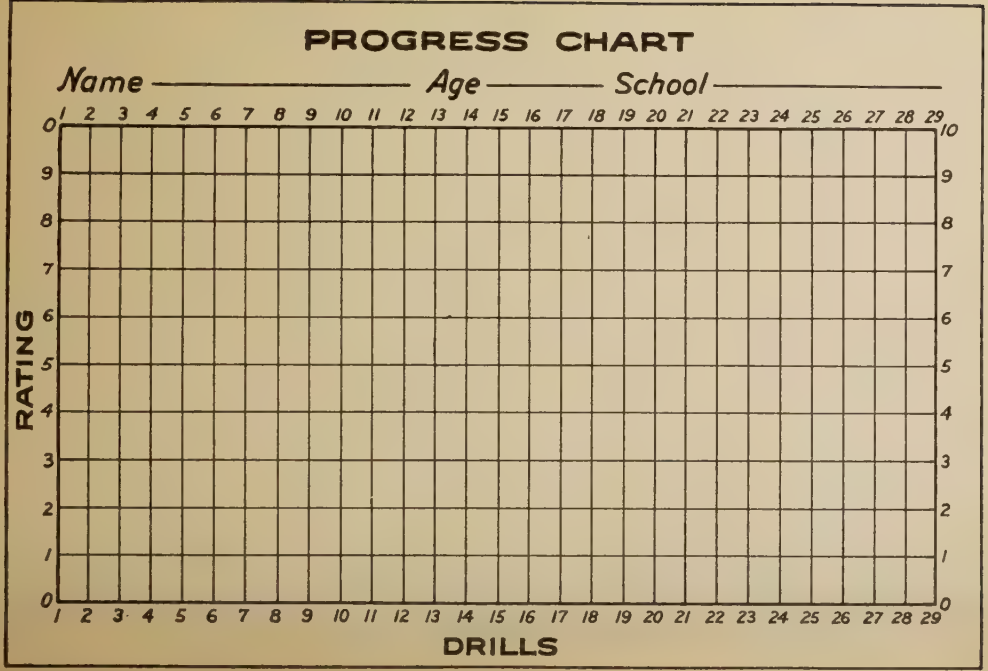
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GENERAL INDEX

(Numbers refer to pages)

Accounts, checking, 109-120; savings, 128-130, 272-273; personal, 260-263; with day-books, 268-269
Acute angle, explained, 289
Addition of decimals, Self-Help Lessons on, 428
Addition of denominate numbers, review tests in, 16-17, 183-184; Self-Help Lessons on, 418-421
Addition of fractions, review tests in, 11, 180; Self-Help Lessons on, 391-398
Addition of mixed numbers, review tests in, 11, 180; Self-Help Lessons on, 395-397
Addition of whole numbers, review tests in, 3-4, 175-176; causes of errors in, 133-134; Self-Help Lessons on, 360-363
Aliquot parts, 123-124
Altitude, explained, 291-292, 296-297
Angles, explained, 287, 289
Answers, form of denominate numbers in, 16, 183, 418-419; estimating, 165-167, 169, 294-295, 313, 339; reducing fractions in, 393-395; changing improper fractions to good forms for, 394-395
Area, review tests in finding, 19-20, 187; review of, 243-244; problems in finding, 245-246; of right triangles, 290-293; of rectangles, 292, 296; of parallelograms, 297-299; of any triangle, 302-303; summaries of, 343-344, 356; Self-Help Lessons on, 424-427; table of measure of, 456
Balance, bank, 111, 113-114
Banking, 109-120, 128-130; project in, 120; review test in, 194-196; savings, 272-273
Base, explained, 291, 296
Bill, defined, 197
Business arithmetic, problems in, 127

Canceled check, 110, 113
Cancellation, Self-Help Lessons on, 408-409
Chapter summaries. *See* Summaries, cumulative
Check, bank, 109-111, 113, 115-118
Check book, 109, 115
Checking, review tests in, 22, 191; in division of decimals, 87; Self-Help Lessons on, 388-389
Common denominators, Self-Help Lessons on, 391-393, 398, 399-400
Consumer, explained, 201
Counting, practice in rapid, 440-441, 447-449
Cubic measure. *See* Volume
Cubic units, explained in Self-Help Lessons, 427

Day-book, 268-269
Decimals, review tests in, 21-22, 188-190; multiplication of, 54-59; division of, 82-87; check for division of, 87; remainders as, 91-92; placing decimal points in division of, 96; reviews of, 252-254; summary of, 355; Self-Help Lessons on, 428-431

Denominate numbers, review tests in, 14-18, 182-185; multiplication of, 44-47; division of, 49-51; review of, 211-212; summary of, 357; Self-Help Lessons on, 417-423; practice with, 446
Denominators, common, Self-Help Lessons on, 391-393, 398, 399-400
Deposit, bank, explained, 109
Deposit book, 112, 113
Deposit slip, explained, 112
Difference, 319, 389
Dimensions, meaning of, 425; to be in same units, 425, 426
Discount, explained, 197-198; cash, 197-198; quantity, 198-200; special-sale, 200; trade, 200-202
Dividend, 389
Division of decimals, 82-87, 91-92, 96; project in, 328-329
Division of denominate numbers, 49-51; review test in, 185
Division of fractions, review tests in, 14, 181; Self-Help Lessons on, 411-416
Division of whole numbers, review tests in, 8, 179; review of zero difficulties in, 213-217; causes of errors in, 232-233; summary of, 321; Self-Help Lessons on, 376-387
Divisor, 389
Drawing to scale, 75-77
Drills. *See* Oral Drills and Self-Testing Drills
Dry measure, table of, 456

Errors, causes of, in addition, 133-134; in subtraction, 135; in multiplication, 230-232; in division, 232-233; careless, 310-311
Essentials, seventh-grade, 352-358
Estimating answers, 169, 294, 339, 442-443, 449
Extra Lessons for Good Workers, mentioned, 5, 7, 8; explained, 440

Floor plans, measuring, 245, 246
Forms of denominate numbers, tests in changing, 15-16, 182, 183; Self-Help Lessons on, 417-419
Fractions, review tests in addition of, 11-12, 180; review tests in subtraction of, 12-13, 180; review tests in multiplication of, 13, 181; review tests in division of, 14, 181; reviews of, 89, 125, 212-213, 252; in business, 127; changing to per cents, 142-143, 150-151, 158-159; estimating size of, 209; changing to decimals, 222; summaries of, 326-327, 354; Self-Help Lessons on addition of, 391-398; Self-Help Lessons on subtraction of, 399-403; Self-Help Lessons on multiplication of, 403-411; Self-Help Lessons on division of, 411-416

Graphs, explained, 102-103, 106-107; making and reading, 106-107; reading per cents from, 230

How to use this book, 24-27

Improper fractions, Self-Help Lessons on changing form of, 394-395
Indorsing a check, 117
Interest, explained, 128, 272; how to work, 272-277

Large numbers, 322-323
Length, table of, 426, 456
Liquid measure, table of, 456
Lowest terms, Self-Help Lessons on reducing fractions to, 393-394

Measure, units of, 343, 425, 427; changing units of, 343, 426; complete tables of, 456
Measurement, review tests in, 18-20, 186-188; a project in, 77; review of, 243-248; problems in,

303-307; summary of, 343-344; Self-Help Lessons on, 424-428; project in, 444-446. *See also* Area and Volume
Mensuration. *See* Measurement
Minuend, 319, 389
Mistakes, careless, 310-311
Mixed numbers, multiplying by large, 159-162; Self-Help Lessons on changing to improper fractions, 394-395; Self-Help Lessons on addition of, 395-397; Self-Help Lessons on subtraction of, 401-403; Self-Help Lessons on multiplication of, 406-411; Self-Help Lessons on division of, 411-416. *See also* Fractions
Monthly statement (bank), 113-115
Multiplicand, 389
Multiplication, review tests in whole-number, 7, 178; of fractions, review tests in, 13, 181; of denominate numbers, 44-47; of decimals, 54-59; by 10, 100, and 1000, 63-67, 71-74; of large mixed numbers, 159-162; causes of errors in whole-number, 230-232; summary of, 320; Self-Help Lessons on whole-number, 369-376; Self-Help Lessons on fraction and mixed-number, 403-411
Multiplier, 389

National capitol, measuring the, 444-446
Net profit, explained, 201
Numbers, rapid work with, 440-441, 447-449
Number relations (practice work), 9-10, 441-442, 443, 451-452

Obtuse angle, 289
Oral drills, mixed fundamentals, 48, 67, 80, 98, 121, 152, 203, 250, 270, 282, 300, 345
Overdrawing a bank account, 115

Parallel lines, explained, 288, 295
Parallelograms, explained, 288-289, 295-297; area of, 297-299
Per annum, explained, 272, 276
Percentage, meaning of, 140-141; practice with, 142-152, 154-155; "hard spot in," 155-156; two methods of finding a per cent of any number, 157-159, 161, 164; review tests in, 192-194; in discount, 197-202; reviews of, 205-206, 224-229, 254-255, 278-280; finding what per cent one number is of another, 219-224; problems in, 235, 237; changing mixed per cents to decimals, 238-239; as interest, 272-277; summary of, 331-335, 358
Perimeter, 424
Perpendicular, 291, 297
Place value, Self-Help Lesson on, 430-431
"Pointing off", explained, 56
Problems, 28-30, 46-47, 66-67, 73-74, 93-95, 106-107, 127, 147-150, 169-170, 228-230, 235, 237, 245, 246, 247-248, 298-299, 303-305, 306-307. *See also* Problem Scales
Problem Scales, explained, 39-40; located as follows: *I*, 40; *II*, 51; *III*, 60; *IV*, 78; *V*, 97; *VI*, 103; *VII*, 130; *VIII*, 170; *IX*, 207; *X*, 236; *XI*, 248; *XII*, 255; *XIII*, 266; *XIV*, 280; *XV*, 308; *XVI*, 316; *XVII*, 324; *XVIII*, 337; *XIX*, 340; *XX*, 347; *XXI*, 350
Problem solving, advice on, 36-39, 165-167, 169, 264-265, 312-315
Product, 389
Profit, net, explained, 201
Progress Chart, explained, 31-33; blanks for pupils, 463-466
Pupil's Self-Help Index. *See* Self-Help Index

Quantity discounts, 198-200
Quotient, 389
Quotient figures, Self-Help Lessons on finding, 379-383

Reciprocal, 411
Rectangle, explained, 289, 424; area of, 292, 296, 343; Self-Help Lessons on finding area of, 425-426
Reducing fraction to lowest terms, Self-Help Lessons on, 393-395
Remainders as decimals, 91-92
Retail price, explained, 200
Retailer, explained, 201
Reviews, general, 41-44, 88. *See also* Summaries
Review work, planning, 1-2, 4, 25-26, 35-36, 174-175, 359, 457
Roman numerals, 143
Rules, for multiplication of decimals, 56; for multiplying by 10, 100, 1000, 65; for dividing decimals, 84; for remainders as decimals, 92; for finding what per cent one number is of another, 221; for changing fractions to decimals, 222; for finding area of rectangle, 292, 296, 425; for finding area of right triangle, 292, 296; for finding area of parallelogram, 297, 298; for finding quotient figures, 380, 384, 386; for finding common denominators, 392; for finding lowest terms, 393; for changing mixed numbers to improper fractions, 407; for multiplying mixed numbers, 410; for dividing fractions, 412; for finding volume of rectangular solid, 427, 428

Sales slips, explained, 126; problems on, 127
Savings Bank, 128-130, 272
Scale drawing, 75-77
Self-Help Index, Pupil's, explained, 25-26, 35-36
Self-Help Study and Practice Lessons, mentioned, 4, 174; explained, 359
Self-Testing Drills, explained, 30-33; located as follows: *1*, 34; *2*, 53; *3*, 62; *4*, 68; *5*, 81; *6*, 90; *7*, 99; *8*, 108; *9*, 121; *10*, 136; *11*, 153; *12*, 163; *13*, 168; *14*, 204; *15*, 210; *16*, 218; *17*, 234; *18*, 240; *19*, 251; *20*, 257; *21*, 271; *22*, 283; *23*, 301; *24*, 311; *25*, 330; *26*, 342; *27*, 346; *28*, 349; *29*, 351
Seventh-grade essentials, 352-358
Solid, meaning of, 428. *See also* Volume
Spare time, how to spend, 4-5, 6-7, 8, 440
Special Sale discounts, 200
Square measure. *See* Area
Statement, monthly bank, 113-115
Stub, bank-check, 115-116
Subtraction of decimals, review tests on, 21, 189; Self-Help Lessons on, 429
Subtraction of denominate numbers, review tests in, 17-18, 183-184; Self-Help Lessons on, 421-423
Subtraction of fractions, review tests in, 12-13, 180; Self-Help Lessons on, 399-403
Subtraction of mixed numbers, review tests on, 12, 180; Self-Help Lessons on, 401-403
Subtraction of whole numbers, review tests in, 5-6, 177; causes of errors in, 135; Self-Help Lessons on, 364-368
Subtrahend, 319, 389
Sum, 389
Summaries, cumulative, 27, 69-70, 100-101, 137-139, 172-173, 241-242, 258-259, 284-286; of ad-

dition, 318; of subtraction, 319; of multiplication, 320; of division, 321; of fractions, 326-327, 354; of division of decimals, 328-329; of decimals, 355; of percentage, 331-335, 358; of mensuration 343-344, 356-357; of whole number processes, 353; of denominate numbers, 357

Tables of measure, 456
Terms of a fraction, Self-Help Lessons on reducing, 393-395
Tests, basic review, in whole numbers, 3-8, 175-179; in fractions, 11-14; 180-181; in denominate numbers, 14-18; 182-185; in mensuration, 18-20, 186-188; in decimals, 21-22, 188-190; in checking, 22, 191; in vocabulary, 22-24; in percentage, 192-194; in banking practice, 194-196
Time, table of measures of, 456
Trade discounts, 200-202
Triangle, defined, 287; right, defined, 287; area of right, 290-293, 296; area of any, 302-303

Units of Measure. *See* Measure

Vocabulary of arithmetic, review tests in, 22-24; summary of, 335-336; Self-Help Lessons on, 389-390, 424-425
Volume, review tests in finding, 20, 188; rule for measuring, 427; review of, 244, 247; summary of, 357; Self-Help Lessons on, 427-428

Weight, table of measure of, 456
Wholesaler, explained, 201

Zero difficulties, review of, 213-217
Zero in quotient, Self-Help Lessons on, 383-386